CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

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Winter 2010

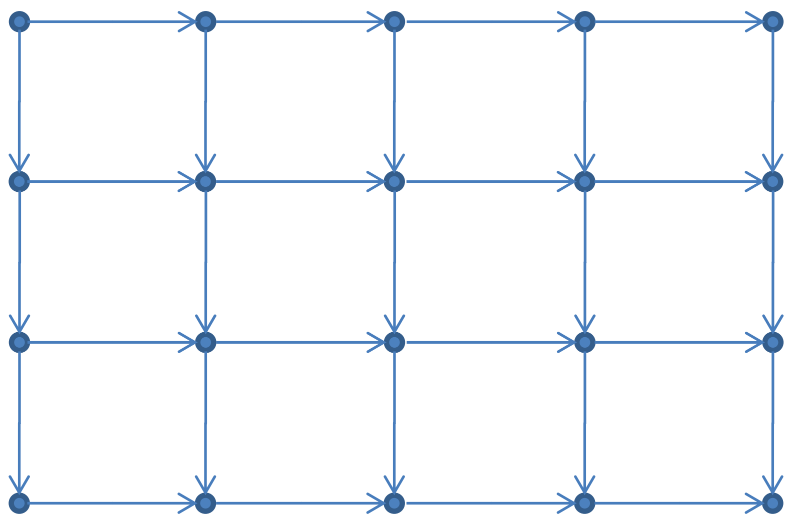
Problem Set #4

Issued: 26 Jan 09
Due: 1 Feb 09

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Solve the following optimization problem using Dynamic Programming.

   The objective is to travel from the upper left intersection to the lower right intersection using minimum cost (fuel, time, etc.). At each intersection, the allowable decisions are to travel down (south) or right (east). Two different costs are given as matrices on the course website; you should write a computer code to do the optimization (can you write this to handle any arbitrary sized grid?) Separate matrices are given for traveling south on each link (3×5) or eastward (4×4). What is the minimum cost and optimal route for each of the example costs?



2. Derive the LQR solution for a finite-horizon discrete-time optimal control problem using Bellman's equation. That is, given the system dynamics

$$x_{k+1} = Ax_k + Bu_k$$

   and time-varying value:

$$V(x,k) = \sum_{i=k}^{N-1} (x_k^T Q x_k + u_k^T R u_k) + x_N^T Q_N x_N$$

   derive the control law $u_k = K_k x_k$ that minimizes this cost, assuming $V(x_k, k) = x_k^T P_k x_k$, starting from Bellman's equation: (where $L(x, u, k)$ is the cost at time-step k).

$$V(x_k, k) = \min_u \left[L(x, u, k) + V(x_{k+1}, k+1)\right]$$

3. Consider the following scalar optimization problem

$$J = \sum_{k=0}^{\infty} (x_k^2 + u_k^2)$$

with dynamics

$$x_{k+1} = x_k + u_k$$

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(a) If u_k is unconstrained, solve the steady-state discrete-time algebraic Riccati equation to determine the optimal cost as a function of x_0.

(b) Now suppose that u_k can only take the values -1, 0, $+1$, and x_k can be any integer. What is the optimal cost $V(x)$ for $x = 0, 1, \ldots 5$? (You should be able to write a recurrence relationship using Bellman's equation. Also note that the problem is symmetric, so $V(-x) = V(x)$.) How does this compare with the unconstrained optimum?