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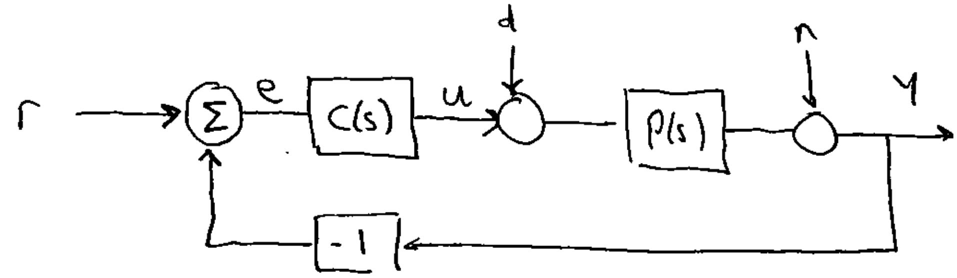
Lecture 6-2: Frequency Response

Linear (or linearized) control system:



Controller design goals:

- Keep $e(t)$ "small" in presence of disturbances (d), sensor noise (n) and changes in reference (r)
- Time domain: given $r(t)$, $d(t)$ or $n(t)$, would like $\underbrace{\|e\|}_{\text{"size" of error signal}} < \underbrace{\gamma}_{\text{"gain"}} \underbrace{\|d\|}_{\text{"size of disturbance"}}$ (or n or r)
- Frequency domain: $|H_{ed}(i\omega)|$ small for all ω

$$\Rightarrow d = A\sin(\omega t) \rightarrow e = A|H_{ed}(i\omega)| \sin(\omega t + \phi(\omega))$$

∴ For control analysis (and design), focus on four key transfer functions

$$\text{Keep small}\begin{cases} H_{er} = \dfrac{-1}{1+PC} \\ H_{ed} = \dfrac{P}{1+PC} \end{cases} \qquad \left.\begin{aligned} H_{ru} &= \frac{C}{1+PC} \\ \underbrace{H_{yr}}_{\text{Keep near 1}} = H_{du} &= \frac{PC}{1+PC} \end{aligned}\right] \text{"Gang of Four"}$$

Example: PI cruise controller

System:

input, disturbance

$$m\frac{dv}{dt} = a_n u\, T(a_n v) - mg C_r \operatorname{sgn}(v) - \tfrac{1}{2}\rho C_d A v^2 - mg\sin\theta$$

$$y = v \leftarrow \text{output}.$$

Linearize around $V_e = 20$ m/sec (~ 40 mph), $\theta = 0$, $n = 4$

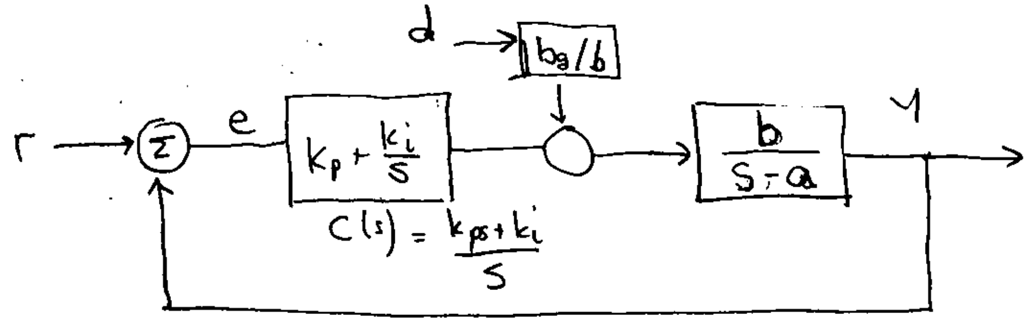
Let $e = v - v_e$, $\tilde{u} = u - u_e$, $d = \theta - 0$

$$\dot{e} = ae + b\tilde{u} + b_g d \qquad P(s)$$

$a = -0.01$
$b = 1.32 \approx 1$
$b_g = 9.8 \approx 10$
Ex 5.11

Controller: $\tilde{u} = k_p e + k_i \int_0^t e$

$k_p = 0.5$
$k_i = 0.1$
Later (Ch 10)



Transfer functions:

$$H_{yr} = \frac{PC}{1+PC} = \frac{b(k_p s + k_i)}{s^2 + (bk_p - a)s + bk_i}$$

Zeros: $0, -k_i/k_p$

Poles: $(a - bk_p)/2 \pm \sqrt{(a-bk_p)^2/4 - bk_i}$

$$H_{ed} = \frac{(b_g/b)P}{1+PC} = \frac{b_g s}{s^2 + (bk_p - a)s + bk_i}$$

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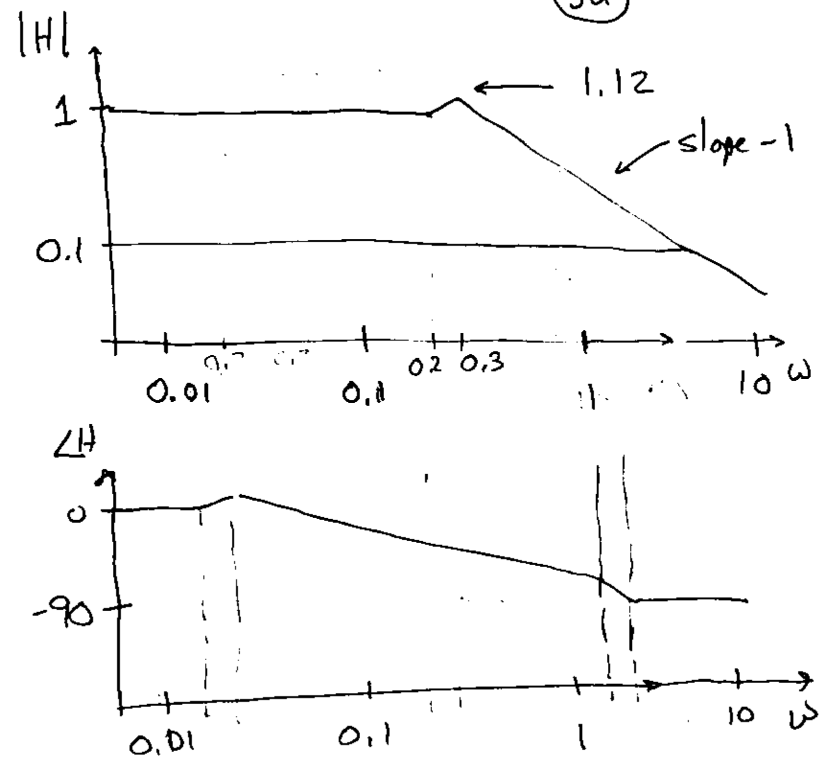
Reference tracking

$$H_{yr} = \frac{b(sk_p + k_i)}{s^2 + (bk_p - a)s + bk_i}$$



Poles: $s^2 + 2\zeta\omega_n s + \omega_n^2$

$$\omega_n^2 = 0.1 \Rightarrow \omega_n \approx 0.32$$

$$2\zeta\omega_n \approx 0.5 \Rightarrow \zeta \approx 0.78$$

Zeros: $s = \dfrac{-k_i}{k_p} = -0.2$

Value at peak:

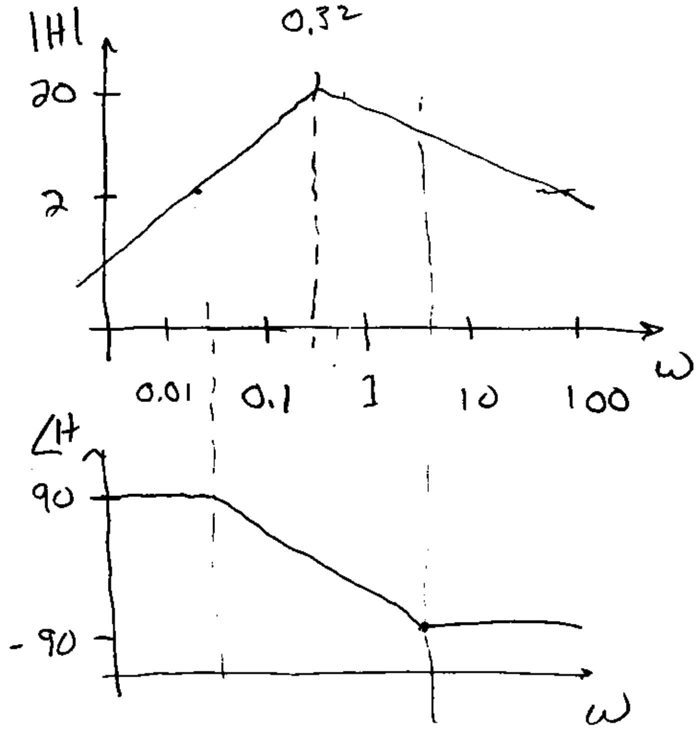
$$|H_{yr}(i\omega_n)| = \left|\frac{b(i\omega_n k_p + k_i)}{2\zeta\omega_n^2}\right| = \frac{b\sqrt{\omega_n^2 k_p^2 + k_i^2}}{2\zeta\omega_n^2} \approx 1.12$$

Implications

1. Get perfect tracking for constant inputs ($\omega = 0$)
2. Oscillating reference at ~ 0.32 rad/sec ($\sim$20 secs/cycle) gives slight error (12%)
3. Higher frequencies can't be tracked; get large errors

Disturbance rejection

$$H_{ed} = \frac{b_g s}{s^2 + (bk_p - a)s + bk_i}$$



Poles: $s^2 + 2\zeta\omega_n s + \omega_n^2$

$\omega_n^2 = 0.1 \Rightarrow \omega_n \approx 0.32$

$2\zeta\omega_n = 0.5 \Rightarrow \zeta \approx 0.75$

Value at peak:

$$|H_{ed}(i\omega_n)| = \frac{b_g \omega_n}{2\zeta\omega_n^2} = \frac{b_g}{2\zeta\omega_n} = \frac{10}{1.5 \times 0.3} \approx 20$$

Implications

1. Constant disturbance $\Rightarrow$ zero error
2. Hills at 0.32 rad/sec ($\approx$ 20 secs/cycle)
   $\Rightarrow$ amplification!
3. How do we design better disturbance rejection?
   A: Choose $2\zeta\omega_n$ larger $\Rightarrow$ $(bk_p - a)$, $\sqrt{bk_i}$ larger
   $\therefore$ increase k_p or k_i (or both)

BUT will see next week there are limits...

skip

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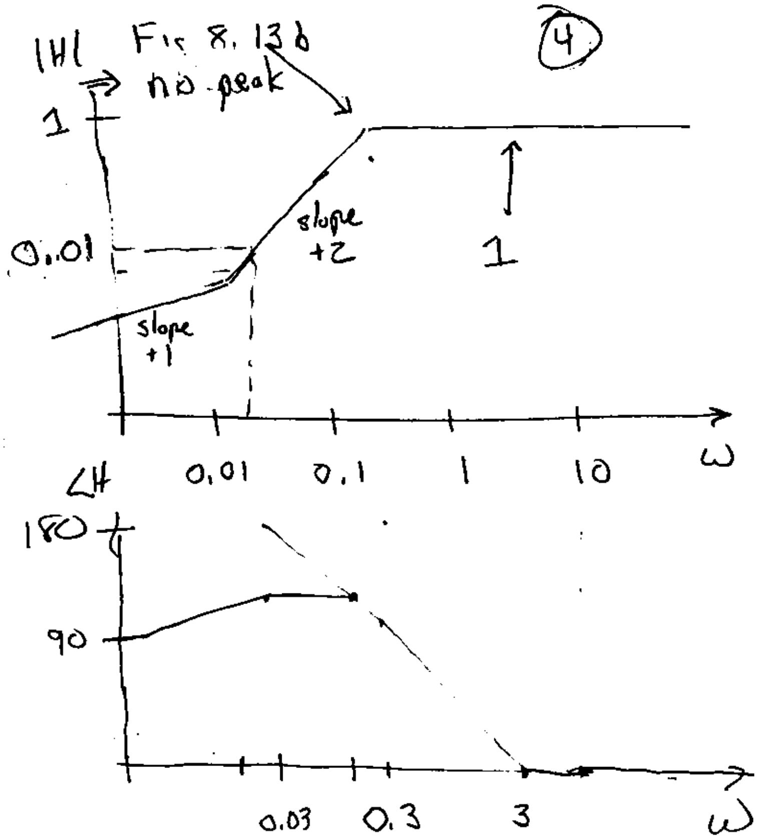
Reference error



$$H_{er} = \frac{s(s-a)}{s^2 + (bk_p - a)s + bk_i}$$

Poles: $s^2 + 2\zeta\omega_n s + \omega_n^2$ -

$\omega_n^2 = 0.1 \Rightarrow \omega_n \approx 0.32$

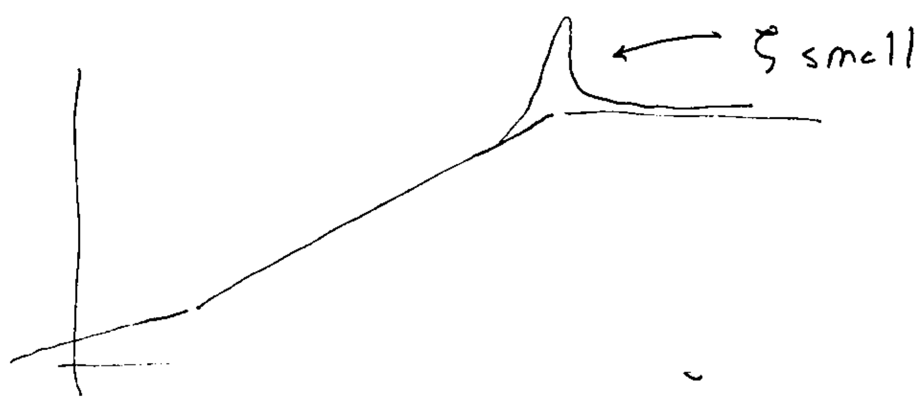
$2\zeta\omega_n \approx 0.5 \Rightarrow \zeta \approx 0.78$

Implications

- Constant reference ⇒ zero error (integral action)
- High frequency reference (2π rad/sec = 1 Hz)
- What if you want more performance?

  A: move ω_n higher ⇒ increase k_i

  But increasing ω_n decreases ζ ⇒ get resonance



Homework: similar analysis with additional dynamics for the engine (and slightly diff parameters)

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Big picture

$$\dot{x} = f(x, u, d) \qquad \dot{x}_d = f(x_d, u_d, 0)$$
$$y = h(x) + n \qquad r = h(x_d, 0)$$

} Trajectory generation

Error dynamics

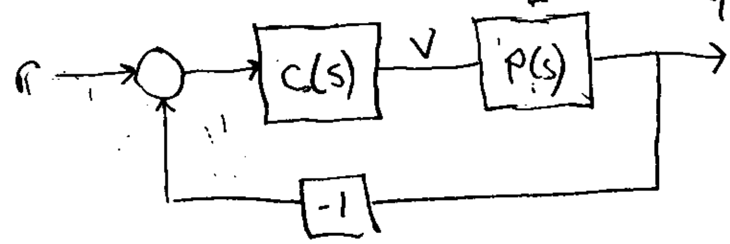
$$e = x - x_d \qquad \dot{e} = f(x,u,d) - f(x_d, u_d, 0)$$
$$v = u - u_d \qquad\quad = f(x_d + e, u_d + e, d) - f(x_d, u_d, 0)$$
$$\qquad\qquad \approx A(x_d)\, e + B(x_d)\, v + F(x_d)\, d$$

$$A(x_d) = \frac{\partial f}{\partial x}\Big|_{(x_d, u_d)} \qquad B(x_d) = \frac{\partial f}{\partial u}\Big|_{(x_d, u_d)} \quad \text{etc}$$

linearized error dynamics

Controller design

$$v = C(s)(r - y)$$



Overall controller: $u = u_d + v$

What's next

1. Need to make sure closed loop system is stable:

$$H_{er} = \frac{1}{1 + PC} \Rightarrow \text{zeros of } 1 + PC \text{ in LHP}$$

2. Need to design $C(s)$ to achieve performance specs (trajectory tracking, disturbance/noise rejection)
3. Study robustness of closed loop: what if $P(s) \to \tilde{P}(s)$