CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

D. G. MacMartin
Fall 2014

Problem Set #7

Issued: 18 Nov 14
Due: 26 Nov 14

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Consider the problem of stabilizing the orientation of a flying insect, modeled as a rigid body with moment of inertia $J = 0.41$ and damping constant $D = 1$.[1] We assume there is a small delay $\tau = 0.01$ seconds given by the neural circuitry that implements the control system. The resulting transfer function for the system is taken to be

$$P(s) = \frac{1}{Js^2 + Ds} e^{-\tau s}.$$

   (a) Suppose that we can measure the orientation of the insect relative to its environment and we wish to design a control law that that gives zero steady state error, less than 10% tracking error from 0 to 0.5 Hz and has a phase margin of at least 60°. Convert these specifications to appropriate bounds on the loop transfer function and sketch the resulting constraints on a Bode plot.

   (b) Using a lead compensator, design a controller that meets the specifications in part (a). Provide whatever plots are required to verify that the specification is met. You may use a Padé approximation for the time delay, but make sure that it is a good approximation over a frequency range that includes your gain crossover frequency.

   (c) Plot or sketch the Nyquist plot corresponding to your controller and the process. You can again use a Padé approximation for the time delay.

   (d) Extra credit: genetically modify a fly to implement your controller, using the fly visual system as your input.

2. The paper "Dynamics of the coupled human-climate system resulting from closed-loop control of solar geoengineering" is posted on the course homepage under announcements. Consider the frequency response shown in Figure 1 and described by equation (3); you can assume the parameter $C = 0$. The control law in this paper is implemented in discrete-time (control updated once per year), giving a one-year time delay that is not included in the frequency response in Figure 1. If the desired closed-loop bandwidth corresponds to a period of roughly 10 years, why do you think the authors used a PI controller rather than (a) PID, (b) P only, (c) I only? (That is, describe in words why each of these other options would be inappropriate or yield poor performance.)

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[1] Based loosely on "Biologically Inspired Feedback Design for Drosophila Flight", M. Epstein, S. Waydo, S. B. Fuller, W. Dickson, A. Straw, M. H. Dickinson and R. M. Murray, 2007 American Control Conference.

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1. Consider a control system with process and controller dynamics given by

$$P(s) = \frac{1}{s(s+c)} \qquad C(s) = k.$$

where $k > 0$.

   (a) Show that the closed loop (tracking) response of the system can be written as

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

   and give formulas for ζ and ω_0 in terms of c and k.

   (b) Show that the phase margin for the system is given by

$$\varphi_m = \tan^{-1}\Big(\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}}\Big)$$

   (Hint: compute the frequency at which $|L(i\omega)| = 1$ and then find the phase at that frequency.)

   (c) Show that the overshoot for the closed loop step response is given by

$$M_p = \begin{cases} e^{-\pi\zeta/\sqrt{1-\zeta^2}} & \text{for } |\zeta| < 1 \\ 0 & \text{for } \zeta \geq 1. \end{cases}$$

   (Hint: use the form of the solution from equation (6.24) and search for the shortest time when $\dot{y}(t) = 0$.)

   (d) Use the formulas from parts 1b and 1c to plot M_p as a function of φ_m for ζ in the range $0 < \zeta \leq 1$.

2. Consider the problem of stabilizing the orientation of a flying insect, modeled as a rigid body with moment of inertia $J = 0.41$ and damping constant $D = 1$.[2] We assume there is a small delay $\tau = 0.01$ seconds given by the neural circuitry that implements the control system. The resulting transfer function for the system is taken to be

$$P(s) = \frac{1}{Js^2 + Ds} e^{-\tau s}.$$

[2]Based loosely on "Biologically Inspired Feedback Design for Drosophila Flight", M. Epstein, S. Waydo, S. B. Fuller, W. Dickson, A. Straw, M. H. Dickinson and R. M. Murray, 2007 American Control Conference.

(a) Suppose that we can measure the orientation of the insect relative to its environment and we wish to design a control law that that gives zero steady state error, less than 10% tracking error from 0 to 0.5 Hz and has an overshoot of no more than 10%. Convert these specifications to appropriate bounds on the loop transfer function and sketch the resulting constraints on a Bode plot. (Hint: Try using problem 1 to convert the overshoot requirement to a phase margin requirement.)

(b) Using a lead compensator, design a controller that meets the specifications in part (a). Provide whatever plots are required to verify that the specification is met. You may use a Padé approximation for the time delay, but make sure that it is a good approximation over a frequency range that includes your gain crossover frequency.

(c) Plot or sketch the Nyquist plot corresponding to your controller and the process. You can again use a Padé approximation for the time delay. Show the gain and phase margin on your plot.

(d) Plot the "gang of 4" for the system. If any of the magnitudes of the closed loop transfer functions are substantially greater than one in some frequency range, explain the consequences of this in terms of one of the input/output responses of your system. (You are not required to fix these problems.)

(e) Extra credit: genetically modify a fly to implement your controller, using the fly visual system as your input.

3. After designing your own controller for the fly, take someone else's controller and compare it with your own (if you don't know anyone in the class who designed a different controller, we will post the parameters of one on the course website, but you should design your own for problem 2 before looking at this one). Plot (open-loop) Bode plots for both your own and the other controller, and plot the (closed-loop) step response to a commanded input for both controllers. Comment on the differences. Which controller is "better", or are they simply different?

4. The paper "Dynamics of the coupled human-climate system resulting from closed-loop control of solar geoengineering" is posted on the course homepage under announcements. Consider the frequency response shown in Figure 1 and described by equation (3); you can assume the parameter $C = 0$. The control law in this paper is implemented in discrete-time (control updated once per year), giving a one-year time delay that is not included in the frequency response in Figure 1. If the desired closed-loop bandwidth corresponds to a period of roughly 10 years, why do you think the authors used a PI controller rather than (a) PID, (b) P only, (c) I only? (That is, describe in words why each of these other options would be inappropriate or yield poor performance.)