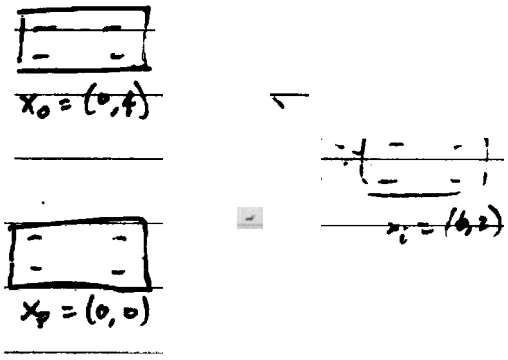
Trajectory Design

How do we choose $r(t)$ to accomplish some goal?

Eg. drive from



Need to choose $r(t)$ to respect dynamics, and actuator saturation or other constraints (eg. obstacles).

Eg. seek x_d, u_d such that $\dot{x}_d = f(x_d, u_d)$ $x_d(0) = x_0$, $x_d(T) = x_f$

⇒ Two-point boundary value problem hard!

More on this later

For now, note that with

$$\left.\begin{aligned} \dot{x} &= \cos\theta\, v \\ \dot{y} &= \sin\theta\, v \\ \dot{\theta} &= \frac{v}{\ell}\tan\phi \end{aligned}\right\} \quad \frac{\dot{y}}{\dot{x}} = \frac{\sin\theta}{\cos\theta} \rightarrow \theta_d = \tan^{-1}\left(\dot{y}_d/\dot{x}_d\right)$$

$$\& \quad v_d = \dot{x}_d/\cos\theta_d$$

$$\& \quad \phi_d = \tan^{-1}\left(\frac{\ell\dot{\theta}_d}{v_d}\right)$$

So given $x_d(t), y_d(t)$, can determine all other (state & input) variables.

This property is known as differential flatness

Remarks:

1) See Text for more details & definition
2) Key: design Trajectory in differentially flat outputs (# = # inputs) and map to appropriate inputs

Defn

A nonlinear system $\dot{x} = F(x,u)$ is <u>differentially flat</u> if there exists a function α such that

$$z = \alpha(x, u, \dot{u}, \ldots u^{(p)})$$

and we can write the solutions of the differential equation as functions of z and a finite number of derivatives

$$x = \beta(z, \dot{z}, \ldots z^{(q)})$$
$$u = \gamma(z, \dot{z}, \ldots z^{(q)})$$

E.g. For kinematic car $z = \alpha\left(\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, \begin{bmatrix} v \\ \phi \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix}$

Remarks

1. For a differentially flat system, the flat outputs z completely define the feasible trajectories
2. The number of flat outputs = number of system inputs
3. General theory for determining if a system is flat is hard (guess & check)
4. General classes of systems that are flat:
   - Reachable linear systems
   - Mechanical systems with m configuration variables, m inputs
   - Feedback linearizable nonlinear systems
5. Design trajectory in flat outputs:

   $x(0) = \beta(z(0), \dot{z}(0) \ldots)$, $x(T) = \beta(z(T), \ldots)$

   Choose e.g. $z(t) = \sum_{i=1}^{N} a_i \psi_i(t)$ for some smooth basis functions

   and solve for a_i to satisfy $x(0)$, $x(T)$