CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

Problem Set #5

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Note: Please put the number of hours that you spent on this homework set (including reading) on the back of the first page of your homework.

1. In this problem you will identify the spectrum from data, and identify a state-space model for a system that generates the same spectrum. The data set is available on the course web page (pressure data recorded on the primary mirror of the Gemini South Observatory). The data were recorded at 10 Hz for 5 minutes (3000 data points). Note that several of the steps below would be easier if we derived all the theory in discrete-time (since the data is). Also note that there are several places where factors of 2π can easily be missed; do your best, but we won't take marks off if you are off by a factor of this somewhere.

   (a) Compute the mean and variance of the data (in Matlab, `mean`, and `std` are useful; the latter computes the standard deviation, which is the square root of the variance.) Plot a histogram of the data to get an empirical probability distribution, and compare with a normal (Gaussian) distribution with the same mean and variance. (You can use the Matlab command `probplot` to do this.) (You might also find it interesting to visualize the cumulative probability distribution (e.g. use `cdfplot`, and compare with `normcdf(sort(x),mean(x),std(x))`.) For the remaining steps, subtract the mean from the data (`x=x-mean(x);`).

   (b) Compute and plot the correlation function of the data (the Matlab command `xcorr` is useful here). (Note that the correlation is only significant for lags between about ± 100.)

   (c) Compute and plot the spectrum of the data. There are two ways to do this: compute the FFT of the correlation function (how the spectrum is rigourously defined), or compute from the data directly (how the spectrum is typically obtained in practice; this is easier, and should yield the same answer after scalings are accounted for). The Matlab command `pwelch` is useful to avoid having to sort out fft scaling and to use averaging to smooth the spectrum (e.g. `[P,F]=pwelch(x,[],[],[],fs);` to use the defaults but also return the frequency vector when sampled at frequency `fs`.) Note that this returns the amplitude in power per Hz, not power per (rad/sec).

   (d) In order to incorporate the knowledge of the disturbance spectrum into a model, it is useful to write down a linear model which gives the same spectrum as the disturbance when driven by a white noise input. Recall that the spectrum should be a real function of frequency; a reasonable (not perfect) fit should be something like $A/(\omega^2+\omega_0^2)$. Estimate A and ω_0 and write down a state-space representation for a system that would generate this spectrum.

   (e) **Optional:** Drive your system with white noise, compute the spectrum of the output, and verify that it is similar to the original spectrum. You can do this with `lsim` with input from `u=(pi*fs*randn(npts,1));`; to get the right input spectrum, the input variance needs to be scaled by the square root of the Nyquist frequency in rad/sec.

2. For the position-control problem

$$m\ddot{z} = u + w$$

a Kalman filter is to be designed to estimate the position and velocity based on the position measurement. Assume that the disturbance force w on the system is white noise with spectral density W. The position is measured with a noisy sensor so

$$y = z + n$$

where n is white with spectral density N.

(a) Solve for the steady-state Kalman filter gains analytically

(b) For $m = 1$ plot the closed-loop estimator pole locations for a range of values of the signal to noise ratio, SNR=$\sqrt{W/N}$.

(c) For several values of SNR, plot the transfer function of the estimator from the measurement to the estimated position and estimated velocity. (Set $m = 1$ to obtain quantitative plots.) Comment on the plots; are they what you would expect?