

CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

D. G. MacMartin
Fall 2014

Problem Set #8

Issued: 24 Nov 14
Due: 5 Dec 14

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. For the control systems below, design a P, PI, PD or PID control law that stabilizes the system, gives less than 1% error at zero frequency and gives at least 30° phase margin. You may use any method (loop shaping, Ziegler–Nichols, eigenvalue assignment, etc) and you only need to design one type of controller (as long as it meets the specification), but be sure to explain why you chose your controller, and include appropriate plots or calculations showing that all specifications are met. For the closed loop system, determine the steady-state error in response to a step input and the maximum frequency for which the closed loop system can track with less than 25% error.

- (a) Disk drive read head positioning system:

$$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}$$

- (b) Drug administration/compartment model (AM08, Section 3.6):

$$P(s) = \frac{1.5s + 0.75}{s^2 + 0.7s + 0.05}$$

2. Consider the dynamics of the magnetic levitation system from lecture. The transfer function from the electromagnet input voltage to the IR sensor output voltage is given by

$$P(s) = \frac{k}{s^2 - r^2}$$

with $k = 4000$ and $r = 25$ (these parameters are slightly different than those used in the MATLAB files distributed with the lecture).

- (a) Design a compensator that stabilizes this process. Compute the poles and zeros for the loop transfer function and for the closed loop transfer function between the reference input and measured output.
- (b) Plot the Nyquist plot corresponding to your compensator and the process dynamics, and verify that the Nyquist criterion is satisfied.
- (c) Plot the log of the magnitude of the sensitivity function, $\log |S(j\omega)|$, versus ω on a *linear* scale and numerically verify that the Bode integral formula is (approximately) satisfied. (Hint: you can do the integration numerically in MATLAB, using the `trapz` function. Make sure to choose your frequency range sufficiently large.)

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2. Consider a first-order system with a PI controller given by

$$P(s) = \frac{b}{s + a} \quad C(s) = k_p \left(1 + \frac{1}{T_i s} \right).$$

In this problem we will explore how varying the gains k_p and T_i affect the closed loop dynamics.

- (a) Suppose we want the closed loop system to have the characteristic polynomial

$$s^2 + 2\zeta\omega_0 s + \omega_0^2.$$

Derive a formula for k_p and T_i in terms of the parameters a , b , ζ and ω_0 .

- (b) Suppose that we choose $a = 1$, $b = 1$ and choose ζ and ω_0 such that the closed loop poles of the system are at $\lambda = \{-20 \pm 10j\}$. Compute the resulting controller parameters k_p and T_i and plot the step and frequency responses for the system.
- (c) Using the process parameters from part (b) and holding T_i fixed, let k_p vary from 0 to ∞ (or something very large). Plot the location of the closed loop poles of the system as the gain varies. You should plot your results in two different ways:
 - A pair of plots showing the real and imaginary parts of the poles as a function of the gain k_p , similar to Figure 4.18a in the text.

- A parametric plot, showing the location of the eigenvalues on the complex plane, as k_p varies. Label the gains at which any interesting features in this plot occur. (This type of plot is called a *root locus* diagram.)

You may find it convenient to use the `subplot` command in MATLAB so that you can present all of your results in a single figure.

3. In this problem we will design a PID compensator for a vectored thrust aircraft (see Example 2.9 in the text for a description). Use the following transfer function to represent the dynamics from the lateral input to the roll angle of the aircraft:

$$P(s) = \frac{r}{Js^2 + cs + mgl} \quad \begin{array}{ll} g = 9.8 \text{ m/s}^2 & m = 1.5 \text{ kg} \\ l = 0.05 \text{ m} & J = 0.0475 \text{ kg m}^2 \end{array} \quad \begin{array}{l} c = 0.05 \text{ kg/s} \\ r = 0.25 \text{ m} \end{array}$$

(these parameters correspond to a laboratory-scale experiment that we used to have at Caltech). Design a feedback controller that tracks a given reference input with the following specifications:

- Steady-state error of less than 1%
 - Tracking error of less than 5% from 0 to 1 Hz (remember to convert this to rad/s).
 - Phase margin of at least 30°.
- (a) Plot the open loop Bode plot for the system and mark on the plot the various frequency domain constraints in the above specification, as we did in class.
 - (b) Design a compensator for the system that satisfies the specification. You should include appropriate plots or calculations showing that all specifications are met.
 - (c) Plot the step and frequency response of the resulting closed loop control. For the step response, compute the steady-state error, rise time, overshoot and settling time of your controller.

(Hint: you may not need all of the terms in a PID controller.)

4. Åström and Murray, Exercise 10.10. Use $k_t = 1$ and simulate the response of the system to step changes in the reference signal of magnitude 1, 1.5, and 3, rather than magnitude 1, 2 and 3 as stated in the on-line textbook. Simulate the response both with and without anti-windup compensation.
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