1. **Perko, Section 3.4, problem 1:** Show that \( \gamma(t) = (2 \cos 2t, \sin 2t) \) is a periodic solution of the system
\[
\begin{align*}
\dot{x} &= -4y + x \left( 1 - \frac{x^2}{4} - y^2 \right) \\
\dot{y} &= x + y \left( 1 - \frac{x^2}{4} - y^2 \right)
\end{align*}
\]
that lies on the ellipse \((x/2)^2 + y^2 = 1\) (i.e., \(\gamma(t)\) represents a cycle \(\Gamma\) of this system). Then use the corollary to Theorem 2 in Section 3.4 to show that \(\Gamma\) is a stable limit cycle.

2. **Perko, Section 3.4, problem 3a:** Solve the linear system
\[
\dot{x} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} x
\]
and show that at any point \((x_0, 0)\) on the \(x\)-axis, the Poincare map for the focus at the origin is given by \(P(x_0) = x_0 \exp(2\pi a / |b|)\). For \(d(x) = P(x) - x\), compute \(d'(0)\) and show that \(d(-x) = -d(x)\).

3. **Perko, Section 3.5, problem 1:** Show that the nonlinear system
\[
\begin{align*}
\dot{x} &= -y + xz^2 \\
\dot{y} &= x + yz^2 \\
\dot{z} &= -z(x^2 + y^2)
\end{align*}
\]
has a periodic orbit \(\gamma(t) = (\cos t, \sin t, 0)\). Find the linearization of this system about \(\gamma(t)\), the fundamental matrix \(\Phi(t)\) for the autonomous system that satisfies \(\Phi(0) = I\), and the characteristic exponents and multipliers of \(\gamma(t)\). What are the dimensions of the stable, unstable and center manifolds of \(\gamma(t)\)?

4. **Perko, Section 3.5, problem 5a:** Let \(\Phi(t)\) be the fundamental matrix for \(\dot{x} = A(t)x\) satisfying \(\Phi(0) = I\). Use Liouville’s theorem, which states that
\[
\det \Phi(t) = \exp \int_0^t \text{trace} A(s) ds,
\]
to show that if \(m_j = e^{\lambda_j T}, j = 1, \ldots, n\) are the characteristic multipliers of \(\gamma(t)\) then
\[
\sum_{j=1}^n m_j = \text{trace} \Phi(T)
\]
and
\[ \prod_{j=1}^{n} m_j = \exp \int_{0}^{T} \text{trace} A(t) \, dt. \]

* Hint: recall that the determinant of a matrix is equal to the product of its eigenvalues, and the trace of a matrix is equal to the sum of the eigenvalues.

5. **Perko, Section 3.9, problem 4a**: Show that the limit cycle of the van der Pol equation
\[
\begin{align*}
\dot{x} &= y + x - x^3/3 \\
\dot{y} &= -x
\end{align*}
\]

must cross the vertical lines \( x = \pm 1 \).

* Hint: you can use the fact (shown in Perko, Section 3.8) that a limit cycle exists for the van der Pol equation and that it is unique.