CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

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Winter 2010 | | Due: 25 Jan 10

Note: Please put the number of hours that you spent on this homework set (including reading) on the back of the first page of your homework.

1. (OBC, 3.1) Consider a nonlinear control system

$$\dot{x} = f(x, u)$$

with linearization

$$\dot{x} = Ax + Bu.$$

Show that if the linearized system is reachable, then there exists a (local) control Lyapunov function for the nonlinear system about the equilibrium point at the origin. (Hint: start by proving the result for the linear system, and show that the same CLF works *locally* for the nonlinear system.)

2. (OBC, 3.2) Consider the optimal control problem given in Example 2.2:

$$\dot{x} = ax + bu, \qquad J = \tfrac{1}{2}\int_{t_0}^{t_f} u^2(t)\,dt + \tfrac{1}{2}cx^2(t_f),$$

where $x \in \mathbb{R}$ is a scalar state, $u \in \mathbb{R}$ is the input, the initial state $x(t_0)$ is given, and $a, b \in \mathbb{R}$ are positive constants. We take the terminal time t_f as given and let $c > 0$ be a constant that balances the final value of the state with the input required to get to that position. The optimal control for a finite time $T > 0$ is derived in Example 2.2. Now consider the infinite horizon cost

$$J = \tfrac{1}{2}\int_{t_0}^{\infty} u^2(t)\,dt$$

with $x(t)$ at $t = \infty$ constrained to be zero.

(a) Solve for $u^*(t) = -bPx^*(t)$ where P is the positive solution corresponding to the algebraic Riccati equation. Note that this gives an explicit feedback law ($u = -bPx$).

(b) Plot the state solution of the finite time optimal controller for the following parameter values

$$a = 2 \qquad b = 0.5 \qquad x(t_0) = 4$$
$$c = 0.1,\ 10 \qquad t_f = 0.5,\ 1,\ 10$$

(This should give you a total of 6 curves.) Compare these to the infinite time optimal control solution. Which finite time solution is closest to the infinite time solution? Why?

Using the solution given in equation (2.5), implement the finite-time optimal controller in a receding horizon fashion with an update time of $\delta = 0.5$. Using the parameter values in part (b), compare the responses of the receding horizon controllers to the LQR controller you designed for problem 1, from the same initial condition. What do you observe as c and t_f increase?

(Hint: you can write a MATLAB script to do this by performing the following steps:

(i) set $t_0 = 0$

(ii) using the closed form solution for x^* from problem 1, plot $x(t)$, $t \in [t_0, t_f]$ and save $x_\delta = x(t_0 + \delta)$

(iii) set $x(t_0) = x_\delta$ and repeat step (ii) until x is small.)