CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

D. G. MacMynowski | **Problem Set #8** | Issued: 21 Nov 11
Fall 2011 | | Due: 30 Nov 11

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. For the control systems below, design a P, PI, PD or PID control law that stabilizes the system, gives less than 1% error at zero frequency and gives at least 30° phase margin. You may use any method (loop shaping, Ziegler–Nichols, eigenvalue assignment, etc) and you only need to design one type of controller (as long as it meets the specification), but be sure to explain why you chose your controller, and include appropriate plots or calculations showing that all specifications are met. For the closed loop system, determine the steady-state error in response to a step input and the maximum frequency for which the closed loop system can track with less than 25% error.

   (a) Disk drive read head positioning system:

   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}$$

   (b) Drug administration/compartment model (AM08, Section 3.6):

   $$P(s) = \frac{1.5s + 0.75}{s^2 + 0.7s + 0.05}$$

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CDS 110a

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2. Consider a first-order system with a PI controller given by

   $$P(s) = \frac{b}{s+a} \qquad C(s) = k_p\left(1 + \frac{1}{T_i s}\right).$$

   In this problem we will explore how varying the gains k_p and T_i affect the closed loop dynamics.

   (a) Suppose we want the closed loop system to have the characteristic polynomial

   $$s^2 + 2\zeta\omega_0 s + \omega_0^2.$$

   Derive a formula for k_p and T_i in terms of the parameters a, b, ζ and ω_0.

   (b) Suppose that we choose $a = 1$, $b = 1$ and choose ζ and ω_0 such that the closed loop poles of the system are at $\lambda = \{-20 \pm 10j\}$. Compute the resulting controller parameters k_p and T_i and plot the step and frequency responses for the system.

   (c) Using the process parameters from part (b) and holding T_i fixed, let k_p vary from 0 to ∞ (or something very large). Plot the location of the closed loop poles of the system as the gain varies. You should plot your results in two different ways:

   - A pair of plots showing the real and imaginary parts of the poles as a function of the gain k_p, similar to Figure 4.18a in the text.

- A parametric plot, showing the location of the eigenvalues on the complex plane, as k_p varies. Label the gains at which any interesting features in this plot occur. (This type of plot is called a *root locus* diagram.)

You may find it convenient to use the `subplot` command in MATLAB so that you can present all of your results in a single figure.

3. Consider a unity feedback control system with plant dynamics given by

$$P(s) = \frac{1}{(s+\alpha)^2}$$

If you use a computer for any of the following, you must explain all of your answers, and sketch all plots by hand. (For Part (c) you may find some calculations easier to do with the aid of a computer; you may use Matlab system objects and commands such as `margin` and `bode` to help you design your controller.)

(a) What is the smallest steady-state error to a unit step input that can be achieved using position feedback $C(s) = k_p$ while meeting a 60° phase margin requirement?
(Hint: it is straightforward to do this by hand; you may use a computer (e.g. with $\alpha = 1$) to help you think about the problem, but you need to show appropriate plots or calculations to explain your answer.)

(b) For $\alpha = 1$, design a controller that meets the following performance specification:

- Steady-state error to a unit step input is less than 1%.
- Tracking error of less than 26% up to 0.1 rad/sec.
- Phase margin of 45 degrees.

Sketch a Bode plot and use it to illustrate that the requirements are met.

(c) For $\alpha = 1$, design a controller that meets the requirements if the performance specification is changed so that a 1% tracking error is required at frequencies up to 0.1 rad/sec. Make sure that your controller is proper ($\lim_{s\to\infty} C(s)$ is finite), and that you explain why you chose all of your design parameters. You are not required to find the optimal design that meets all requirements with the smallest possible gain, however, points will be deducted if your design significantly exceeds requirements.

Estimate the maximum time delay that could be included in the feedback path before the closed-loop system would not be stable with your design.