

CALIFORNIA INSTITUTE OF TECHNOLOGY  
Control and Dynamical Systems

**CDS 101**

D. G. MacMartin  
Fall 2014

**Problem Set #6**

Issued: 10 Nov 14  
Due: 19 Nov 14

**Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).**

1. Plot the (open loop) Nyquist and Bode plots for the following systems and compute the gain and phase margin of each. You should annotate your plots to show the gain and phase margin computations. For the Nyquist plot, mark the branches corresponding to the following sections of the Nyquist “D” contour: negative imaginary axis, positive imaginary axis, semi-circle at infinity (the curved part of the “D”).

- (a) Disk drive read head positioning system, using a lead compensator (described in Chapter 11):

$$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}, \quad C(s) = 1000 \frac{s + 1}{s + 10}.$$

- (b) Second-order system with PD compensator:

$$P(s) = \frac{100}{(100s + 1)(s + 1)}, \quad C(s) = s + 10.$$

Note: you may find it easier to sketch the Nyquist plot from the Bode plot (taking some liberties with the scale) rather than relying on MATLAB.

2. In this problem we will design a PI controller for a cruise control system, building on the example shown in class. Use the following transfer function to represent the vehicle and engine dynamics:

$$P(s) = \frac{Tba/m}{(s + a)(s + c/m)}$$

where  $b = 25$  is the transmission gain,  $T = 200$  is the conversion factor between the throttle input and steady state torque,  $a = 0.2$  is the engine lag coefficient,  $m = 1000$  kg is the mass of the car, and  $c = 50$  N s/m is the viscous damping coefficient.

- (a) Consider a proportional + integral controller for the car,

$$C(s) = k_p + \frac{k_i}{s}.$$

Fill in the following table (make sure to show your work), where  $g_m$  is the gain margin,  $\varphi_m$  the phase margin, and SSerr the steady state error.

$k_p$	$k_i$	Stable?	$g_m$	$\varphi_m$	SSerr
0.5	0.1				
0.05	1				
0.05	0.001				
0.005	0.001				

For each entry in the table, plot the pole zero diagram (pzmap) for the *closed loop* system and the step response. (Note that the steady state error is zero in each stable case, due to the integral term in the control law. The gain and phase margins are not defined if the system is unstable.)

(Suggestion: look for relationships between the various quantities you are computing and plotting. This problem should give you some insight into the relationship between some of the quantities.)

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$$C(s) = k_p + \frac{k_i}{s}.$$

Fill in the following table (make sure to show your work), where  $g_m$  is the gain margin,  $\varphi_m$  the phase margin, SSerr the steady state error, BW the bandwidth (you can use the bandwidth command in Matlab, but you need to do so for the closed-loop system),  $T_r$  the rise time, and  $M_p$  the overshoot (see Fig. 5.9 on p. 151 of the text, you do not need to be exact).

$k_p$	$k_i$	Stable?	$g_m$	$\varphi_m$	SSerr	BW	$T_r$	$M_p$
0.5	0.1							
0.05	1							
0.05	0.001							
0.005	0.001							

For each entry in the table, plot the pole zero diagram (pzmap) for the *closed loop* system and the step response. (Note that the steady state error is zero in each stable case, due to the integral term in the control law. The gain and phase margins are not defined if the system is unstable, nor are bandwidth, rise time, or overshoot.)

(Suggestion: look for relationships between the various quantities you are computing and plotting. This problem should give you some insight into the relationship between some of the quantities.)

- Continuing the previous problem, we will now insert a small amount of time delay into the feedback path of the system. A pure time delay of  $\tau$  seconds satisfies the equation

$$y(t) = u(t - \tau)$$

This system is a linear input/output system and it can be shown that its transfer function is

$$G(s) = e^{-s\tau}.$$

Unfortunately, MATLAB is not able to perfectly represent a time delay in this form, and so we have to use a “Padé approximation”, which gives a constant gain transfer function with phase that approximates a time delay. Using a 2nd order Padé approximation, we can approximate our time delay as

$$G(s) = \frac{1 - \tau s/2 + (\tau s)^2/12}{1 + \tau s/2 + (\tau s)^2/12}$$

This function can be computed using the `pade` function in MATLAB (although the numerator and denominator are scaled slightly differently).

Assume that there is a time delay of  $\tau$  seconds, which we will insert between the output of the plant and the controller.

- For the case  $k_p = 0.05$ ,  $k_i = 0.001$ , insert time delays of  $\tau = 0.25$  s and  $\tau = 0.75$  s. Using a Padé approximation, compute the resulting gain and phase margin for each case and compute the overshoot and settling time (2%) for the step responses.
  - Optional:* Plot the Nyquist plot with the exact time delay and compare with the Pade approximation.
- Consider a system with the loop transfer function

$$L(s) = \frac{k}{s(s-1)(s+5)}.$$

This transfer function has a pole at  $s = 1$ , which is inside the Nyquist contour. Draw the Nyquist plot for this system and determine if the closed loop system is stable.

5. In homework #4 you designed an output-feedback for the Whipple bicycle model that combined an observer with a state feedback  $u = -K\hat{x}$ . Ignoring the reference input ( $r = 0$  or  $k_r = 0$ ), compute the transfer function of your controller (from input  $y$  to output  $u$ ) and plot the Bode plot (frequency response) of your controller. Combine this with the transfer function for the (linearized) bicycle model and plot the Bode and Nyquist plots for the loop transfer function, and compute the gain and phase margins.