CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

Problem Set #6

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Due: 22 Feb 10

Note: Please put the number of hours that you spent on this homework set (including reading) on the back of the first page of your homework.

1. At power-up, an autonomous vehicle has no information about its position or velocity (initial covariance is infinite). Use the predictor-corrector form of the discrete-time Kalman filter equations to simulate the convergence of the covariance estimate, given noisy measurements of the acceleration from an on-board IMU and less-frequent measurements of position using GPS.

   Assume the single-axis vehicle dynamics are given by

   $$m\ddot{z} = -c\dot{z}$$

   where the normalized damping is given by $d = c/m = 0.01\,s^{-1}$. The GPS measures the position z at a sample rate of 5 Hz and an accuracy (standard deviation of each estimate) of 1.2 m; assume the errors are uncorrelated. The IMU measures the acceleration $\ddot{z}$ at a sample rate of 20 Hz and an accuracy of 10 m-g ($0.0981\,\mathrm{m/s^2}$). Assume there are no disturbance forces acting on the vehicle.

   Convert the system to the equivalent discrete-time system, and compute the time-history of the optimal estimation-error covariance. Plot the diagonal elements of $P_{k|k}$ and $P_{k+1|k}$.

   To avoid using the information form of the equations (involving P^{-1}), initialize the estimation with a measurement from both the GPS and the IMU.