CDS 101/110a: Lecture 8-1
PID Control

Douglas G. MacMartin

Joel Burdick

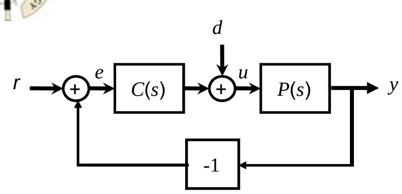
Goals:

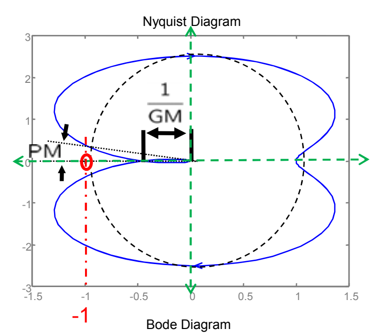
- *Continuation* frequency-domain performance specification
- Introduce PID (Proportional + Integral + Derivative) feedback
- Show how to use (*tune*) PID to achieve a performance specification

Reading:

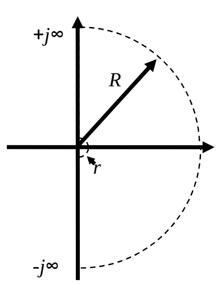
- Åström and Murray, *Feedback Systems,* Ch 10
- *Advanced*: Lewis, Chapters 12-13

Last Week: Loop Analysis





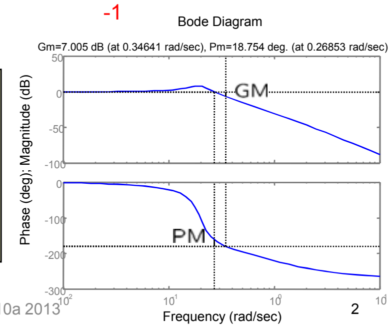
- L(s) = P(s)C(s): zeroes 1+L(s) are poles of TF
- Nyquist criteria for loop stability
- Gain, phase margin for robustness



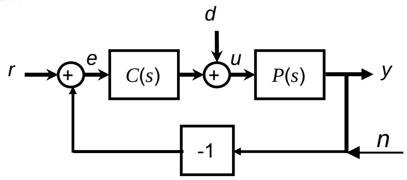
Thm (Nyquist).

P	# RHP poles of $L(s)$
N	# CW encirclements
Z	# RHP zeros of $1+L$

$$Z = N + P$$



Design based on loop transfer function



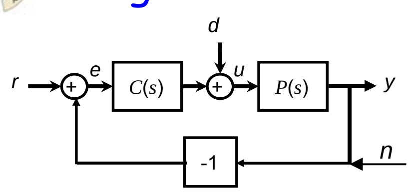
$$L(s) = P(s)C(s)$$

$$H_{er} = \frac{1}{1+L} \qquad H_{yr} = \frac{L}{1+L}$$

$$H_{yd} = \frac{P}{1+L} \qquad H_{yn} = \frac{-L}{1+L}$$

"Gang of Four"

Design based on loop transfer function



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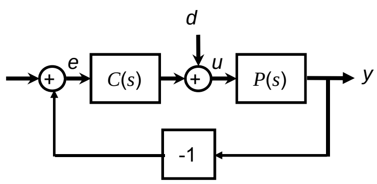
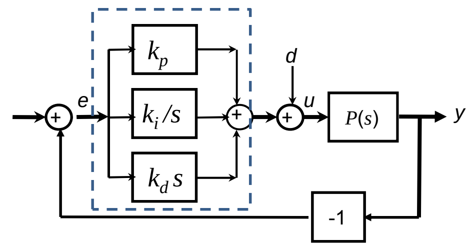
- Stability depends only on *L = PC* (last week)
  - Robustness requires reasonable gain and phase margin
- Performance depends (mostly) on *L = PC*
  - When L is large, tracking performance and disturbance rejection is good
  - When L is small, sensor noise rejection is good, actuator response is small.
  - Typically care about tracking and disturbance response at low frequencies
- Use ***Loop Analysis*** to design C(s):
  - **PID Control** (*Today*)
  - **Loop Shaping** (Wednesday)

Proportional Integral Derivative (PID) Control

$$u(t) = \underbrace{k_p e(t)}_{\text{Proportional}} + \underbrace{k_i \int_0^t e(\tau)\, d\tau}_{\text{Integral}} + \underbrace{k_d \frac{de(t)}{dt}}_{\text{Derivative}}$$

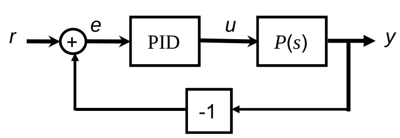




Extremely common and popular

- *"Based on a survey of over 11,000 controllers in the refining, chemicals and pulp and paper industries, 97% … utilize PID feedback."* **L. Desborough and R. Miller, 2002**
- Many variations possible

Overview: PID control



$$u = k_p e + k_i \int e\, dt + k_d \dot{e}$$

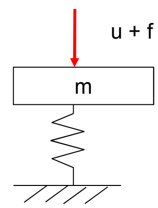
$$C(s) = \frac{k_i}{s} + k_p + k_d s$$

- Intuition
  - Proportional term: provides inputs that correct for "current" errors
  - Integral term: ensures steady state error goes to zero
  - Derivative term: provides "anticipation" of upcoming changes (or damping in some cases)
- A bit of history on "three term control"
  - First appeared in 1922 paper by Minorsky: "Directional stability of automatically steered bodies" under the name "three term control"
  - Also realized that "small deviations" (linearization) could be used to understand the (nonlinear) system dynamics under control
- Utility of PID
  - For many systems, only need PI or PD (special case)
  - Many tools for tuning PID loops and designing gains
  - Found in biological Systems

Time-domain motivation

- Mass-spring system: $m\ddot{z} + c\dot{z} + kz = u + f$
- PD control: $u = -(k_p z + k_d \dot{z})$
- Closed-loop: $m\ddot{z} + (c + k_d)\dot{z} + (k + k_p)z = f$
  - Derivative gain acts like increasing damping
    - Increases system stability (greater phase margin)
  - Proportional gain acts like increasing stiffness
    - No matter how large the stiffness, still a non-zero response to disturbance force
  - Steady-state (for constant disturbance force *f*)

$$\dot{z} = 0 \quad \Rightarrow \quad \lim_{t\to\infty} z(t) = \frac{1}{k + k_p} f, \text{ and } \lim_{t\to\infty} u(t) = -\frac{k_p}{k + k_p} f$$



- Integral control: $\dot{q} = z$

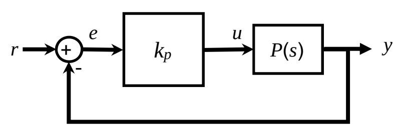
$$u = -(k_i q + k_p z + k_d \dot{z})$$

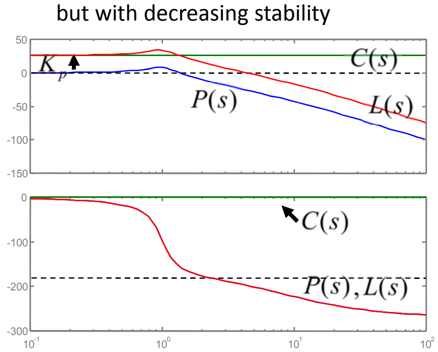
  - Steady-state (assuming stability) then for constant *f*

$$\dot{q} = 0 \quad \Rightarrow \quad \lim_{t\to\infty} z = 0, \text{ and } \lim_{t\to\infty} u(t) = -f$$

Proportional Feedback

- Simplest controller choice: $u = k_p e$
  - Effect: lifts gain with no change in phase
  - Good for plants with low phase up to desired bandwidth
  - Bode: shift gain up by factor of k_p
  - Step response: better steady state error, but with decreasing stability





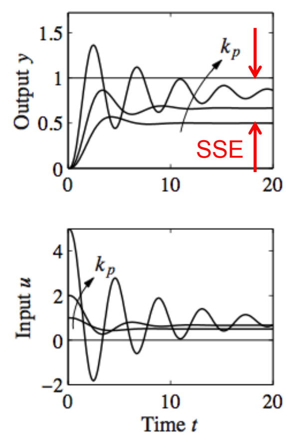
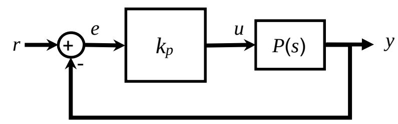


figure 10.2(a), p. 295

Proportional Feedback (continued)

$$H_{yr} = \frac{PC}{1+PC}$$



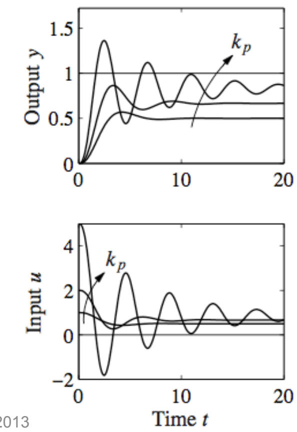
Unit Step Steady State Error

$$1 - H_{yr}(0) = \frac{1}{1+k_p P(0)} \neq 0$$

Error decreases with gain, but not zero. To avoid SSE:

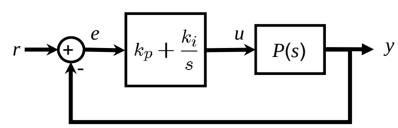
$$\text{u(t)} = k_p e(t) + u_{ff}$$

u_{ff} is a *feed-forward* term, called a *reset*. Not good in practice.

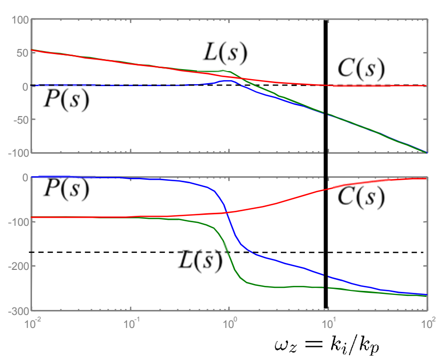


Proportional + Integral (PI) Compensation

- Use to eliminate steady state error
  - Effect: lifts gain at low frequency
  - Gives *zero* steady state error
  - Bode: infinite SS gain + nonzero phase margin
  - Step response: zero steady state error, with smaller settling time, but more overshoot



$k_p > 0$, $k_i > 0$



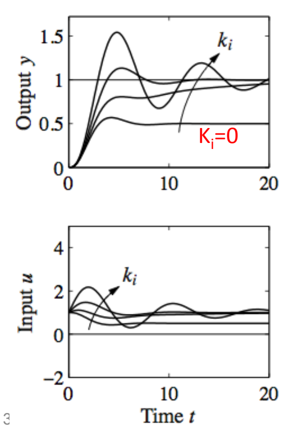
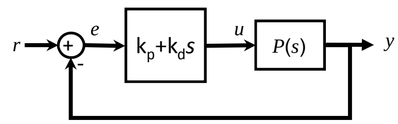


figure 10.2(b), p. 295

Proportional + Derivative (PD) Control

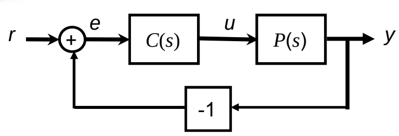


PD controller can be written as:

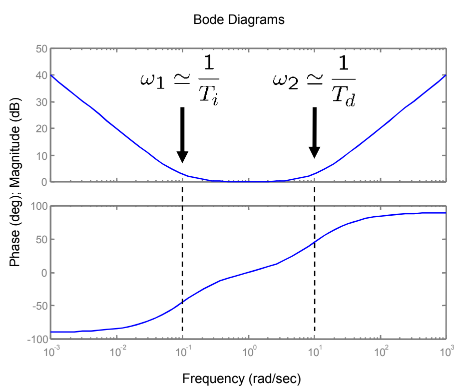
$$u = k_p e + k_d \frac{de}{dt} = k_p(e + T_d \frac{de}{dt}) = k_p e_p$$

Where e_p is the *prediction error,* which is the linear approximation of error at time T_d

Proportional + Integral + Derivative (PID)



$$C(s) = k_p + k_i \frac{1}{s} + k_d s$$
$$= k\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= (kT_d)\frac{(s+\alpha_i)(s+\alpha_d)}{s}$$



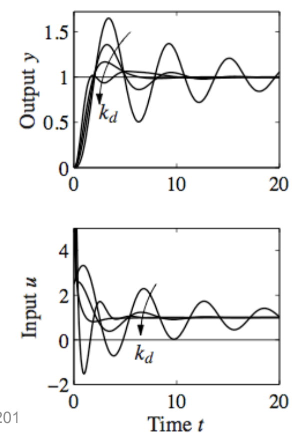
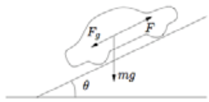


figure 10.2(c), p. 295

Example: Cruise Control using PID - Specification



Car going up hill (adapted from Ex. 5.11, p. 158)

Car Dynamics under Cruise Control

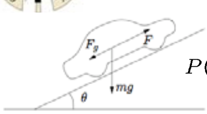
$$m\frac{dv}{dt} = \underbrace{\alpha_n u T(\alpha_n v)}_{\text{Engine Force}} - \underbrace{mgC_r sgn(v)}_{\text{Rolling Friction}} - \underbrace{\frac{1}{2}\rho C_v A v^2}_{\text{Air Drag}} - \underbrace{mg\sin\theta}_{\text{Gravity}}$$

Dynamics linearized about equilibrium

Transfer Function (including dynamics of cruise control (diff from book))

$$P(s) = \frac{1/m}{s+b/m} \times \frac{r}{s+a}$$

Example: Cruise Control using PID - Specification



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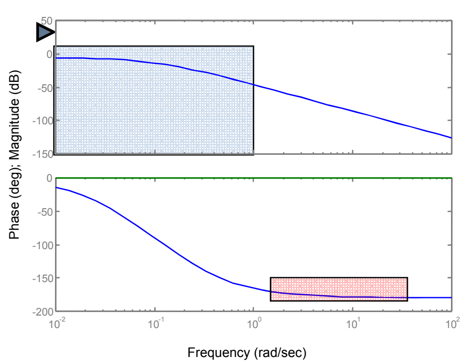
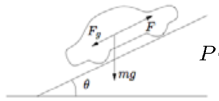
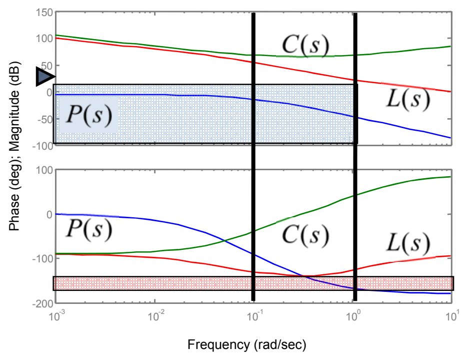


- Performance Specification
  - 1% steady state error
    - ⇒ Zero frequency gain > 100
  - 10% tracking error up to 1 rad/sec
    - ⇒ Gain > 10 from 0-1 rad/sec
  - 45˚ phase margin
    - ⇒ Gives good relative stability
    - ⇒ Provides robustness to uncertainty
    - ⇒ But overshoot will be ~25%
- Observations
  - Purely proportional gain won't work: to get gain above desired level will not leave adequate phase margin
  - Need to increase the phase from ~0.5 to 2 rad/sec and increase gain as well

Example: Cruise Control using PID - Design



$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s+a}$$

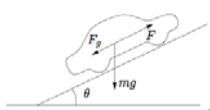


- Approach
  - Use proportional gain to give desired tracking performance
  - Use integral gain to make steady state error small (zero, in fact)
  - Use derivative action to increase phase lead in the cross over region
- Controller
  - Ti = 1/0.1; Td = 1/1; k = 2000

$$C(s) = 2000\frac{s^2 + 1.1s + 0.1}{s}$$
$$= 2200 + \frac{200}{s} + 2000s$$

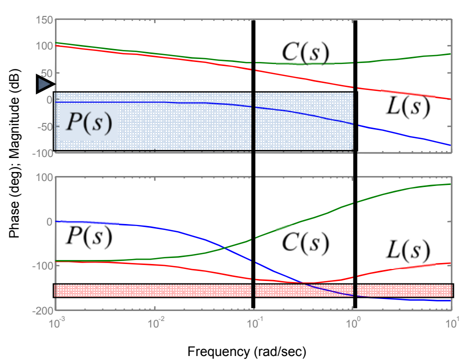
- Closed loop system
  - Very high steady state gain
  - Adequate tracking @ 1 rad/sec
  - ~80° phase margin
  - Verify with Nyquist + Gang of 4

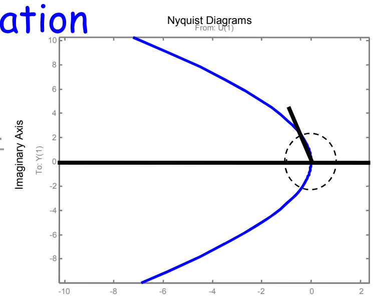
Example: Cruise Control using PID - Verification

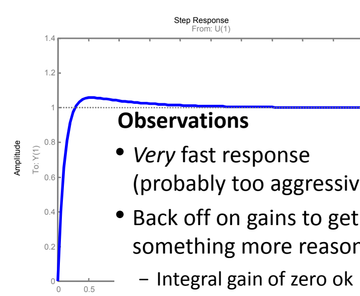


$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s+a}$$

$$C(s) = 2000\frac{s^2 + 1.1s + 0.1}{s}$$



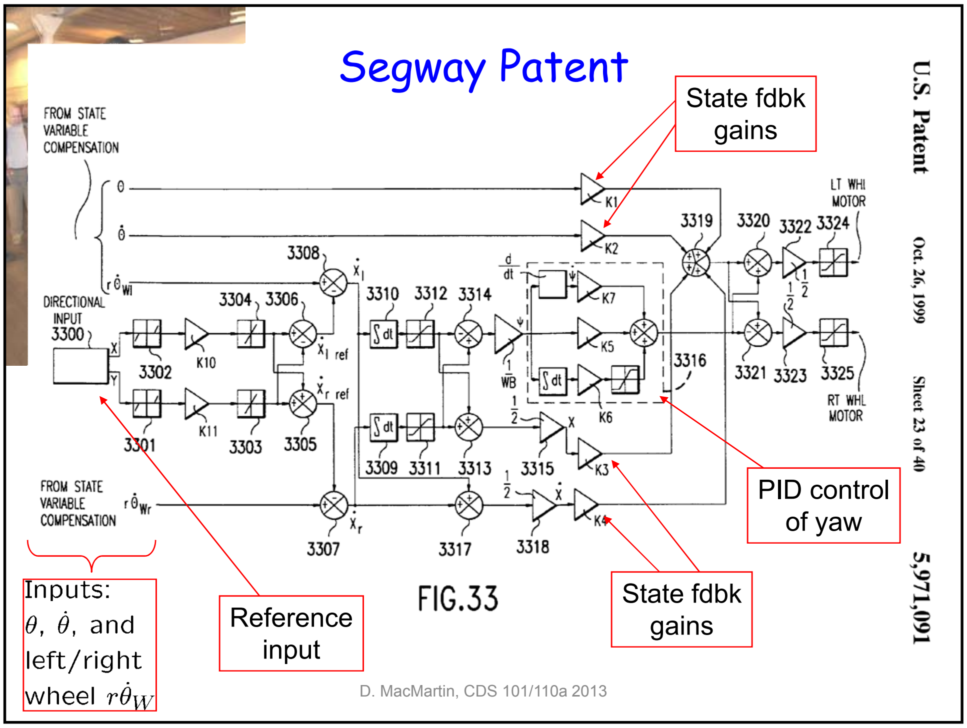




Observations

- *Very* fast response (probably too aggressive)
- Back off on gains to get something more reasonable
  - Integral gain of zero ok
  - Lower rate fdbk gain ok

Segway Patent



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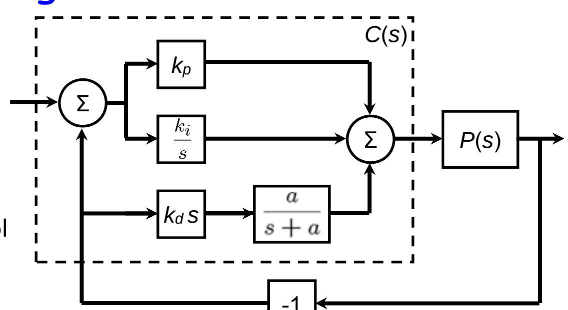
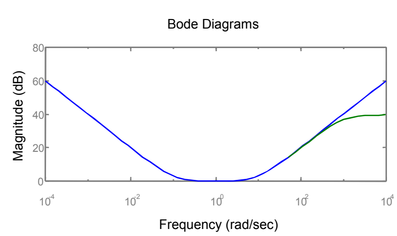
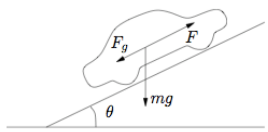
Implementing Derivative Action

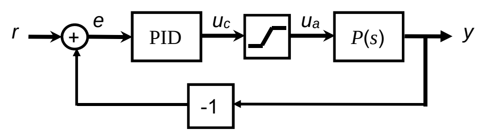
- Problems with derivatives
  - High frequency noise amplified by derivative term
  - Step inputs in reference can cause large inputs
  - Shows up in Gang of Four...
- Solution: modified PID control
  - Use high frequency rolloff in derivative term
    - first order filter will give finite gain at high frequency
    - use higher order filter if needed
  - Don't feed reference signal through derivative block
    - Useful when reference has unwanted high frequency content
    - Alternative solution: reference shaping via two DOF design ($F(s)$ block)
  - Many other variations (see AM08 + refs)



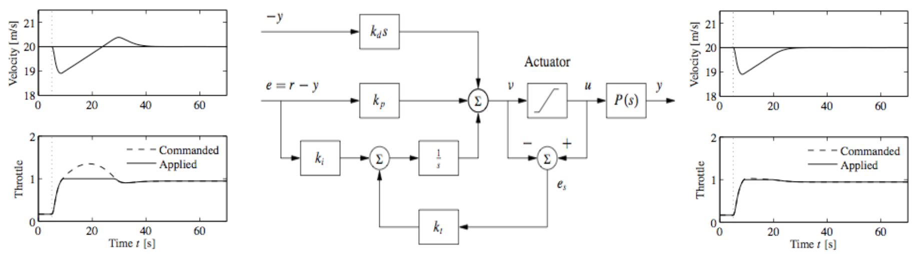


Windup and Anti-Windup Compensation





- Problem
  - Limited magnitude input (saturation)
  - Integrator "winds up") overshoot
- Solution
  - Compare commanded input to actual
  - Subtract off difference from integrator

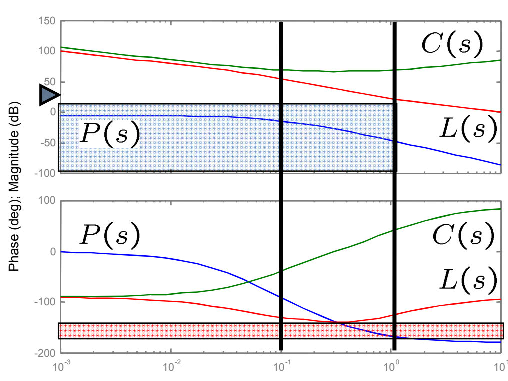


(a) Windup
(b) Anti-windup

Summary: Frequency Domain Design using PID

- Loop Shaping for Stability & Performance
  - Steady state error, bandwidth, tracking

$$H_{ue}(s) = k_p + k_i \frac{1}{s} + k_d s$$



- Main ideas
  - Performance specs give bounds on loop transfer function
  - Use controller to shape response
  - Gain/phase relationships constrain design approach
  - Standard compensators: proportional, PI, PD, PID