CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

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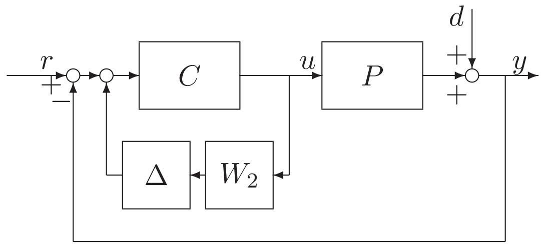
Winter 2010

Problem Set #8

Issued: 1 Mar 10
Due: 8 Mar 10

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. The block diagram below represents an uncertainty in the *compensator* using a feedback representation. Assume $\|\Delta\|_\infty \leq 1$.

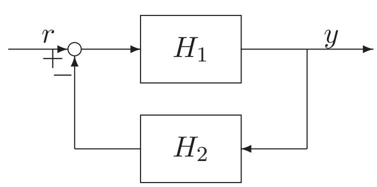


(a) Using the Small Gain Theorem, write a necessary and sufficient condition for robust stability. Assuming that P is biproper (P^{-1} exists), express this condition in a form that does not depend explicitly on the compensator C (i.e., only dependent on W_2, P, S, and T).

(b) **Optional:** If the performance criterion is given by $\|W_1 S\|_\infty \leq \gamma$, derive a sufficient condition for robust performance with respect to the uncertainty Δ. (Show your steps, not just the result!)

2. The Small Gain Theorem gives conditions on the stability of a feedback interconnection that is based on the magnitude of the transfer functions; a similar result also holds based on the phase:

A SISO *positive real* transfer function $H(s)$ is one which has no poles in the right-half plane (or on the $j\omega$ axis) and satisfies $\text{Re}(H(j\omega)) \geq 0\ \forall \omega$. That is, the phase of the transfer function is always between $-90°$ and $+90°$ which you can write as $|\phi(H(j\omega))| \leq \pi/2$. The system is *strictly positive real* if $\text{Re}(H(j\omega)) > 0\ \forall\omega$ which gives $|\phi(H(j\omega))| < \pi/2$. Consider the feedback interconnection of two systems as shown below:



(a) Prove that the feedback interconnection is internally stable if both H_1 and H_2 are positive real systems, and at least one is strictly positive real. (Hint: use a Nyquist argument, and follow the proof of the Small Gain Theorem.)

(b) Prove that if H_2 is positive real but otherwise unknown, that the condition that H_1 be strictly positive real is also necessary for *guaranteed* stability of the feedback interconnection (sufficiency was proven in part (a)).

3. This problem shows that the stability margin is critically dependent on the type of perturbation. The setup is a unity-feedback loop with controller $C(s) = 1$ and process dynamics $\tilde{P}(s) = P(s) + \Delta(s)$, where

$$P(s) = \frac{10}{s^2 + 0.2s + 1}$$

(a) Assume $\Delta(s)$ is a stable transfer function. Compute the largest β such that the feedback system is internally stable for all $\|\Delta\|_\infty < \beta$.

(b) Now suppose that Δ is a real number. Determine the bounds on Δ such that the closed loop system is stable and compare to the first part. (Hint: compute the closed loop transfer function analytically and determine when the eigenvalues go unstable.)