



CDS 101/110a: Lecture 8-2 Limits on Performance

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Goals:

- Describe limits of performance on feedback systems
- Introduce Bode’s integral formula and the “waterbed” effect
- Show some of the limitations of feedback due to RHP poles and zeros

Reading:

- Åström and Murray, *Feedback Systems*, Ch 11

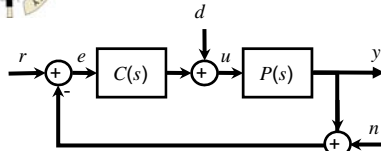
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Algebraic Constraints on Performance



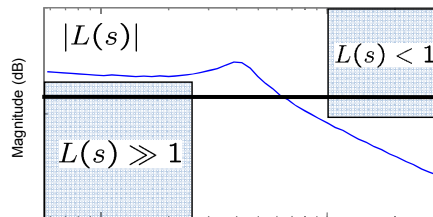
$$H_{er} = \frac{1}{1 + PC} =: S$$

Sensitivity function

$$H_{yn} = \frac{PC}{1 + PC} =: T$$

Complementary sensitivity function

- Goal: keep S & T small
 - S small \Rightarrow low tracking error
 - T small \Rightarrow good noise rejection (and robustness [CDS 110b])
- Problem: $S + T = 1$
 - Can’t make *both* S & T small at the same frequency
 - Solution: keep S small at low frequency and T small at high frequency
 - Loop gain interpretation: keep L large at low frequency, small at high freq.



- Transition between large gain and small gain complicated by stability (phase margin)

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Sensitivity

- From Rowley & Battin, *Fundamentals & Applications of Modern Flow Control*, Ch 5

• Example plotted is:

$$\frac{20}{(s+1)(s+2)(s+3)}$$

- Distance from -1 impacts performance as well as robustness

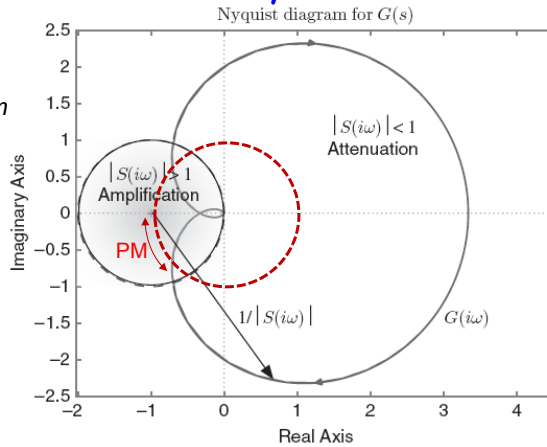


Fig. 4 Nyquist plot of the loop gain $G(s) = P(s)C(s)$ for the system (29). For frequencies for which $G(i\omega)$ enters the unit circle centered about the -1 point, disturbances are amplified and, for frequencies for which $G(s)$ lies outside this circle, disturbances are attenuated relative to open-loop.

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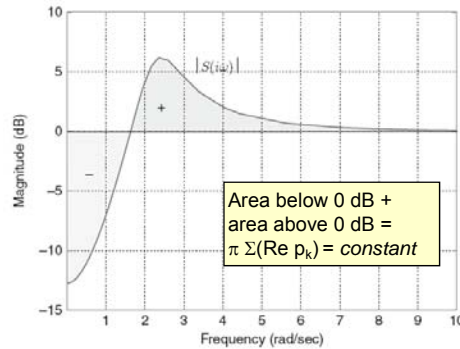
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Bode's Integral Formula: the Waterbed Effect

- Bode's integral formula for $S = 1/(1+PC) = 1/(1+L)$:
 - Let p_k be the *unstable* poles of $L(s)$ and assume relative degree of $L(s) \geq 2$
 - Theorem: the area under the sensitivity function is a conserved quantity:

$$\int_0^\infty \log_e |S(j\omega)| d\omega = \int_0^\infty \log_e \frac{1}{|1+L(j\omega)|} d\omega = \pi \sum \text{Re } p_k$$



Waterbed effect:

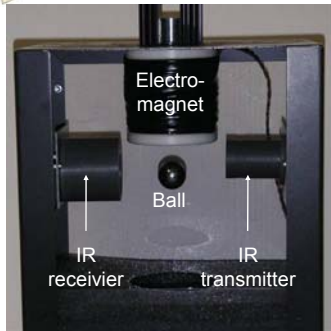
- Making sensitivity smaller over some frequency range requires *increase* in sensitivity someplace else
- Presence of RHP poles makes this effect worse
- Actuator bandwidth further limits what you can do
- Note: area formula is linear in ω ; Bode plots are logarithmic

Fig. 5 Magnitude of $S(i\omega)$, illustrating the area rule (31): for this system, the area of attenuation (denoted $-$) must equal the area of amplification (denoted $+$), no matter what controller $C(s)$ is used.

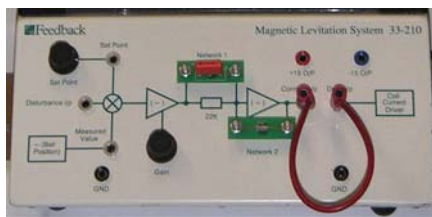
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Example: Magnetic Levitation



- System description
 - Ball levitated by electromagnet
 - Inputs: current thru electromagnet
 - Outputs: position of ball (from IR sensor)
 - States: z, \dot{z}
 - Dynamics: $F = ma, F = \text{magnetic force generated by wire coil}$



- Controller circuit
 - Active R/C filter network
 - Inputs: set point, disturbance, ball position
 - States: currents and voltages
 - Outputs: electromagnet current

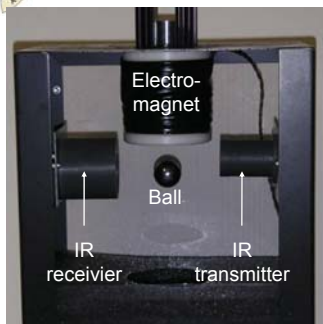
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Equations of Motion



- Process: actuation, sensing, dynamics

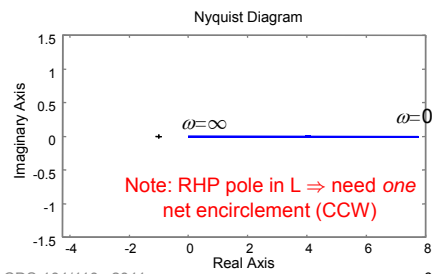
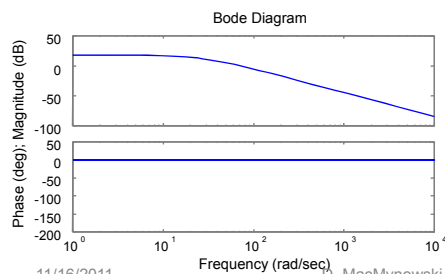
$$m\ddot{z} = mg - k_m(k_A u)^2 / z^2$$

$$v_{ir} = k_T z + v_0$$
 - $u = \text{current to electromagnet}$
 - $v_{ir} = \text{voltage from IR sensor}$

- Linearization:

$$P(s) = \frac{-k}{s^2 - r^2} \quad k, r > 0$$

- Poles at $s = \pm r \Rightarrow \text{open loop unstable}$



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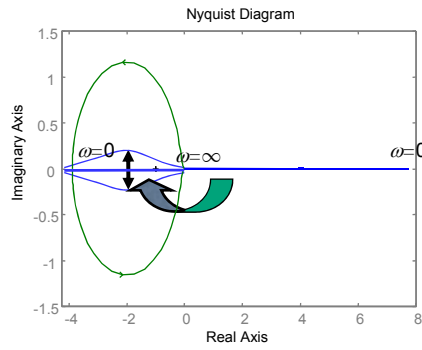
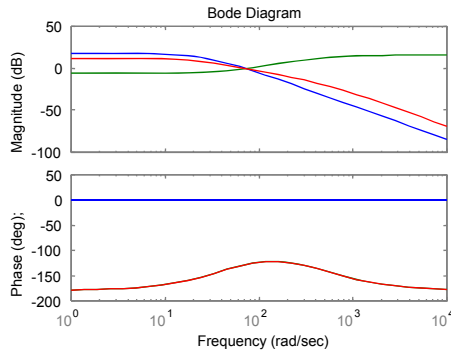
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Control Design

- Need to *create* encirclement
 - Loop shaping is not useful here
 - Flip gain to bring Nyquist plot over -1 point
 - Insert phase to create CCW encirclement
- Can accomplish using a lead compensator
 - Produce phase lead at crossover
 - Generates loop in Nyquist plot

$$C(s) = -k \frac{s + a}{s + b}$$



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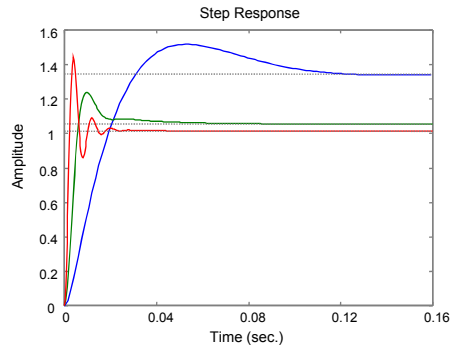
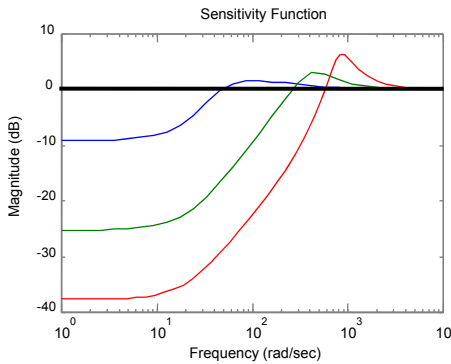
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Performance Limits

- Nominal design gives low perf
 - Not enough gain at low frequency
 - Try to adjust overall gain to improve low frequency response
 - Works well at moderate gain, but notice waterbed effect
- Bode integral limits improvement
 - Must increase sensitivity at some point

$$\int_0^\infty \log |S(j\omega)| d\omega = \pi r$$



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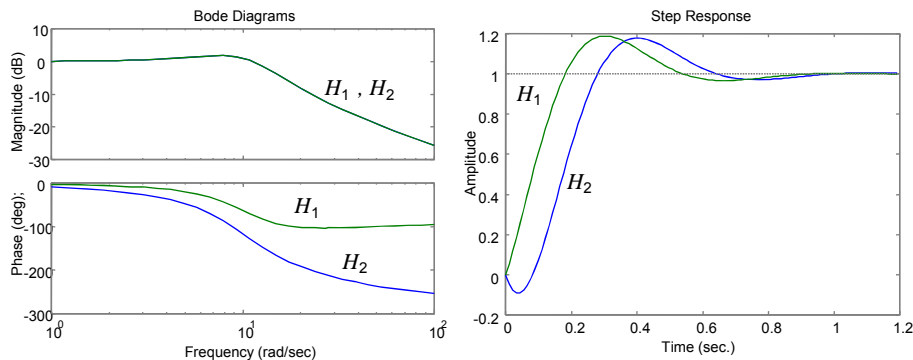


Right Half Plane Zeros

- Right half plane zeros produce “non-minimum phase” behaviour
 - Phase of frequency response has additional phase lag for given magnitude
 - Can cause output to move *opposite* from input for a short period of time

• Example: $H_1 = \frac{s + a}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ vs $H_2 = \frac{s - a}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$= H_1(s) \times \left(-\frac{s - a}{s + a} \right)$$



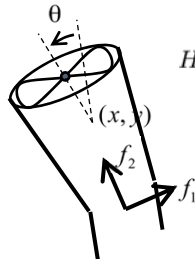
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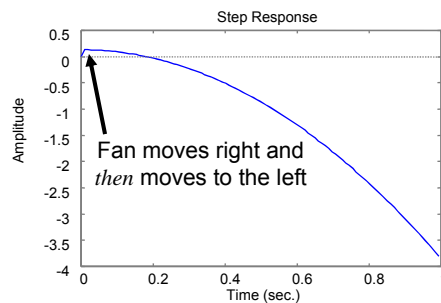
Example: Lateral Control of the Ducted Fan



$$H_{xf_1} = \frac{s^2 - mgl}{Js^2 + ds - mgl}$$

- Poles: 0, 0, $-\sigma \pm j\omega$
- Zeros: $\pm\sqrt{mgl}$

- Source of non-minimum phase behavior
 - To move left, need to make $\theta > 0$
 - To generate positive θ , need $f_1 > 0$
 - Positive f_1 causes fan to move *right* initially
 - Fan starts to move left after short time (as fan rotates)



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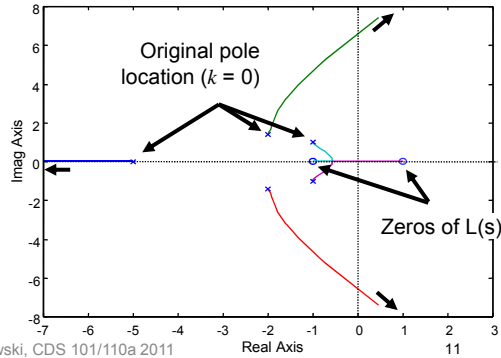


Stability in the Presence of Zeros

- Loop gain limitations
 - Poles of closed loop = zeroes of $1 + L$. Suppose $C = k n_c/d_c$, where k is the gain of the controller

$$1 + L = 1 + k \frac{n_c n_p}{d_c d_p} = 1 + k \frac{n}{d} = \frac{d + kn}{d}$$

- For large k , closed loop poles approach open loop zeros
 - RHP zeros limit maximum gain \Rightarrow serious design constraint!
- Root locus interpretation
 - Plot location of eigenvalues as a function of the loop gain k
 - Can show that closed loop poles go from open loop poles ($k = 0$) to open loop zeros ($k = \infty$)



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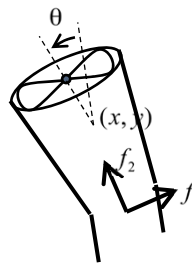
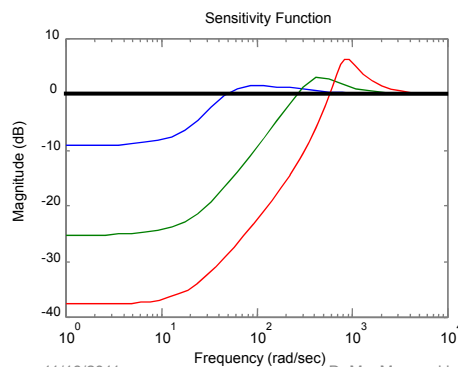


Summary: Limits of Performance

- Many limits to performance
 - Algebraic: $S + T = 1$
 - RHP poles: Bode integral formula
 - RHP zeros: Waterbed effect on peak of S

Main message: try to avoid RHP poles and zeros whenever possible (eg, re-design)

$$\int_0^{\infty} \log_e |S(j\omega)| d\omega = \int_0^{\infty} \log_e \frac{1}{|1 + L(j\omega)|} d\omega = \pi \sum \text{Re } p_k$$



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