CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

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Winter 2010

Problem Set #7

Issued: 22 Feb 10
Due: 1 Mar 10

Note: Please put the number of hours that you spent on this homework set (including reading) on the back of the first page of your homework.

1. For the ducted fan described in HW 2, design an LQG controller, including both a (steady-state) Kalman filter estimator and (steady-state) LQR. There is an effective time delay of roughly 0.03 s that was ignored in HW 2; you can approximate this in your model with a Pade approximation:

$$e^{-sT} \simeq \frac{1 - sT/2}{1 + sT/2}$$

   The Kalman filter will estimate the state associated with this.

   For the control problem, recall from homework #2 that you will probably want to minimize the horizontal position $x^T x$ and the control effort.

   For the estimation problem, assume there is a horizontal wind disturbance force acting on the system in the same way that the control force does, with spectrum $S(\omega) = A/(\omega^2 + 1^2)$ and rms amplitude of 0.1 N. The position x and angle θ are measured through optical rotary encoders; assume the noise on these are white with amplitude $1\,\text{mm}/\sqrt{\text{Hz}}$ and $1\,\text{deg}/\sqrt{\text{Hz}}$ respectively.

   (a) Assemble the system, design the estimator and feedback, and assemble the controller. (You may use the `lqg` command, or the `lqr` and `kalman` (or `lqe`) commands in matlab.) Plot the step response to a commanded input in the horizontal position.

   (b) Now assume that only the state x is available for feedback (this gives you a single-input, single-output controller). Plot the loop transfer function, and compute the gain and phase margin for several different values of control weighting.