CDS 110a: Midterm Review

Fall 2011

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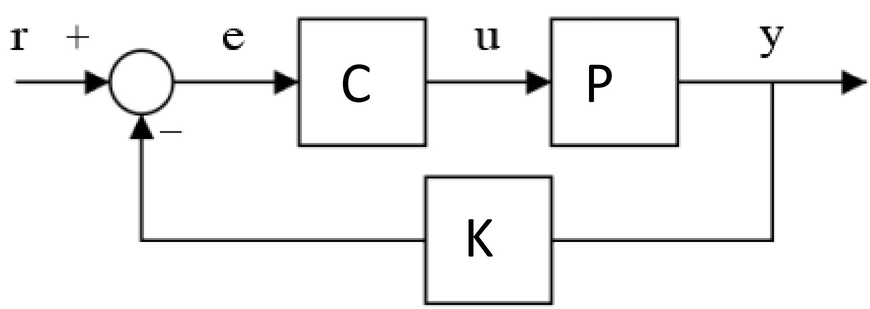
General Information

- Open Book and Notes
- No computer for symbolic manipulations
- No Internet
- MATLAB is not required, but can be used for numerical computations
- 2 exam's in 1
  - The start of each question shows if it is a 101 question, 110 question, or both.
- 2 hour limit for CDS 101, 3 hour limit for CDS 110

System Representations

- Block Diagram
- ODE
- State Space
- You should know how to convert between these forms. In particular higher order ODE's into state space.

Block Diagram Notation



ODE

Continuous Time

$$\dot{x}(t) = f(x(\dot{t}), u(t), d(t))$$
$$y(t) = h(x(t))$$

Discrete time

$$x(k) = f\big(x(k-1), u(k), d(k)\big)$$
$$y(k) = h(x(k), u(k))$$

These Systems are not equivalent, they can have very different properties.

State Space

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

- A = dynamics matrix
- B = control matrix
- C = sensor matrix
- D = direct matrix

Important Properties

- Autonomous Systems
  - No u(t) or d(t)
- Equilibrium Points in Continuous time
  - Find x_e such that $f(x_e) = 0$
- Stability of equilibrium points

Stability

- Lyapunov stability – "If you start close enough to the equilibrium, you remain close"
- Asymptotical Stability – "If you start close enough, you will eventually go to the equilibrium point"
- Unstable equilibrium Point – "There is at least one direction such that no matter how close you start to the equilibrium point, you will go far away from the equilibrium point"

Lyapunov Stability

Theory

$$\forall \varepsilon < \varepsilon_{max}: \exists \delta: \left||x(0) - x_e|\right| < \delta \Rightarrow \left||x(t) - x_e|\right| < \varepsilon$$

Practice

$$V(x) > 0 \; \forall \; x \neq 0$$
$$\dot{V}(x) < 0 \; \forall \; x \neq 0$$

Asymptotical Stability

Theory

$$\forall \varepsilon < \varepsilon_{max}: \left||x(0) - x_e|\right| < \varepsilon \Rightarrow \left||x(t) - x_e|\right| \to 0$$

Practice

Check the eigenvalues of the matrix

Checking Stability

- Linearization
  - Accurate
  - Can be hard to calculate

- Phase Portraits
  - Easy to see stability
  - May not be able to see all equilibrium points

Linearization

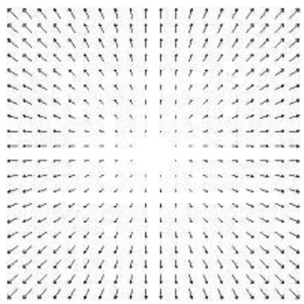
- $f(x, y)$
- $g(x, y)$

- Create the Jacobian matrix at **ALL** equilibrium point (x_e, y_e) the Jacobian may change depending on which equilibrium point you are looking at.

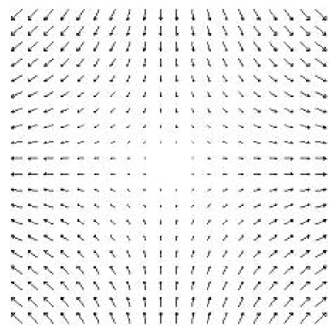
$$\begin{matrix} \frac{\delta f}{\delta x} & \frac{\delta f}{\delta y} \\ \frac{\delta g}{\delta x} & \frac{\delta g}{\delta y} \end{matrix} \text{ at } (x_e, y_e)$$

Eigenvalues, Stability and Phase Portraits

- All eigenvalues are positive
- Unstable



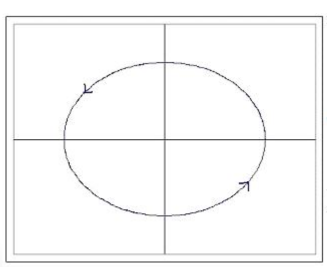
- Positive and Negative real parts for eigenvalues
- Unstable – Saddle Point



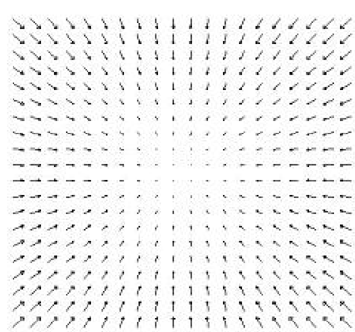
Eigenvalues, Stability and Phase Portraits

- Zero real part
- (Nonlinear Effects

can create stability or instability)

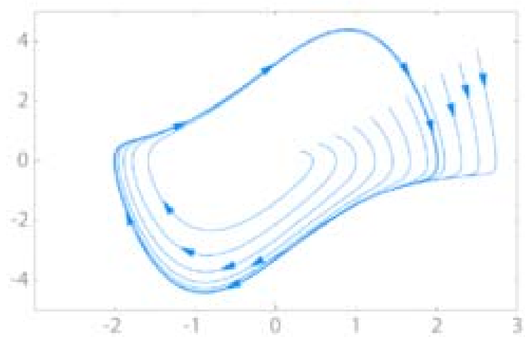


- All eigenvalues are negative
- Asymptotically stable

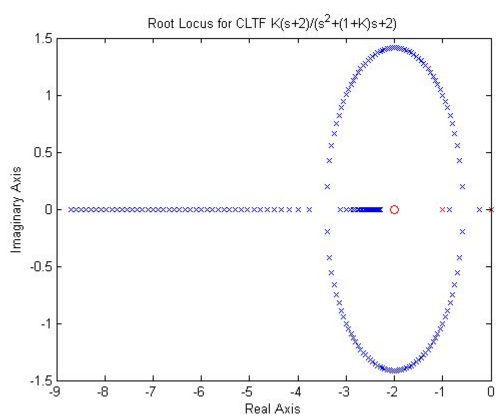


Limit cycles

- Only occur in non-linear systems
- Can not be seen directly though the eigenvalues

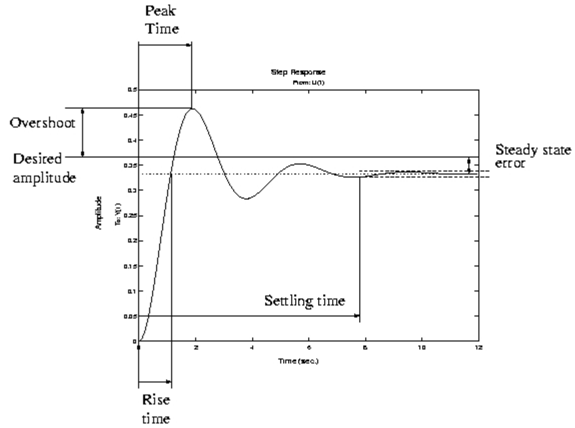


Root Locus



Step Response

- Rise Time
- Settling Time



Reachability

- A system is reachable if the matrix $W_R$is invertible.
- W_R = [B AB … A^{n-1}B]
- Check to see if W_R is full rank
- If a system is reachable it can be stabilized

Observability

- A system is observable if the matrix Q is invertible.
- Q =

[C
CA
…
CA^{n-1}]

- Check to see if Q is full rank

- If a system is observable, it is possible to determine the behavior of the entire system form the system's outputs.

Pole Placement

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

Using u = -K*x

$$\frac{dx}{dt} = (A - BK)x = \tilde{A}x$$

Choose u = -Kx+kr*r

With kr = -1/(C*(A–B∗K)$^{-1}$∗ B)
Ensures that y = r at the equilibrium