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Essays

The Influence of G&H on Nonlinear Dynamics

This paper describes the place of the book by Guckenheimer and Holmes (Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer-Verlag, Berlin, 1983) in the research and literature on nonlinear dynamics.
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Personal Recollections

The book of Guckenheimer and Holmes [1], denoted hereafter as G&H, was hatched by Holmes in the 1970s, shortly after his move from the UK to Cornell. His enthusiasm for the subject and his propensity for writing were unstoppable. He visited me in Berkeley in 1980 for about six months, during which time our own papers [2] on the interaction between geometric mechanics and, for example, Melnikov's method were born, along with applications to, for instance, chaotic oscillations of a forced beam. The latter, together with work of Kopell and her coworkers [3] around the same time, provided the first definitive proof of chaos in an interesting partial differential equation (PDE). I do not know exactly how many other papers Holmes wrote during that visit—I just remember being stunned when one day he handed me a long paper on chaotic oscillations of surface waves. I wondered how in the heck he managed to do that with what seemed to me like a house full of small children at home and, at the same time, working long hours during the day with me at the university.

I remember vividly sitting in a lecture of Holmes at Berkeley (around 1977 or so) during a time when he was "bringing the Poincaré-Melnikov method to the West." Being engineering oriented, he was pretty clearly wondering why so much attention was given to qualitative work on homoclinic tangles (abstracted to Smale's horseshoe construction) and not so much was given to how you really show that this occurs in specific systems, along with how to estimate the width of the stochastic layer and so on. He was right on target: the Western literature clearly had some catching up to do. In particular, I recall saying to myself during that lecture "I have to learn some of these techniques from this guy." The idea for his visit was then clearly settling in, and it materialized a couple of years later.

I do not personally know the exact history of how Holmes hooked up with Guckenheimer to write the book [1], but it occurred around this time. Guckenheimer had recently moved to the Mathematics Department at UC Santa Cruz and was a strong disciple of the Smale school, which Holmes admired greatly; Guckenheimer's thesis [4] was written under the direction of Smale. Their collaboration of course made for a perfect union of complimentary viewpoints. Guckenheimer eventually moved to Cornell, where he was a colleague of Holmes for a short period; to keep things complicated, Holmes later moved to Princeton.

Imperfect Measures of Success

If one looks up G&H on Google Scholar, then it certainly ranks number 1 in citations for either Guckenheimer or Holmes, with 2398 citations (as of this writing). There are similar large numbers on MathSciNet (368), although that number is abnormally low as many of the citations come from areas that are not covered by MathSciNet. Comparing this to typical book citations, it is evident that this book [1] has had an enormous influence in bridging the mathematics-engineering gap. A brief analysis of these citations shows that they are positive. Most of them are to the research literature and vary from traditional research papers in dynamical systems (for instance, papers on proving the existence of chaos in a particular system) to papers on control and robustness, economics, gene regulatory networks, etc.

Chillingworth's review MR1139515 [5] of the 1990 reprinted version of G&H is worth quoting: "Although the application of refined geometrical and topological techniques in the theory of nonlinear oscillations has a long pedigree in Soviet mathematics, this book (which appeared at almost the same time as the English translation of V. I. Arnold's Geometrical methods in the theory of ordinary differential equations [Springer, New York, 1988]) was the first major text in the Western literature to try to bridge the gap between the engineering and "applied" tradition of nonlinear oscillations and the modern mathematical theory of dynamical systems. Predictably, it had a mixed reception (see the review of the first edition). As many engineers and scientists found the mathematical terminology daunting, the book may initially have done more to educate "pure" mathematicians in the richness of applied problems than the other way round. However, cultural differences in this field are rapidly eroding and the work is firmly established as a much-cited and authoritative source of information on many of the main topics and techniques in applied dynamical systems."

The Mixed Reception

As Chillingworth's review points out, the book [1] did have an uphill battle to fight. In fact, the authors were courageous to publish it. As often happens, even today, research mathematics by its nature has a hard time selling itself in a technology-oriented world. The process by which new mathematical concepts become standard in an applied area is a subtle one. For example, geometric concepts as basic as invariant manifolds had no place in many applied mathematicians vocabularies. I recall a rather traditional person once saying something to the effect that "these new fangled methods are not worth anything; I can do it all better by my well-proven techniques." The opposition to dynamical systems techniques, such as notions of chaos and strange attractors, was nothing short of fierce. These things took time to fade and for the new approaches to take hold and to really prove themselves. For such a monumental undertaking, this was perhaps natural, while at times, painful.

One of the areas of the most opposition to the sorts of ideas espoused in G&H was fluid mechanics. Ironically, this is now generally acknowledged to be of the nicest application areas of dynamical systems. For example, the well-known paper of Ruelle and Takens [6], which was soundly rejected by the fluids community, was eventually seen as full of pioneering (albeit speculative) ideas. The experiments of Gollub and Swinney [7] provided some of the first experimental evidence that the ideas perhaps were not so goofy after all—but they were not warmly embraced by the fluids community. This is a good example of the backdrop that

G&H had to face head on; in fact, it did its part to establish the legitimacy of the dynamical systems viewpoint. But it took almost 20 years for the *Journal of Fluid Mechanics* editorial board to get past this and accept dynamical systems methods as mainstream. Finally, these methods are mainstream, although science has also spoken loudly that subjects, such as turbulence, do not fall as easily as Ruelle and Takens might have naively thought.

Reasons for the Ultimate Success

Dynamical systems in the "West," of course, has a long and distinguished history, building on, as the book [1] calls them, "pioneers in a chaotic land," such as Duffing, Lorenz, and van der Pol, not to mention Poincaré, Lefschetz, Smale, Cartwright, etc. On the other hand, one has to also remember that this was also a time when communication between Soviet and Western scientists was, to put it mildly, strained. The influence of dynamical systems in the Soviet Union in terms of teaching and research was, in many ways, further along than in the West, with such influential researchers as Andronov, Minorsky, Melnikov, Arnold, etc. Their books, such as those of Andronov, Vitt, and Khaĭkin [8] and Andronov and Chaikin [9][1] as well as Minorsky [10] were well developed and dealt, in a fairly comprehensive way, with what we would call today "applied dynamics" or "nonlinear science"[2]—an approach that reaches out to the other sciences and engineering in a significant way.

As an indication of the reality of these strains, I happened to be in the company of Smale at the 1974 International Congress of Mathematics in Vancouver when he was talking to a group of Soviet mathematicians during a coffee break. Smale pointedly, but politely, asked Anosov why Arnold was not at the meeting to give his own lecture (Brieskorn gave it for him). While slowly smoking a cigarette, Anosov proudly replied something like this: "Arnold has been saying some things that he shouldn't have." Well, that was my fast lesson in American-Soviet relations at the time.

Based on a 1979 conference at Asilomar in the Monterey Bay area, Holmes edited the collection *New Approaches to Nonlinear Problems in Dynamics* (see [11]). It contained a beautiful balance of theoretical developments as well as interesting and varied applications. This meeting and book helped to assess the current state of affairs as well as shape the vision of where the subject was going.

The book of G&H was timely and filled a definite void in the West. Although there were excellent books by Hale, Coddington and Levinson, and others on differential equations, there was still a need for a book from the "applied dynamics" perspective that brought together the classical approaches as well as the more modern dynamical systems methods of Poincaré, Smale, etc, that balance the geometric and analytic approaches, together in a way that John, Phil, and others found helpful in applications. Obviously the book filled this gap in a successful way.

Influence

What influence did it have? Have another look at Google, but this time at Google Books and look up, for instance, books containing "nonlinear oscillations" in the title. You will see a vast array of books. There is a lot of works to scan, but it is evident that, at least in the Western literature, G&H was the source of inspiration for many of them. A whole new generation of books and interest in the subject, while already there implicitly, was revitalized, and G&H definitely was a major positive influence that shaped the scene as we see it today. The push to make contacts with other disciplines grew in strength and, by the early 1990s, SIAM established the dynamical systems activity group (Holmes is now the activity group's chair), and the associated Snowbird meeting has become a primary forum for applied dynamics worldwide. John Guckenheimer and Phil Holmes, all of us—as well as science herself—thank you for everything you have done for the subject and for enduring the bumps along the way.

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[1]Apparently, Vitt was a victim of the Stalinist purges and died in the Gulag; see Dalmedico and Gouzevitch [12] for further information.

[2]As far as I know, the name "Nonlinear Science" was created by a University of California Committee around 1982 that included Henry Abarbanel, David Campbell (then from the Center for Nonlinear Studies in Los Alamos), John Guckenheimer, the author and others, in some early steps to create a network of "Institutes for Nonlinear Science" in the UC system.

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