



Classical Control Design Guidelines & Tools (L10.2)

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- Summarize frequency-domain control design guidelines and approach

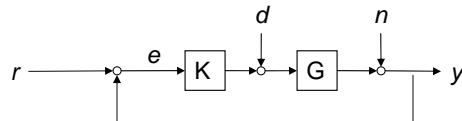
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Transfer Functions



$$\frac{e}{r} = \frac{1}{1 + GK} = S \quad \text{Tracking error (Sensitivity)}$$

$$\frac{y}{r} = \frac{GK}{1 + GK} = T \quad \text{Tracking response (Complementary Sensitivity)}$$

$$\frac{u}{r} = \frac{K}{1 + GK} = KS \quad \text{Actuator response}$$

$$\frac{e}{d} = \frac{G}{1 + GK} = GS \quad \text{Disturbance response}$$

$$\frac{(y/d)_{CL}}{(y/d)_{OL}} = S \quad \text{Improvement in disturbance rejection}$$

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Design Guidelines

1. Design the loop transfer function $L=GK$, not the closed-loop
 - a) L large: good performance
 - b) L small: good robustness
2. Steady-state error based on $L(0)$
 - a) Zero steady-state error to step requires integrator in $L(s)$
 - b) Zero steady-state error to ramp requires two integrators
 - c) Note that more than two integrators is harder
 - Conditionally stable (finite negative gain margin)
3. Cross-over with slope of -1
 - = 0 then there is no crossover
 - = -1 then phase is -90°
 - = -2 then phase is $-180^\circ \Rightarrow$ Zero phase margin
 - a) Behaviour near crossover is what influences stability
 - b) Usual problem is *losing* phase (higher order dynamics, filters, time delays,...)

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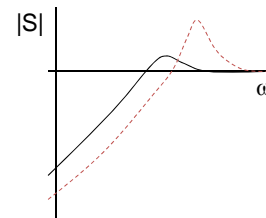
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Guidelines, cont'd

4. Phase margin required for both robustness and performance
 - a) Typically want $30^\circ < PM < 60^\circ$
(30 is typically absolute minimum, generally no advantage to more than 60)
 - b) Can trade bandwidth for phase margin
 - Higher bandwidth \rightarrow faster response
 - Lower phase margin \rightarrow worse overshoot



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Guidelines, cont'd

5. Main design tools:

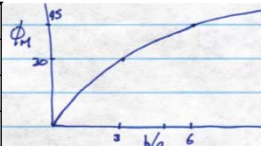
a) Lead: $K(s) = \frac{s+a}{s+b}$, $a < b$

Adds phase, maximum phase ϕ_m added at

$$\omega = \sqrt{ab} \quad \phi_m = 90^\circ - 2 \tan^{-1} \sqrt{a/b}$$

E.g.

ϕ_m	b/a
30°	~3
45°	~6
60°	~14



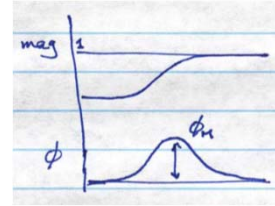
or $K(s) = \frac{\alpha Ts + 1}{Ts + 1}$

a) Lag:

$$K(s) = \frac{s+a}{s+b}, \quad b < a < \omega_c$$

a/b = increase in error constant

Use for steady-state performance



c) PID:

$$K(s) = \frac{k_i/s + k_p + k_d s}{s^2 k_d + s k_p + k_i}$$

- May not need all three terms
- For second order system $G(s) = (s^2 + 2\zeta\omega_n s + \omega_n^2)^{-1}$ need derivative term to ensure slope is -1 if crossover is above ω
- For simple system $G(s) = 1$, then integral gain may suffice

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Guidelines, cont'd

6. Bandwidth constrained by RHP zero:

$$G(s) = -\frac{s-a}{(s+b)^2} = \left[\frac{s+a}{(s+b)^2} \right] \cdot \left[-\frac{s-a}{s+a} \right] = G_{mp} \cdot G_{ap}$$

Minimum-phase,
same magnitude as G

"all-pass": unity magnitude,
pure phase lag. E.g. at $s=ja/2$,
 $\Delta\phi = 30^\circ + 30^\circ = 60^\circ$

Also note that $S(a) = 1$

- Maximum bandwidth for non-minimum phase systems
- Minimum bandwidth requirement for unstable systems

7. Time delay $= e^{-s\tau} \simeq -\frac{s-a}{s+a}$, $a = 2/\tau$

- Phase lag = 360° at $f = 1/\tau$ (one full cycle)
= 36° at $f = 1/10$ of $1/\tau$
- 1st order Padé error only ~1° at $f=0.1/\tau$ (or 4° at $\omega=1/\tau$)

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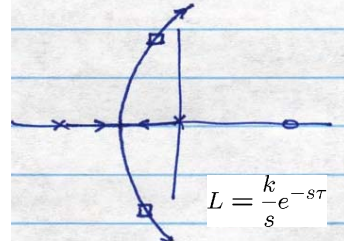
Guidelines, cont'd

8. Closed-loop performance typically looks close to a 2nd order system
- There is typically a dominant pair of poles that limit ability to increase loop gain any further
 - These are typically associated with phase margin at crossover:

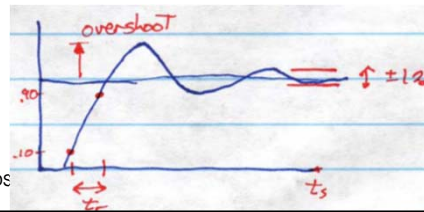
$$S(s) \simeq \frac{n(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- where $\omega_n \sim$ loop crossover frequency
 $\zeta \sim \phi_m/100$ (phase margin in degrees)

- Step response can be approximated (but use Matlab for accuracy)
 - $t_r \sim 2.2\tau$ (where $\tau = 1/\omega_c$)
 - $t_s \sim 4\tau$ if critically damped
 - $t_s \sim 4/(\zeta\omega_n)$
 - Overshoot $P \sim 1 + e^{-\pi/\tan\theta}$, $\theta = \cos^{-1}\zeta$
 (e.g. PM=30°, P=1.37)



k = 0.5	k = 1	k = 1.5
$\phi_m = 62^\circ$	= 37°	= 16°
$\zeta = 0.75$	= 0.35	= 0.14
$\omega_c = 0.5$	= 1	= 1.5
$\omega_n = 1$	= 1.4	= 1.7



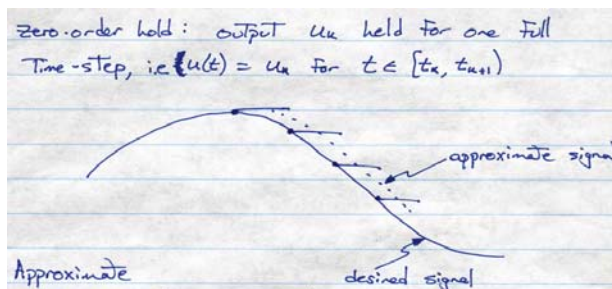
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Guidelines...

9. Digital implementation:
- Add time delay to plant model
 - Design in continuous-time
 - Convert to discrete-time to implement
 - Approximate ZOH by delay of T/2, Include compute delay
 Include anti-aliasing and reconstruction filters
- } OK if $f_s > 10f_c$ where f_s is the sample rate and f_c is loop cross-over frequency



Converting to discrete-time: use "c2d" with "tustin" option

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Cont'd...

10. Actuator saturation: $\frac{u}{r} = \frac{K}{1 + GK}$
- If $|GK|$ small, should keep $|K|$ small also
 - If saturating within control bandwidth, then $u=u_{\text{sat}}$ probably best anyway
This effectively reduces the gain! Be careful with conditionally stable control...
 - Be careful about integrator windup
11. MIMO: via sequential loop closure only
- Associate actuators with sensors
 - E.g. aircraft flight control:
$$\begin{bmatrix} \text{climb rate} \\ \text{velocity} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} \text{elevator} \\ \text{thrust} \end{bmatrix}$$

Close one loop
(e.g. for system from thrust to velocity)
and design second loop (e.g. elevator to climb rate)
with first loop closed (Works fine if $G_{21}=0$)
12. Limits on performance, e.g. $S + T = 1$,
$$\int_0^\infty \log |S(j\omega)| d\omega = \pi \sum \text{Re}(p_k) \geq 0$$

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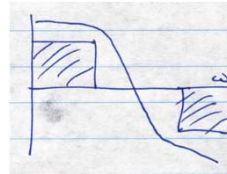
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Design Approach

- Convert performance specifications into specifications on $L(s)$
$$\left| \frac{e}{r} \right| < \frac{1}{a} \Rightarrow |L| > a$$
- Add integrator(s) if needed for steady-state performance
- Estimate crossover frequency
 - Sufficient to meet specification (e.g. if need 10% error at freq x , crossover will be $\sim 10x$)
 - Specification is often "as good as possible":
 - If bandwidth limited by phase lag (frequent), construct phase budget:
 - crossover slope of -1 gives 90° phase
 - Lead of 30° is (relatively) easy to add
 - For 60° phase margin, can tolerate 60° phase lag due to time delay, non-minimum phase part of plant, actuator lag (minimum phase, but inverting will typically lead to saturation)
 - May be limited by actuator saturation, model uncertainty, or sensor noise...



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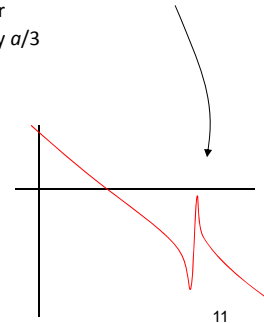
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Design Approach, cont'd

4. Crossover at slope of -1
 - a) Choose appropriate number and frequency of poles and zeros (invert minimum-phase part of plant)... same as PID design knobs
5. Clean up
 - a) Add lead to fix phase at cross-over and obtain desired phase margin
 - b) Add lag at low frequencies if need to boost low frequency gain
 - c) Add notch filters to deal with problem high-frequency lightly-damped modes and recover gain margin
 - d) Add roll-off to ensure compensator is proper or strictly proper
 - Single pole at frequency a gives 18° phase lag at frequency $a/3$
 - e) Adjust lead (and maybe bandwidth) to compensate for phase lag added in steps (b)-(d)
6. Plot "gang of four" frequency and step response
7. Simulate and iterate
8. Test and iterate



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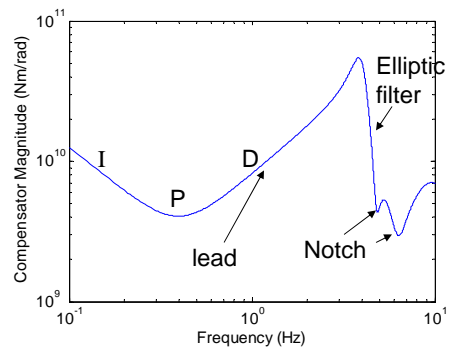
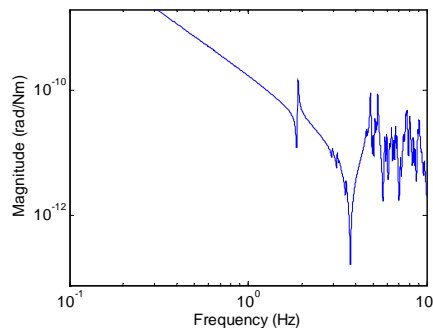
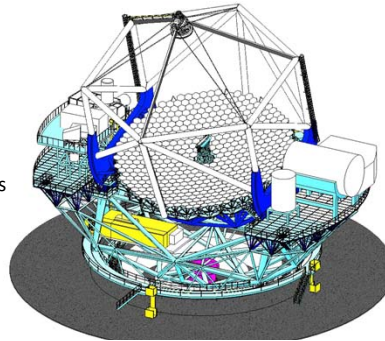
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Example

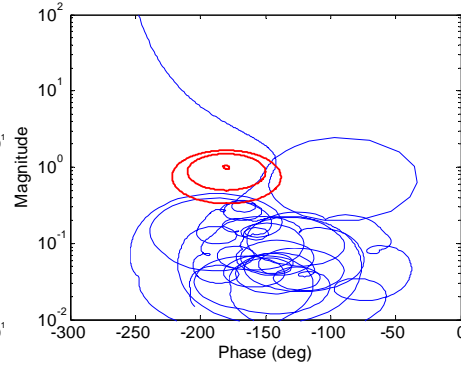
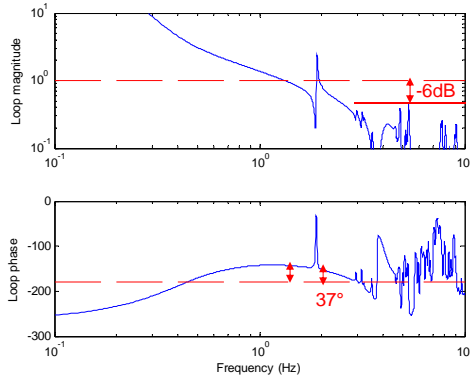
- Thirty meter telescope pointing control
 - Input is drive torque about elevation axis
 - Output is (collocated) rotation from encoder output
 - Need notch filters to compensate for resonant peaks





Loop Transfer Function

- Loop crossover frequency = 1.3 Hz
- Bandwidth: -3dB sensitivity = 0.63 Hz
- 37° phase margin, 6 dB gain margin
- Nichols chart: Same as Nyquist, but log-magnitude and phase instead of real/imag
- Red lines plotted at $|S|=2$ and $|S|=1.5$



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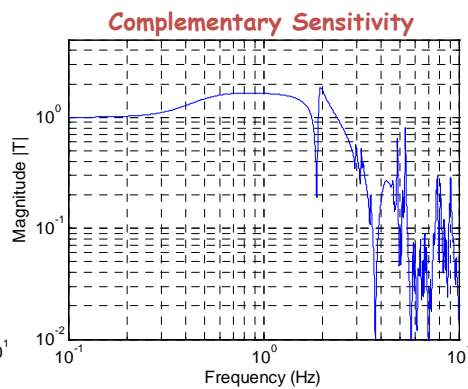
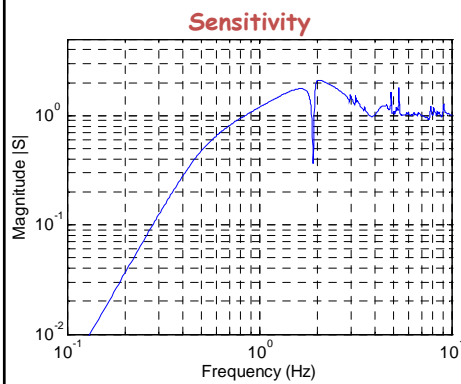
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Performance

- Good disturbance rejection up to ~ 0.5 Hz
- Good tracking performance up to ~ 0.3 Hz



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Summary

- Loop shaping (frequency-domain) design provides intuition about what can be achieved and how to achieve it
 - Convert performance specifications into specification on $L(s)$
 - Figure out how to stabilize unstable poles (e.g. with Nyquist plot)
 - Figure out required bandwidth
 - Cross-over with slope of -1
 - Add phase if needed
 - Plot gain of 4
 - If uncertain about stability, plot Nyquist.