



## A low-dimensional model of separation bubbles

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### ABSTRACT

In this work, motivated by the problem of model-based predictive control of separated flows, we identify the key variables and the requirements on a model-based observer and construct a prototype low-dimensional model to be embedded in control applications.

Namely, using a phenomenological physics-based approach and dynamical systems and singularity theories, we uncover the low-dimensional nature of the complex dynamics of actuated separated flows and capture the crucial bifurcation and hysteresis inherent in separation phenomena. This new look at the problem naturally leads to several important implications, such as, firstly, uncovering the physical mechanisms for hysteresis, secondly, predicting a finite amplitude instability of the bubble, and, thirdly, to new issues to be studied theoretically and tested experimentally.

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## 1. Introduction

### 1.1. Motivation and objective

In recent years there has been an increasing demand to extend the range of aircrafts' flight conditions to high angle-of-attack regimes and high-amplitude maneuvers. The latter usually lead to intense flow separation and dynamic vortex shedding which in turn generate destructive pitching moments, sharp increases in drag, and losses in lift. Therefore, the only way to enlarge the flight envelope is to design efficient ways of controlling flow separation. The classical approach is based on open-loop control, which is achieved either by mechanical or fluidic actuation according to operating schedules (lookup tables) constructed using extensive and costly experimental studies. On the other hand, feedback control schemes do not require operating schedules and, being more efficient and reliable [1], also naturally allow one to address the optimization issue.

However, feedback control is more demanding theoretically since it requires an embedded model which predicts the behavior of the physical system at hand. In the case of flow separation, the

general equations of fluid motion – the Navier–Stokes equations (NSEs) – are known and can, in principle, produce accurate prediction of the flow structure, but because the real boundary and initial conditions are noisy and cannot be precisely measured and because the NSEs cannot be solved in real time in flight, this approach is impractical. However, an accurate real-time solution is not actually necessary, since in reality one can use sensors on the boundary of lifting surfaces, which in turn read off a certain amount of extra information from the physical system and therefore should allow one to weaken the requirements on the model accuracy. Thus, one is naturally led to look for *coarse* models, which should also be *low dimensional* for computational real-time efficiency. Construction of such models is the main objective of our work.

Historically, the importance of low-dimensional modeling of unsteady aerodynamic characteristics – aerodynamic forces and moments acting on an aircraft – for control purposes, stability analysis, and dynamic simulations has been realized for a long time and the appropriate models were developed; see, for example, [2–7]. However, the necessity to model the separation and vortex shedding dynamics was realized just recently in view of the increasing demand for high angle-of-attack regimes [2]. Early attempts to develop dynamical models are based on the anzatz that the phenomenon of flow separation behaves linearly for small variations of the parameters involved [5–7], which has many limitations, as will be clear from the subsequent discussion when we establish the requirements on a model aimed for robust control.

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## 1.2. Previous works

In one of the early works, [2], the coordinate of a separation point  $x \in [0, 1]$  is taken as an internal state-space variable and the ad hoc linear first-order equation is used to account for the movement of the separation point for unsteady flow conditions:

$$\tau_1 \frac{dx}{dt} + x = x_0(\alpha - \tau_2 \dot{\alpha}),$$

where we kept the original notations:  $\tau_i$  are relaxation times, and  $x_0(\alpha)$  is the stationary value of the separation point position for a given angle of attack  $\alpha$ . Then the value  $x = 1$  corresponds to attached flow, while  $x = 0$  corresponds to leading edge separation. A similar approach was used in the construction of the models for separation phenomena, for example, [5] and the ONERA model [6]. In particular, the key feature of the state-of-the-art low-dimensional model used in a closed-loop control of dynamic stall with pulsed vortex generator jets due to Magill et al. [5] is a choice of the governing physical parameters, such as lift  $Z$  and separation state  $B$  with  $B = 0$  corresponding to fully attached flow and  $B = 1$  to fully separated flow. Steady states,  $B_s(\alpha)$  and  $Z_s(\alpha)$ , represent the baseline case and the measured steady lift, respectively, as functions of the angle of attack  $\alpha$ . The experimentally measured function  $Z_s(\alpha)$  may contain a hysteretic behavior and thus is an empirical way of accounting for a hysteresis, as suggested by Magill et al. [5]. Exploiting the physical arguments: (i) lift  $Z \sim$  circulation  $\Gamma(\alpha)$ ; (ii) relaxation to a baseline state  $\lim_{t \rightarrow +\infty} B(t) = B_s(\alpha)$ ; (iii) rise in lift when a dynamic vortex is shed  $Z \sim B_t$ , one arrives at the simplest low-order model with adjustable parameters,

$$B_{tt} = -k_1 B_t + k_2 [B_s(\alpha) - B], \quad (1a)$$

$$Z_t = k_3 B_{tt} + k_4 [Z_s(\alpha) - Z] + \Gamma_\alpha \alpha_t. \quad (1b)$$

As one can notice, all these models are linear.

In the case of aerodynamic models for forces and moments, the addition of nonlinear terms is known to extend the range of flight conditions to high angle-of-attack regimes and maneuvers [4], which is usually done in an ad hoc manner, by simply adding polynomial terms with unknown coefficients without any physical insight or justification. Systematic application of local and global bifurcation theories to the aerodynamic models is just beginning to be appreciated [3]. To the authors' knowledge there have been no similar attempts in these directions for models of flow separation. In fact, the systematic methods available to formulate such models are very limited and the connection of known models to physics is rather far from desired. A commonly used approach is to first generate experimental data and then to extract the model by a projection onto proper orthogonal decomposition (POD) modes using, for example, balanced truncation or similar methods. This technique is known to be incapable of capturing the dynamics with a few modes for the wide range of governing parameters in view of the open flow nature of the problem, in particular. In addition, while POD models are based on the most energetic modes, they do not provide deep insights into the physics of the flow.

## 1.3. Model requirements and methodology

In this work we develop a model which is not a mnemonic device encoding the experimental observations, as ad hoc models would be, but is *physically motivated* and thus is more robust in reflecting the actual behavior for a wide range of flight and control parameters.

In view of the necessary coarse nature of the model and the application objectives, one has to decide which aspects of the dynamics should be modeled reasonably accurately. With the target of producing a model, upon which an observer in a closed-loop control scheme can be based for a wide range of physical

parameters, in this work we identify the crucial elements of the dynamics of separation bubble, namely *bifurcation* and *hysteresis*, which need to be reflected in the model. These elements reflect the fundamental *nonlinear* nature of the physical problem, which apparently cannot be captured with linear models.

As follows from the above discussion, being nonlinear, low dimensional, and physically motivated are *key requirements* for a model. In this work, we shall make use of *phenomenological* modeling, which has been successful in many other problems, such as Duffing's equation for the buckling of elastic beams [8], simple maps to describe a dripping faucet [9], which even capture the observed chaotic behavior to a great extent, and bubble dynamics in time periodic straining flows [10], to name a few. As will be clear from the text later, besides appealing to a phenomenological physically motivated analysis of empirical facts we also provide a basis for it in dynamical systems and singularity theories. A symbiosis of these two methodologies yields a coherent picture of the phenomena. It should also be stressed that in view of the coarse nature of the model that is sought, we effectively construct an "approximate global normal form", which reflects the key global features of the physical system, necessary for control purposes.

## 1.4. Paper outline

To achieve the above objectives, we first will identify the central idea of our approach in Section 2, and then appeal to the tools of the bifurcation and singularity theory [11], as will be made precise in Section 3. The outline of the paper is as follows. In Section 3, we discuss the first nonlinear aspect of separation bubbles, namely the bifurcation phenomenon and the way to model it. In Section 4, we explore the basic physics of hysteresis phenomena, and suggest a single model capable of capturing both bifurcation and hysteresis.

## 2. Central idea

A central notion and object, whose dynamics we study, is a *separation bubble*, whose main features are as follows. First of all, separation of the boundary layer develops due to an adverse pressure gradient [12] which occurs when the angle of attack of an airfoil is sufficiently large, see Fig. 1(a), and may be followed by re-attachment as in Fig. 1(b), thus forming a typical flow around an airfoil. The region encompassed by the boundary layer is termed a separation bubble after the work of Jones [13] and, as shown in Fig. 1, it can be closed or open. Classification of separation bubbles concerns their laminar or turbulent nature, but at a coarse level topologically laminar and turbulent bubbles do not differ, and thus we will not be distinguishing between various cases, but rather treat a generic case. It should be noted that, in certain physical situations, a bubble needs to be understood in a time-averaged sense [14]. Given the notion of a separation bubble, our dynamical systems model will aim to capture its characteristics, which are important for controlling separation phenomena.

A central idea of this work is to approach the modeling of separation bubble phenomena by identifying the key crucial elements of the bubble dynamics, namely bifurcations and hysteresis, in the appropriate portion of configuration space, as sketched in Fig. 2. In this figure we show the minimal dimension of the *configuration space*, defined by the bubble size  $x$ , the angle of attack  $\alpha$ , and the actuation amplitude  $w$ ; that is, we will be looking for the minimal model determined by the dependence of the bubble size on the angle of attack and actuation amplitude.

This minimal dimension is motivated by the fact that while generally there are other parameters involved, such as the Reynolds number  $Re$ , the critical angle of attack  $\alpha_c$ , and the airfoil thickness  $h$ , the resulting model will still have wide applicability. This can be understood based on the aerodynamic properties

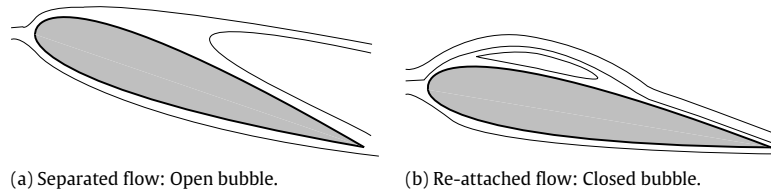


Fig. 1. On the notion of separation bubble.

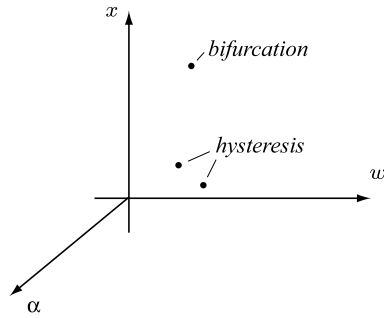


Fig. 2. A sketch of the key dynamic elements—bifurcation and hysteresis—to be captured by the minimal number of parameters, namely the bubble size  $x$ , the angle of attack  $\alpha$ , and the actuation amplitude  $w$ .

of airfoils. To explain this, we draw critical curves, i.e., when separation takes place depending upon  $Re$ ,  $\alpha_c$ , and  $h$  in Fig. 3.

As illustrated by Fig. 3(a), in the case of real airfoils, separation occurs at finite Reynolds numbers even at zero critical angle of attack; the higher  $\alpha_c$  the lower the critical Reynolds number  $Re_c$ ; also, the thicker the airfoil, the lower  $Re_c$ . Fig. 3(b) demonstrates the fact that the thinner an airfoil, the larger the critical angle of attack that is required to achieve separation at a given Reynolds number  $Re^*$ . Finally, in the  $\alpha_c$ – $Re$  plane in Fig. 3(c) one can observe that, for fixed airfoil thickness  $h^*$ , separation can occur at zero  $\alpha_c$ , which requires high enough Reynolds numbers. Since in reality the Reynolds numbers are huge (e.g. for real aircraft  $Re$  varies between  $10^6$  and  $10^{11}$ ), one concludes that limiting ourselves to “thick airfoils”, which can, in fact, be regarded as real airfoils since they have to carry structural load and fuel, is not a serious restriction in this first step towards low-dimensional modeling of separation phenomena.

Having identified the key elements of the bubble dynamics in the configuration space, we construct a model by successively addressing bifurcation phenomena in Section 3 and hysteresis phenomena in Section 4.

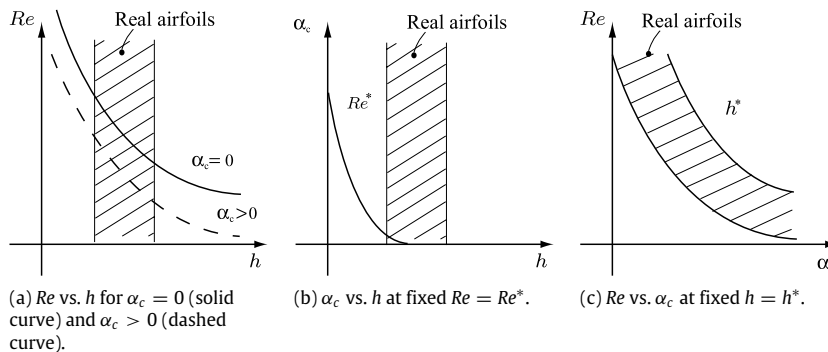


Fig. 3. The placement of real airfoils in the space defined by the Reynolds number  $Re$ , the critical angle of attack  $\alpha_c$ , and the airfoil thickness  $h$ : the critical curves corresponding to the instant when separation occurs.  $Re^*$  and  $h^*$  are typical fixed values of these parameters.

### 3. Bifurcation in the dynamics of separation bubble

#### 3.1. On the notion of bifurcation

As was noted in Section 2, bubbles can be either in a closed or open state. This allows us to introduce the first key element of the low-dimensional modeling, namely it must capture this *basic bifurcation* from an *open* to a *closed* state, as shown schematically in Fig. 4, which is also known as *bursting* [15].

Notably, the fact that this is the primary bifurcation was realized just recently [16]. While from the vast literature one can get the impression that one separated flow is not like any other, here we take a different point of view, i.e. we treat the coarse behavior of separation bubbles as (generic) phenomena that can be modeled by a single low-dimensional dynamical system.

#### 3.2. Quantifying separation bubbles

To quantify the behavior of a separation bubble, consider the coordinate  $x$ , measuring the distance along the airfoil from the bubble onset to the bubble reattachment, as shown in Fig. 4. The bubble dynamics in the first approximation can be described by two parameters: the location of separation,  $x_s$ , and of reattachment,  $x_r$ , which can move under the change of flight and control parameters. In some cases, e.g. the Glauert Glas II airfoils, the separation point  $x_s$  remains fixed for all practical purposes. Therefore, we will start by considering only the behavior of the reattachment point, which experiences a primary bifurcation in the above sense; extending the model to include variation of  $x_s$  will require the addition of a reliable separation criterion. As an alternative to  $x_r$ , one could also utilize the bubble area. From now on we will use  $x$  as a variable representing the bubble state.

#### 3.3. On the physical nature of bifurcation

The mechanism by which the excitation affects the flow lies in the generation of instabilities, and thus of large coherent structures [17] transferring high momentum fluid towards the

Fig. 4. Basic setup and primary bifurcation.

Fig. 5. On the mechanism of actuation.

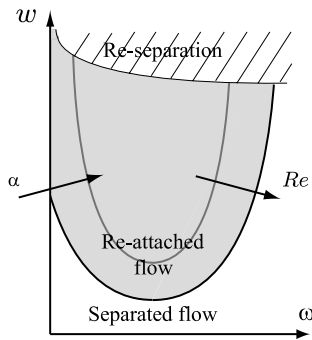


Fig. 6. Effect of time-varying actuation: criticality of the actuation amplitude  $w$  and the frequency  $\omega$ . The shaded region corresponds to a reattached flow (closed bubble). Arrows indicate the change in location of the transition curve with increasing  $Re$  and  $\alpha$ .

surface, see Fig. 5(b), and therefore leading to reattachment, as indicated in Fig. 5(c). Since actuation exploits the instabilities of the shear layer [18], the response to actuation depends on both amplitude  $w$  and frequency  $\omega$ , and therefore is nonlinear. The latter again indicates, now from the point of view of actuation control mechanisms, that the low-dimensional model must be nonlinear. As follows from experiments with synthetic (zero mass flux) jet actuation, the critical phenomena are as sketched in Fig. 6, where the shaded region corresponds to a reattached flow (that is, a closed bubble) and the arrows indicate a change in location of the transition curve with an increase in  $\alpha$  and  $Re$ , respectively. The size of the bubble,  $x$ , has a specific dependence on the amplitude  $w$  and frequency  $\omega$  of actuation, i.e.  $\partial x/\partial w < 0$ ,  $\partial x/\partial \omega < 0$ , when moving away from the origin  $(w, \omega) = \mathbf{0}$  in Fig. 6. In this work we focus on the case of time-invariant actuation,  $\omega = 0$ , although the time-varying case will be commented on later in this section.

Finally, it is notable that the criticality and hysteresis phenomena depend on the connectedness of the flow domain: the bubble experiences bifurcation only in the case of flow around an airfoil, as in Fig. 7(b), while in the case of a hump model in Fig. 7(a), which is frequently used in experiments, e.g. [19], there is no bifurcation. Thus, there are two basic configurations in which the behavior of

the separation bubble differs: the *hump model* and the *airfoil model*. Namely, in the hump case  $x(w)$  is smooth, while in the case of an airfoil  $x(w)$  is discontinuous. Also, as will be important in Section 3, the hysteresis phenomena are present only in the airfoil case. Here, in view of its practical importance, we naturally focus on the airfoil case.

### 3.4. Modeling the bubble bifurcation

In developing a model, we are guided by the principle of a minimal complexity together with the physical requirements one has to meet. At the methodological level, there are two basic ways to account for the form of  $x(w)$ , which has both the saturation and criticality shown in Fig. 7: (a) to design an *algebraic* relation  $f(x, w) = 0$ , or (b) to introduce a *dynamic* description  $f(x, \dot{x}, \ddot{x}, \dots, w) = 0$ . The latter approach is better suited for dynamics and control purposes, because in the case of active feedback control one would need to deal with a few characteristic times and transient effects, and thus the model should be time dependent. The simplest possible way of introducing time-dependent dynamics is a second-order oscillator model,

$$\ddot{x} - \mu \dot{x} = F(x, w), \tag{2}$$

where  $\mu$  is a damping parameter. The justification for the latter is the fact that both separation and reattachment points may oscillate [14].

In what follows, we first formulate the mathematical requirements on a model in Section 3.4.1, then, by appealing to the ideas of a potential function in Section 3.4.2 and a dynamic bifurcation in Section 3.4.3, we identify a specific form of model (2) in Section 3.4.4.

#### 3.4.1. Mathematical requirements

Naturally, the bubble size  $x$  also depends on a *flight parameter*, in our case the angle of attack  $\alpha$ , which needs to be incorporated in the model (2); thus,  $F$  in (2) is enhanced to  $F(x, w, \alpha)$ . Since we want to minimize the functional complexity but retain the true nonlinear features of the phenomena, the simplest form is a quadratic nonlinearity,

$$F(x, w, \alpha) = x^2 + b(w, \alpha)x + c(w, \alpha), \tag{3}$$









