Dynamical Systems and Control in Celestial Mechanics and Space Mission Design

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Introductory Remarks

- Topics for discussion:
  - solar system dynamics (eg, dynamics of comets)
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  - the role of the three and four body problems
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- solar system dynamics (e.g., dynamics of comets)
- the role of the three and four body problems
- space mission trajectory design
  - and the relationships between these topics
Introductory Remarks

**Some history:**

- **1700–1850**: Euler, Lagrange, Gauss, began to lay the mathematical foundations
- **1880–1890**: Poincaré: fundamental work on the 3-body problem; creates the research area *chaos*
- **1900–1965**: Moser, Conley, and others make fundamental contributions to the 3-body problem
- **1965–present**: Research in the 3 and 4 body problems and other topics in geometric mechanics and associated applications continues by many people: by no means finished!
Introductory Remarks

Current research importance

- design of some NASA mission trajectories—such as the *Genesis Discovery Mission* to be launched July 30, 2001—EXCITING DAY!!
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■ Acknowledgements

- the *Genesis team*, the groups of Kathy Howell (Purdue), Michael Dellnitz (Paderborn), Linda Petzold (UC Santa Barbara), Gerard Gomez, Josep Masdemont, Carles Simo (the Barcelona group), etc.
Some dynamical systems concepts

Simple pendulum

- three kinds of orbits:
  - oscillating orbits
  - running orbits
  - special separating orbits
Some dynamical systems concepts

Simple pendulum

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  - oscillating orbits
  - running orbits
  - special separating orbits

- Via $F = ma$, can visualize solutions as trajectories in the $(\theta, \dot{\theta})$ plane ($\theta$ is the angle of the pendulum from the vertical downward position)

- the resulting phase portrait allows one to put together the basic orbits in one figure:
Dynamical Orbits

Connecting (homoclinic) orbits of a pendulum system:
- Upright pendulum (unstable point)
- Downward pendulum (stable point)

Phase portrait of the simple pendulum

$\theta = \text{pendulum angle}$

$\dot{\theta} = \text{velocity}$

Connecting (homoclinic) orbit at the saddle point.
Invariant Manifolds

Higher dimensional analog of the invariant curves

Invariant manifolds
Periodic Orbits

- Can replace equilibria by *periodic orbits*:

  Stable Manifold (orbits move toward the periodic orbit)

  Unstable Manifold (orbits move away from the periodic orbit)

Invariant manifolds attached to a periodic orbit
Three body problem

- General Three Body Problem

- Three bodies move under mutual gravitational interaction
General Three Body Problem

- Three bodies move under mutual gravitational interaction
- Some interesting new orbits discovered in the last few years by Richard Montgomery, Alain Chenciner, Carles Simo. Movies by Randy Paffenroth (Caltech) generated using AUTO
Three body problem
Three body problem
Three body problem
Three body problem

- Restricted Circular Problem

- the two primaries move in circles; the smaller third body moves in the field of the primaries (without affecting them); view the motion in a rotating frame

- we consider the planar and the spatial problems
Three body problem

- *Restricted Circular Problem*

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- we consider the *planar* and the *spatial* problems

- there are places of *balance*; eg, a point between the two bodies where the attraction balances
Three body problem

■ Restricted Circular Problem

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□ There are five such equilibrium points:
  • Three collinear (Euler, 1750) on the $x$-axis—$L_1, L_2, L_3$
  • Two equilateral points (Lagrange, 1760)—$L_4, L_5$. 
Equilibrium points for the three body problem
if a spacecraft is at $L_1$ or at $L_2$, it will stay there

one can go into orbit about the $L_1$ and $L_2$ points, even though there is no material object there!
Three body problem

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- if a spacecraft is at $L_1$ or at $L_2$, it will stay there
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- some of these orbits are called Liapunov orbits, others are called halo and Lissajous orbits.
- just as in the pendulum, one can draw the invariant manifolds associated to $L_1$ (and $L_2$) and the periodic orbits surrounding them.
- these invariant manifolds play a key role in what follows
Invariant manifolds for the 3-body problem
Three body problem

- consider the **planar case**; the **spatial case** is similar
- **Kinetic energy** (wrt inertial frame) in rotating coordinates:
  \[ K(x, y, \dot{x}, \dot{y}) = \frac{1}{2} \left[ (\dot{x} - \omega y)^2 + (\dot{y} + \omega x)^2 \right] \]
- **Lagrangian** is K.E. – P.E., given by
  \[ L(x, y, \dot{x}, \dot{y}) = K(x, y, \dot{x}, \dot{y}) - V(x, y); \quad V(x, y) = -\frac{1 - \mu}{r_1} - \frac{\mu}{r_2}. \]
- **Euler–Lagrange equations**: 
  \[
  \ddot{x} - 2\omega \dot{y} = -\frac{\partial V_{\omega}}{\partial x}, \quad \ddot{y} + 2\omega \dot{x} = -\frac{\partial V_{\omega}}{\partial y}
  \]
  where the **effective potential** is
  \[ V_{\omega} = V - \frac{\omega^2(x^2 + y^2)}{2}. \]
**Effective potential**

- In the circular planar restricted three body problem, and in a rotating frame, the equations for the third body are those of a particle moving in an effective potential plus a magnetic field (goes back to work of Jacobi, Hill, etc.)
Three body problem

- Invariant Manifolds of Periodic Orbits

- red = unstable, green = stable
These manifold tubes play a crucial role in what passes through the resonance (transit orbits) and what bounces back (non-transit orbits). Transit possible if you are “inside” the tube, otherwise nontransit—important for transport issues.
we consider the historical record of the orbit of comet Oterma from 1910 to 1980

- first in an inertial frame (fixed relative to the stars)
- and then a rotating frame
- very special case of pattern evocation

similar pictures for many other comets
Comet Oterma
Comet Oterma
Mission Purpose: To gather solar wind samples and to return them to Earth for analysis
Genesis Discovery Mission

- **Mission Purpose:** To gather solar wind samples and to return them to Earth for analysis

- **Mission Constraints/Features:**
  - Return in Utah during the daytime
  - Descend with a parachute for a *helicopter snatch*
  - *lunar swingby contingency* in case of bad weather
  - *Energy efficient* (small thrust required): makes use of the dynamical sensitivity to design a low-cost trajectory
Genesis Discovery Mission
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Mission Trajectory

Four phases:
Mission Trajectory

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1. from low Earth orbit, insertion onto an $L_1$ halo orbit
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Mission Trajectory

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2. using saddle point controllers, remain on the halo orbit for about 2 years (4 revolutions)
Genesis Discovery Mission

Mission Trajectory

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3. return to a near halo orbit around $L_2$ via a near \textit{hetero-clinic connection}
**Mission Trajectory**

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  1. from low Earth orbit, insertion onto an $L_1$ halo orbit
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  2. using *saddle point controllers*, remain on the halo orbit
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  4. return to Earth on *an impact orbit* (guided by the unstable manifold of a halo orbit around $L_2$).
Genesis Discovery Mission

Mission Trajectory

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Final trajectory computation takes into account all the major bodies in the solar system.
The Genesis trajectory
Genesis Discovery Mission

View of the Genesis trajectory in the plane
Genesis orbit and the Sun-Earth dynamical structure
Some *planet-impacting asteroids* use invariant manifolds as a pathway from nearby heliocentric orbits. This phenomena has been observed in the impact of comet *Shoemaker-Levy 9* with Jupiter.

Some NEO’s are subject to similar dynamics and are the most dangerous ones; perhaps the KT impact event was one of these too!

These ideas apply to *any planet or moon system*!
Lunar Missions

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we approach this problem by

- *systematically implementing* the view that the Sun-Earth-Moon-Spacecraft 4-body system can be modelled as *two coupled 3-body systems*
- and using invariant manifold ideas
Lunar Missions

- Idea: *put two Genesis-type trajectories together*; we transfer from
  - the *Sun-Earth-spacecraft* system to
  - the *Earth-Moon-spacecraft* system
Lunar Missions

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□ *But*: takes longer (6 months as opposed to 5 days). [OK for cargo ships, but not human missions]
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- Fuel savings *and* the time of flight in other missions (e.g., to Jupiter’s moon’s) is more dramatic

- Schematic of the idea
Lunar Missions

periodic orbit around the Sun-Earth $L_2$

energetically inaccessible region
Lunar Missions

"Shoot the Moon" Lunar Capture Trajectory (seen in Earth-Centered Inertial Frame)
Lunar Missions

"Shoot the Moon" Lunar Capture Trajectory (seen in Sun-Earth Rotating Frame)
Construction of new trajectories that visit the Jovian system.
Construction of new trajectories that visit the Jovian system.

Example 1: Europa → Io → Jupiter collision

1. Begin tour
2. Europa encounter
3. Jump between tubes
4. Io encounter
5. Collide with Jupiter
Jovian Lunar Tour

- **Strategy:**
  - use burns (controls) that enable a transfer from one three body system to another
    - from the *Jupiter–Europa–spacecraft* system to
    - the *Jupiter–Io–spacecraft* system
  - this strategy is similar to that used in the lunar missions together with some symbolic dynamics
  - trajectories do well on fuel savings
  - here is a close-up of the Io encounter
Close-up of the Io encounter
Example 2: Ganymede → Europa → orbit injection around Europa
Jovian Lunar Tour

Ganymede Rotating Frame

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Jovian Lunar Tour

Europa Rotating Frame
Uses of Optimal Control

- **Halo Orbit Insertion**

- After launch, the *Genesis Discovery Mission* will get onto the stable manifold of its eventual periodic orbit around $L_1$

- Errors in, eg, launch velocity, means that there must be corrective manoeuvres

- The software **COOPT** is very useful in determining the necessary corrections (burn sizes and timing) systematically for a variety of launch conditions

- It gets one onto the orbit at the right time, while minimizing fuel (what is being optimized)
Uses of Optimal Control

- A number of unusual features, such as the nature of the boundary conditions
- A very nice mixture of dynamical systems (providing guidance and first guesses) and optimal control
- See the talk of Linda Petzold in the satellite dynamics minisymposium (work with Radu Serban, Martin Lo, Wang Sang Koon, JEM and Shane Ross)
Uses of Optimal Control

[Diagram of a 3D coordinate system with points and lines indicating different planes and axes.]
Uses of Optimal Control

- Satellite reconfiguration, stationkeeping and de-configuration

- This application makes use of the software NTG (Non-linear Trajectory Generation) developed at Caltech

- Details were in the minisymposium talk: Richard Murray (together with Mark Milam and Nicolas Petit)

- This involves near Earth spacecraft clusters
Uses of Optimal Control

Why clusters?

- Clusters can achieve the same resolution as a large telescope using vision systems coordinated in software and modern optics
- *coordinated clusters* can obtain unprecedented resolution for both Earth-pointing systems and those pointing into deep space
Uses of Optimal Control

- Two basic problems
  - Formation maintenance: keep the satellites in relative position—use small controls
  - Formation changes: get the formation as a whole to reposition itself for the next task—use larger controls

- Especially for reconfiguration, one wants to do this optimally (again, minimize fuel)

- Handles constraints, such as imaging and communication constraints very nicely
Formation maintenance with guaranteed Earth coverage.
Formation Flying Methodology

- **Active Formation Methodology**

- **Passive Formation Methodology**
Stationkeeping
Reconfiguration
Deconfiguration
Terrestrial Planet Finder

- **Goal**: probe for Earth-like planets using a large baseline group of satellites—this time a *deep space cluster*
- Orbiting around $L_2$ is a candidate position: away from the Earth.
- Each halo orbit is surrounded by a torus that provides a natural dynamical formation
- Very nice visualizations of this by Ken Museth, Martin Lo and Al Barr; see Martin’s talk in the satellite minisymposium
Getting to TPF and Beyond

- **The $L_1$ Gateway Station**

- A gateway at the *Earth-Moon $L_1$ point* is of interest as a semi-permanent *manned site*.

- Can be used for going to the moon, servicing TPF and possibly for missions to other planets.

- Efficient transfers can be created using the 3-body and invariant manifold techniques that our group has developed.
Getting to TPF and Beyond

People (for Moon, Telescope Servicing, and Mars)

Earth-Moon L₁

Earth-Sun L₂

Telescopes (low-energy transfer)

Moon

Asteroids

Mars
More Information

- http://www.cds.caltech.edu/~marsden/
- http://www.cds.caltech.edu/~koon/
- email: marsden@cds.caltech.edu

- two of the main publications:
The End

Typesetting Software: \TeX, Textures, \LaTeX, hyperref, texpower, Adobe Acrobat 4.05
Graphics Software: Adobe Illustrator 9.0.2
\LaTeX\ Slide Macro Packages: Wendy McKay, Ross Moore