Transfer function of a state space system

take the state-space system with zero initial condition

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \\ x(0) = 0 \end{cases}$$

Laplace transform of the first equation (elementwise):

$$sX(s) = AX(s) + BU(s) \quad \Rightarrow \quad (sI - A)X(s) = BU(s).$$

substute $X(s) = (sI - A)^{-1}BU(s)$ into second equation:

$$Y(s) = \underbrace{\left(C(sI - A)^{-1}B + D\right)}_{H_{yu}(s)} U(s)$$

initial condition: for general $x(0) = x_0$,

$$Y(s) = C(sI - A)^{-1}x_0 + \left(C(sI - A)^{-1}B + D\right) U(s)$$

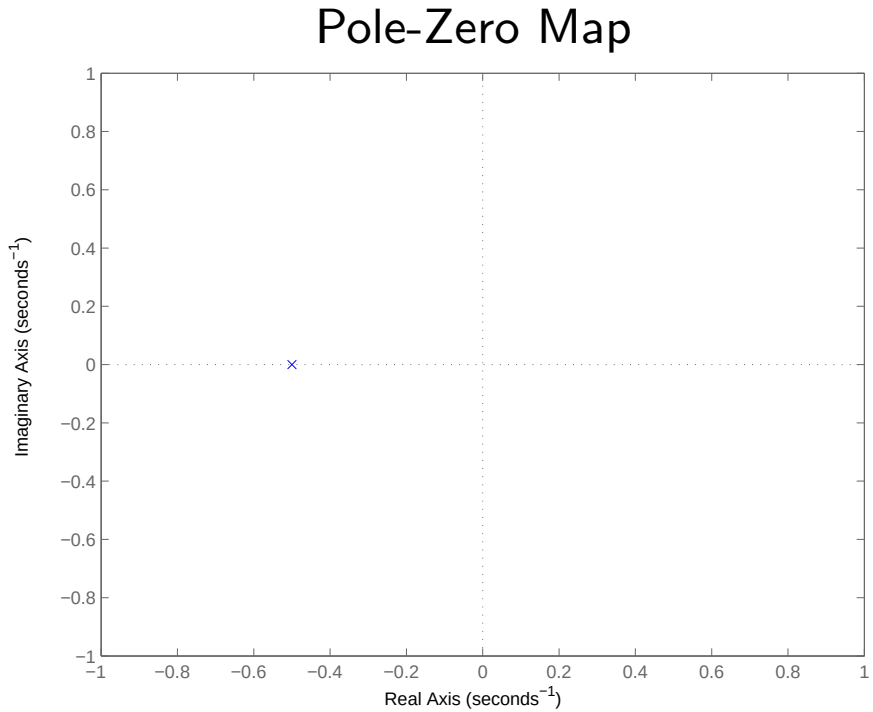
Pole-zero plot

the transfer function

$$H(s) = \frac{1}{1 + \tau s}$$

has one pole at $s = -1/\tau$

```
% time constant (say)
tau = 2;
% define 1/(tau*s+1)
sys = tf([1], [tau, 1]);
% pole-zero plot
pzplot(sys);
xlim([-1,1]);
ylim([-1,1]);
```



Bode magnitude: evaluate on imaginary axis

magnitude of the transfer function evaluated at $s = j\omega$:

$$|H(j\omega)| = \left|\frac{1}{1+j\omega\tau}\right| = \frac{1}{|1+(\omega\tau)^2|^{1/2}}$$

in decibels:

$$\begin{aligned} 20\log_{10}|H(j\omega)| &= \cancel{20\log_{10} 1} - 20\log_{10}(1+(\omega\tau)^2)^{1/2} \\ &= -10\log_{10}(1+(\omega\tau)^2) \end{aligned}$$

- for $\omega\tau \ll 1$, $\log_{10}(1+(\omega\tau)^2) \approx 0$
- for $\omega\tau \gg 1$, $\log_{10}(1+(\omega\tau)^2) \approx 2(\log_{10}\omega + \log_{10}\tau)$

Bode magnitude

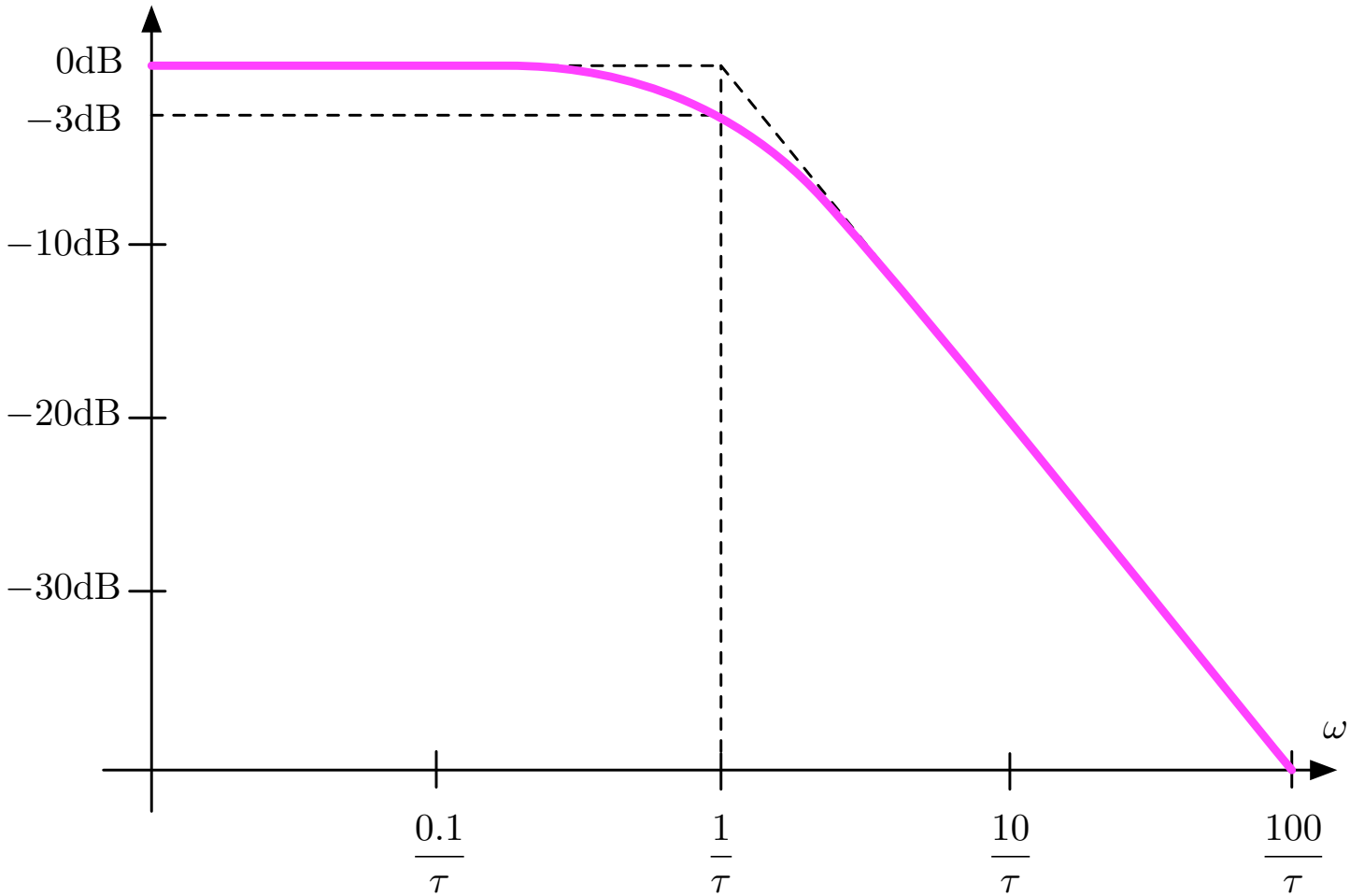
hence the magnitude in decibels is

$$20\log_{10}|H(j\omega)| \approx \begin{cases} 0, & \text{if } \omega \ll 1/\tau \\ -10\log_{10} 2, & \text{if } \omega = 1/\tau \\ -20(\log_{10}\omega + \log_{10}\tau), & \text{if } \omega \gg 1/\tau \end{cases}$$

- $\omega_{3\text{dB}} = 1/\tau$ is the -3dB *half-power* or *break* point (actually $-10\log_{10} 2 \approx -3.0103$dB)
- unit DC gain (0dB)
- magnitude decreases at 20dB per decade for $\omega \gg 1/\tau$

Bode magnitude plot

$$H(j\omega) = \frac{1}{1+j\omega\tau}$$



Bode phase

argument (phase) of the transfer function at $s = j\omega$:

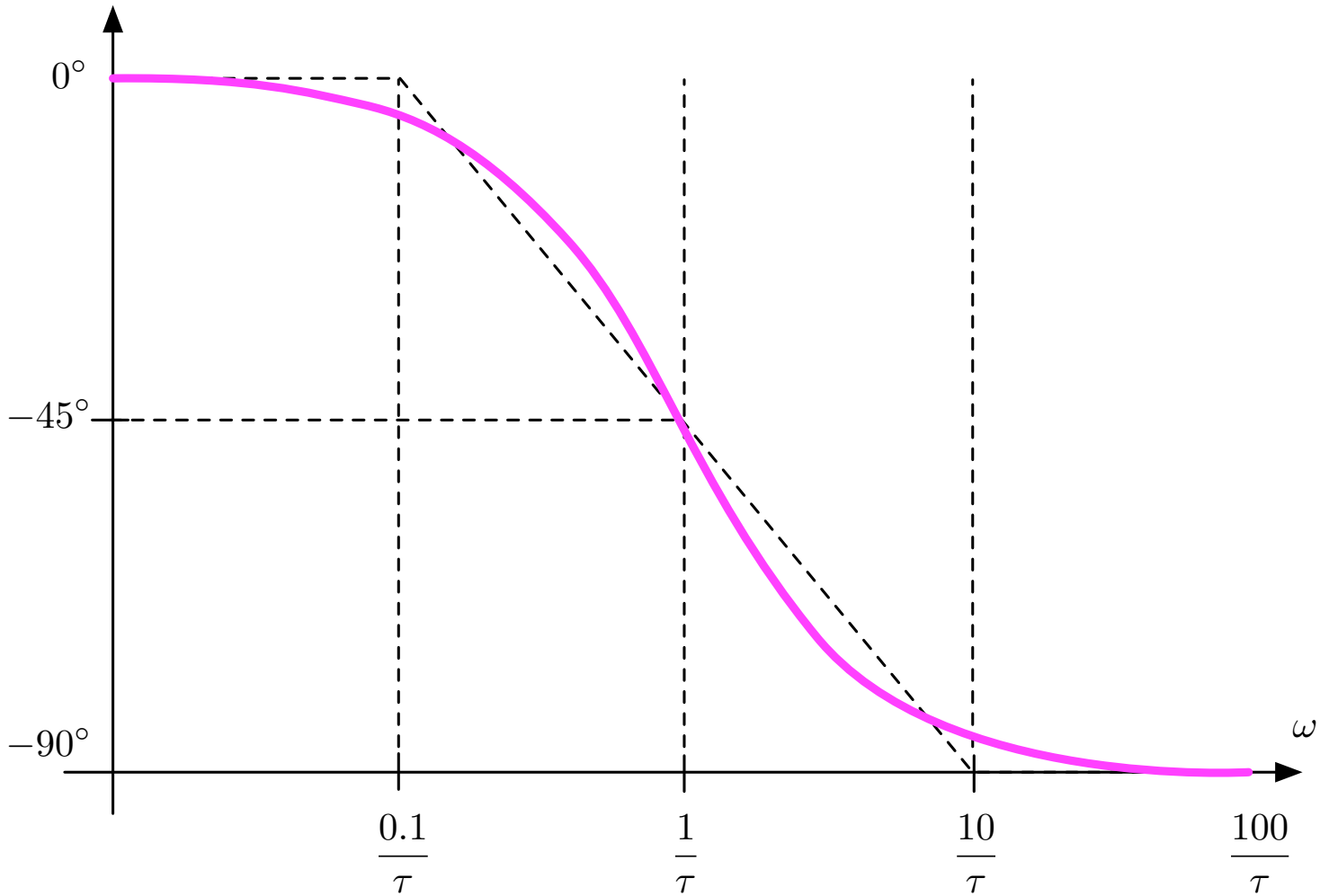
$$\begin{aligned}\angle H(j\omega) &= \angle\left(\frac{1}{1+j\omega\tau}\right)\\ &= \angle 1 - \angle(1+j\omega\tau)\\ &= -\arctan(\omega\tau)\end{aligned}$$

asymptotic approximation

$$-\arctan(\omega\tau) \approx \begin{cases} 0, & \text{if } \omega < 0.1/\tau \\ -\frac{\pi}{4}(1+\log_{10}\omega+\log_{10}\tau), & \text{if } 0.1/\tau \le \omega \le 10/\tau \\ -\frac{\pi}{2}, & \text{if } \omega > 10/\tau \end{cases}$$

Bode phase plot

$$H(j\omega) = \frac{1}{1+j\omega\tau}$$



More complicated example

plot the Bode magnitude and phase plots for the transfer function

$$H(s) = \frac{100(1+s)}{(10+s)(100+s)}$$

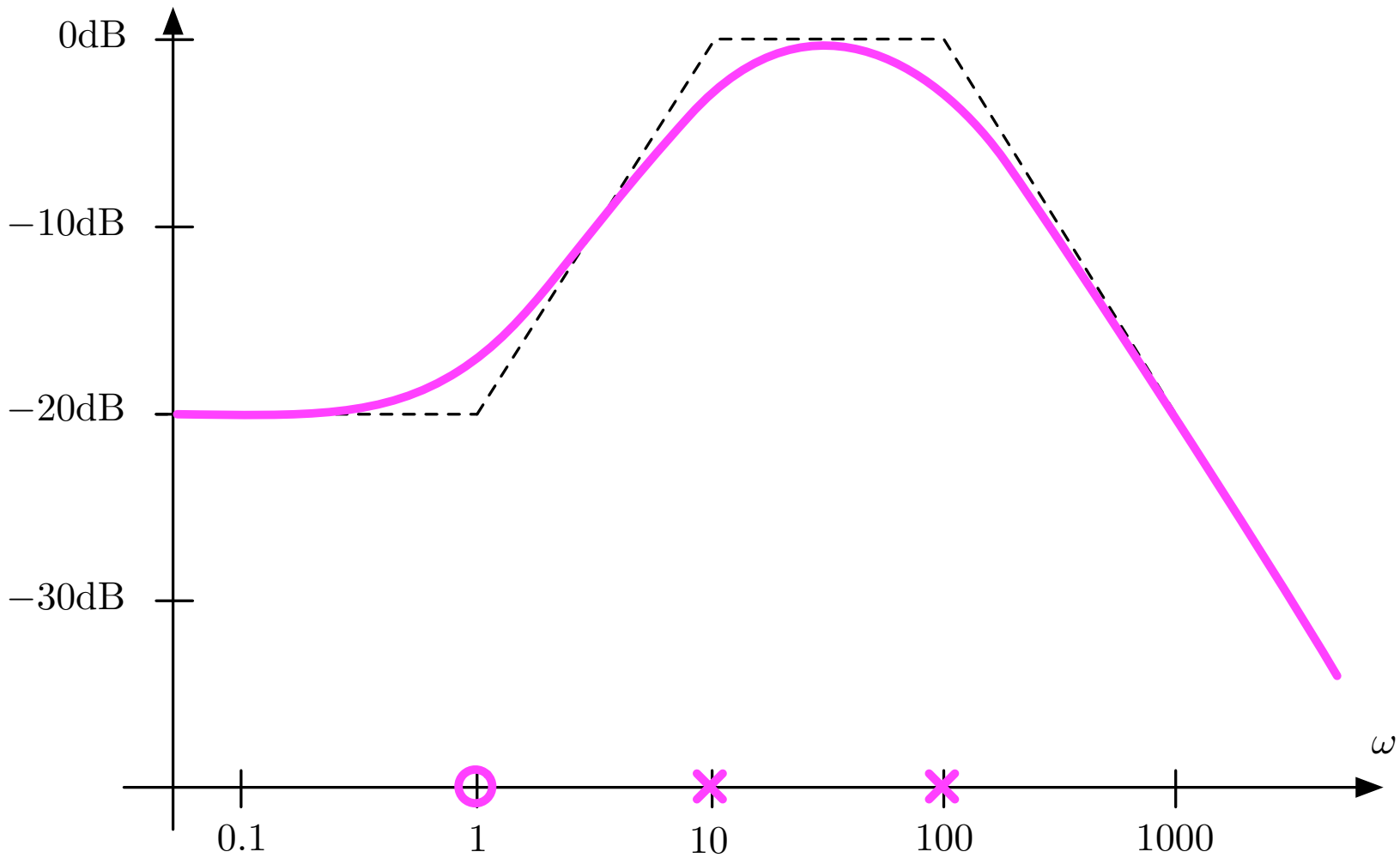
solution: rewrite as

$$H(j\omega) = \left(\frac{1}{10}\right)(1+j\omega)\left(\frac{1}{1+j\omega/10}\right)\left(\frac{1}{1+j\omega/100}\right)$$

- DC gain $20\log_{10}|H(0)| = 20\log_{10}(1/10) = -20\text{dB}$
- zero at $s = -1$
- poles at $s = -10,\ -100$

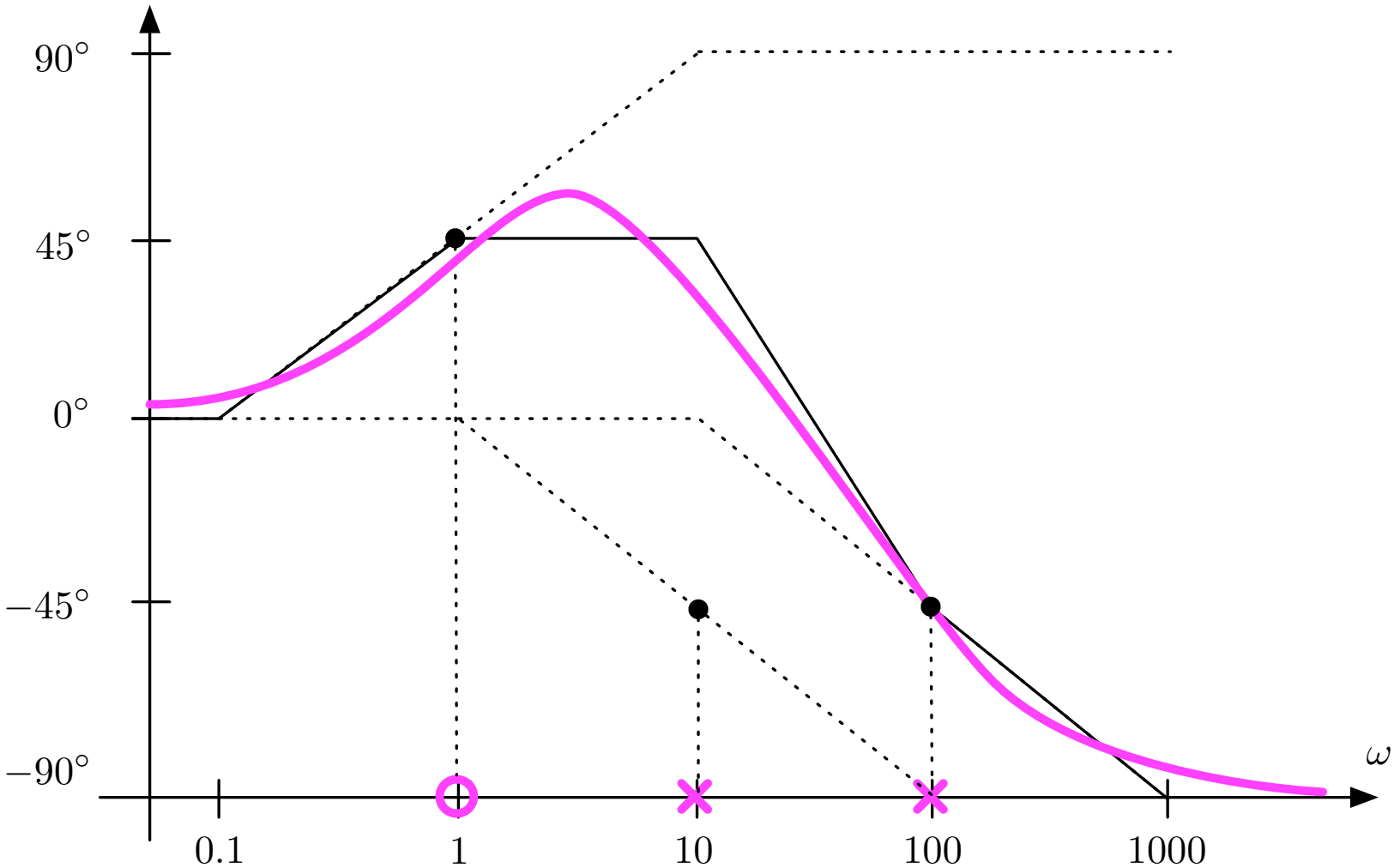
Bode magnitude plot

$$H(j\omega) = \left(\frac{1}{10}\right)(1 + j\omega)\left(\frac{1}{1+j\omega/10}\right)\left(\frac{1}{1+j\omega/100}\right)$$



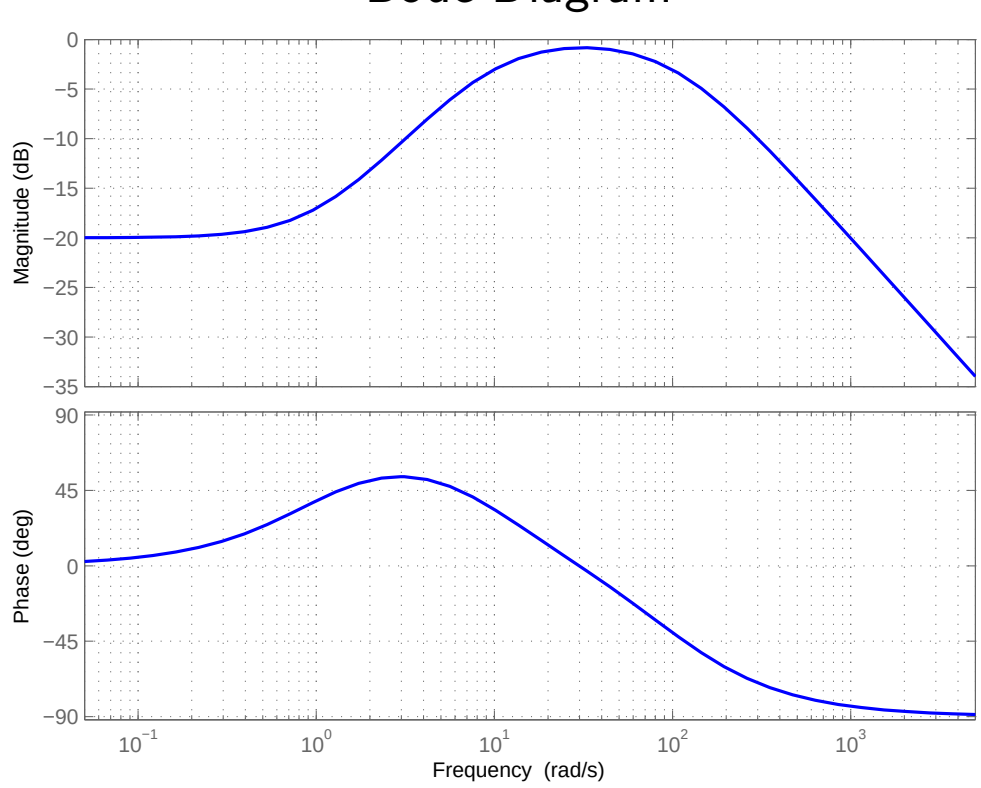
Bode phase plot

$$H(j\omega) = \left(\tfrac{1}{10}\right)(1 + j\omega)\left(\tfrac{1}{1+j\omega/10}\right)\left(\tfrac{1}{1+j\omega/100}\right)$$



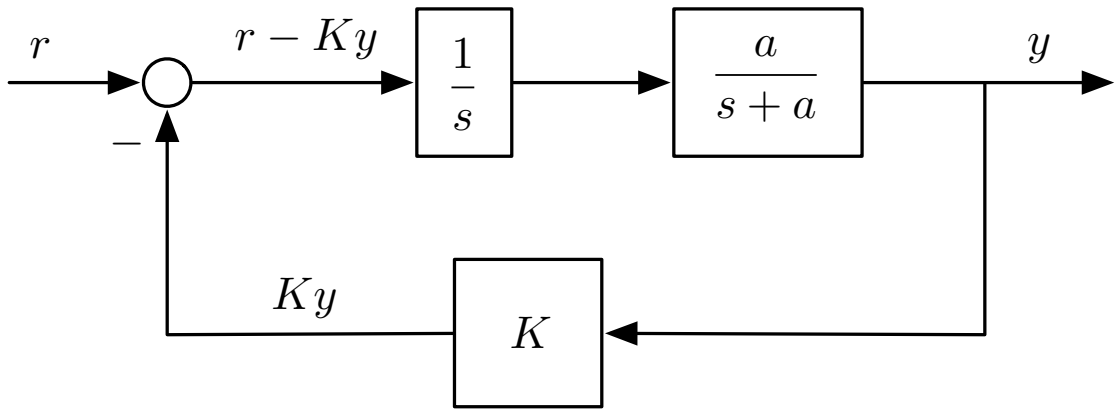
Checking with matlab

```
% define transfer function
sys = zpk([-1], [-10, -100], 100);
% plot magnitude and phase
bode(sys, {0.05, 5000});
grid on;
```



Block diagram algebra

find the transfer function from $r(t)$ to $y(t)$



solution: middle block is a cascade with transfer function $\frac{a}{s(s+a)}$

$$\frac{a}{s(s+a)}(R(s) - KY(s)) = Y(s) \quad \Rightarrow \quad \frac{Y(s)}{R(s)} = \frac{a}{s^2 + as + aK}$$

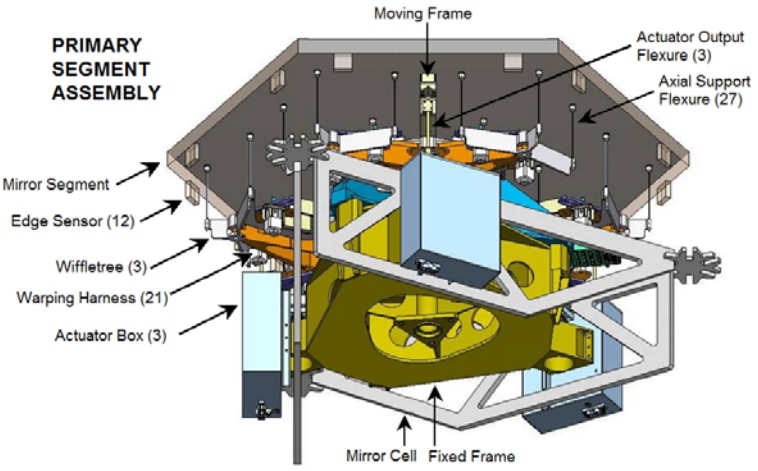
Primary Segment Assembly
Test Results & Model Comparison

Peter Thompson
Martin Regehr
Mark Colavita

M1CS Dynamics Review 2

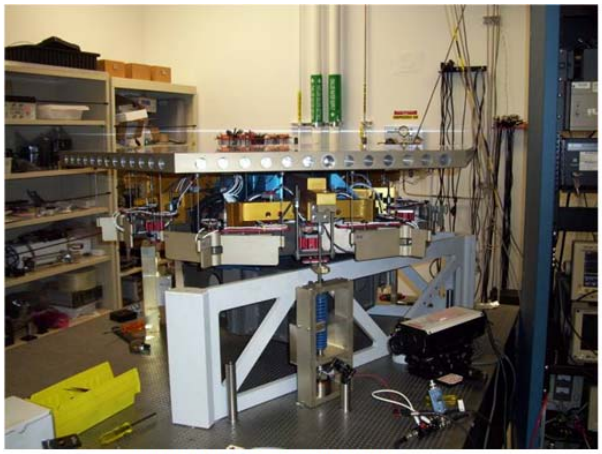
Feb 1-2, 2011

The Model
Primary Segment Assembly

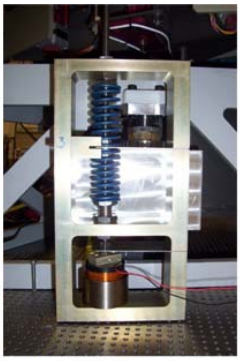


TMT
THIRTY METER TELESCOPE

The Lab at JPL



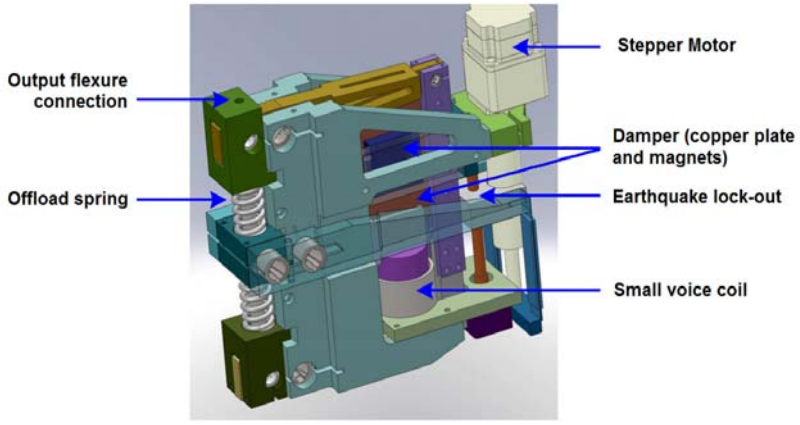
a) Prototype PSA on optical table



b) Dummy actuator with voice coil and offload springs visible

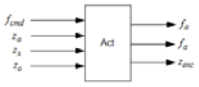
TMT
THIRTY METER TELESCOPE

Marjan P2 Voice Coil Actuator (Concept)

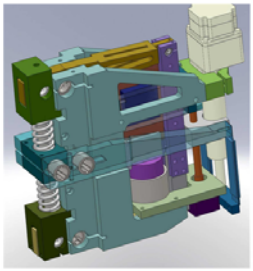


Marjan P2 Soft Actuator Model

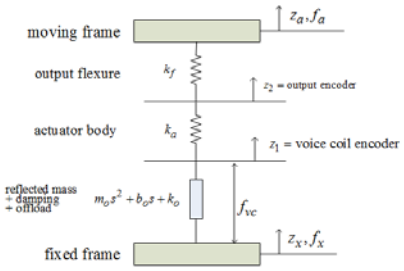
- Used for current review



Block Diagram



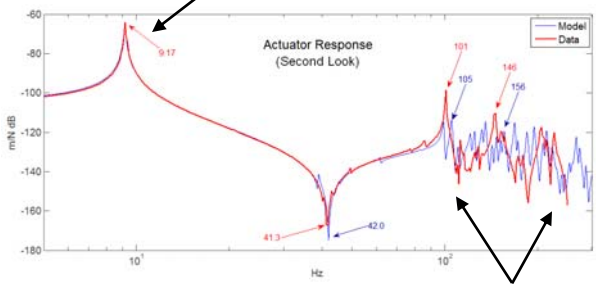
Marjan P2 Actuator



Mechanical Diagram

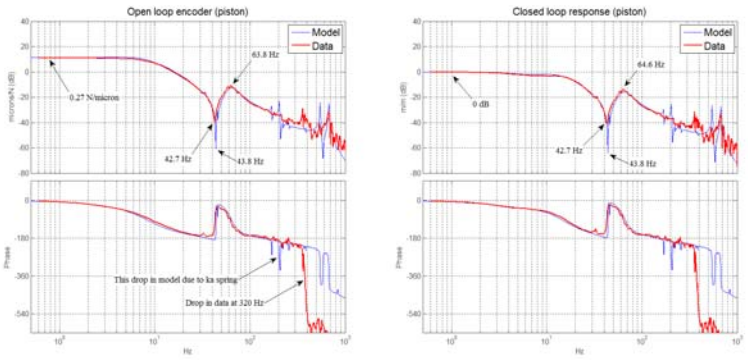
Comparison Structural Model and Measurement (Second Look)

Piston increases due to table and by coincidence matches Tip-tilt



No attempt to improve match > 100 Hz

Marjan P2 Actuator: Output Encoder Response



- Used to verify P2 actuator model
- Shows effect of actuator damping