



CDS 101/110a: Lecture 8-1 Frequency Domain Design

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Goals:

- Describe canonical control design problem and standard performance measures
- Show how to use “loop shaping” to achieve a performance specification
- Work through a detailed example of a control design problem

Reading:

- Åström and Murray, *Feedback Systems*, Ch 11
- *Advanced*: Lewis, Chapter 12

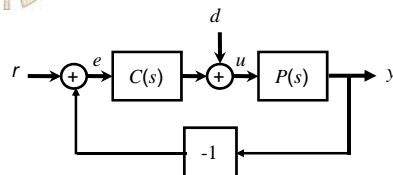
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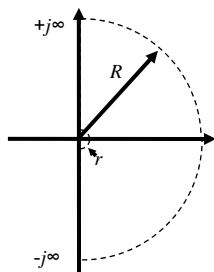
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Last week: Nyquist stability analysis



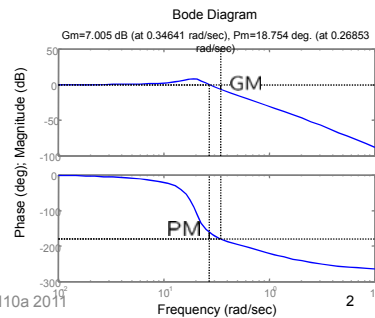
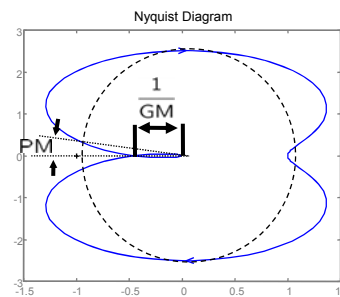
- Nyquist criteria for loop stability
- Gain, phase margin for robustness



Thm (Nyquist).
 P # RHP poles of $L(s)$
 N # CW encirclements
 Z # RHP zeros
 $Z = N + P$

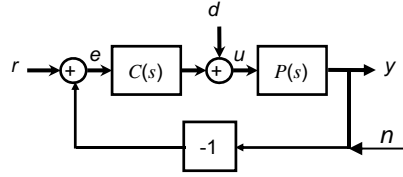
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Design based on loop transfer function



$$L(s) = P(s)C(s)$$

$$H_{er} = \frac{1}{1+L} \quad H_{yr} = \frac{L}{1+L}$$

$$H_{yd} = \frac{P}{1+L} \quad H_{yn} = \frac{-L}{1+L}$$

- Stability depends only on $L = PC$ (last week)
 - Robustness requires reasonable gain and phase margin
- Performance depends (mostly) on $L = PC$
 - When L is large, tracking performance and disturbance rejection is good
 - When L is small, sensor noise rejection is good, actuator response is small.
 - Typically care about the tracking and disturbance response at low frequencies
 - If gain or phase margin is small, tend to get large overshoot and ringing

- Definitions:

$$S(s) = \frac{1}{1+L} := \text{Sensitivity} \quad T(s) = \frac{L}{1+L} = \text{complementary sensitivity}$$

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Frequency Domain Performance Specifications

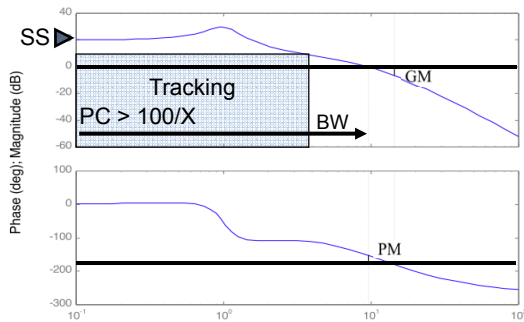
- Specify bounds on the loop transfer function to guarantee desired performance

– Steady state error: $H_{er}(0) = \frac{1}{(1+L(0))} \simeq 1/L(0)$

⇒ sets zero frequency (“DC”) gain ▶

- Tracking: Error less than X% up to frequency ω

⇒ Determines gain bound $|1+L| > 100/X$



- Bandwidth: assuming $\sim 90^\circ$ phase margin

$$\frac{L}{1+L}(i\omega_c) = \left| \frac{1}{1+i} \right| = \frac{1}{\sqrt{2}}$$

⇒ sets loop crossover frequency

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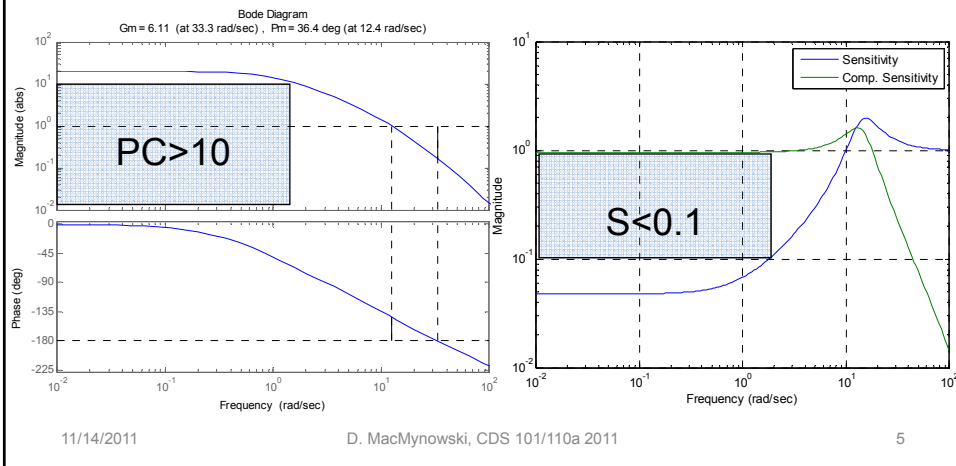
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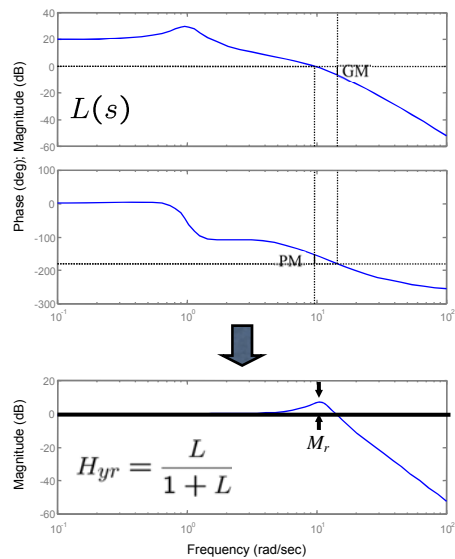
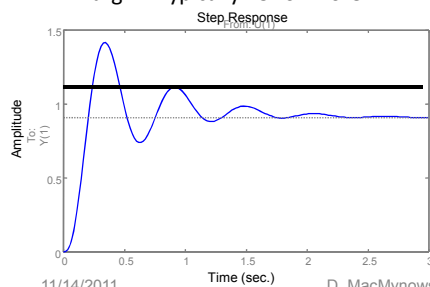
Example:

• Suppose $L(s) = \frac{20}{(s+1)(s/10+1)(s/100+1)}$



Relative Stability: Robustness and Performance

- Loop transfer function close to -1 gives poor robustness *and* performance:
 - System can be stable but still have bad response at certain frequencies
 - Typically occurs if system has low phase margin \Rightarrow get resonant peak in closed loop \Rightarrow overshoot; poor step response
 - Solution: specify minimum phase margin. Typically 45° or more





Summary of specifications

- Steady-state tracking error $< X$ $|L(0)| > 1/X$
- Tracking error $< Y$ up to frequency f_t Hz $|L(i\omega)| > 1/Y$ for $\omega < 2\pi f_t$
- Bandwidth of ω_b rad/sec Roughly, $|L(i\omega_b)| = 1$
 - Strictly, need to specify whether this means $|T| = -3\text{dB}$ or $|S| = -3\text{dB}$
- Overshoot $< Z$ Phase margin $>$
 - Can evaluate after design and confirm
- May specify phase and gain margins directly
 - Typically gain margin of 2 (6dB)
 - Typically phase margin 30-60 degrees

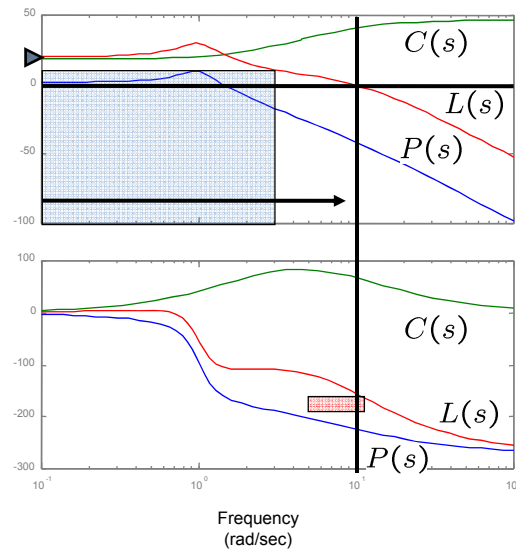
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Graphical Overview of Loop Shaping



• Performance specification

- ▶ Steady state error
- ▣ Tracking error
- ➔ Bandwidth
- ▣ Relative stability

• Approach: “shape” loop transfer function using $C(s)$

– $P(s)$ + specifications given

– $L(s) = P(s) C(s)$

- Use $C(s)$ to choose desired shape for $L(s)$

– Important: can't set gain and phase independently

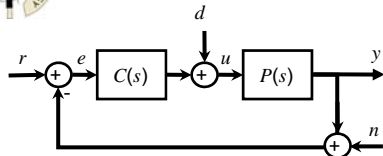
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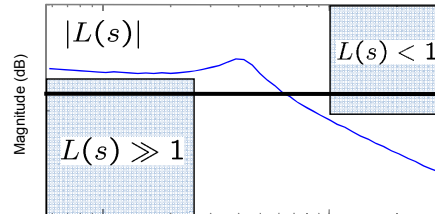
Algebraic Constraints on Performance



$$H_{er} = \frac{1}{1 + PC} =: S \quad \text{Sensitivity function}$$

$$H_{yn} = \frac{PC}{1 + PC} =: T \quad \text{Complementary sensitivity function}$$

- Goal: keep S & T small
 - S small \Rightarrow low tracking error
 - T small \Rightarrow good noise rejection (and robustness [CDS 110b])
- Problem: $S + T = 1$
 - Can't make *both* S & T small at the same frequency
 - Solution: keep S small at low frequency and T small at high frequency
 - Loop gain interpretation: keep L large at low frequency, small at high freq.



- Transition between large gain and small gain complicated by stability (phase margin)

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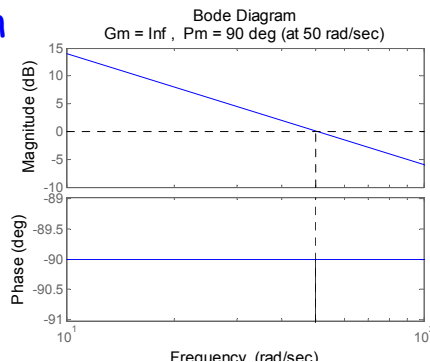
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Process Inversion

- Simple trick: invert out process
 - Write all performance specs in terms of the desired loop transfer function
 - Choose $L(s)$ to satisfies specifications
 - Choose controller by inverting $P(s)$
$$C(s) = L(s)/P(s)$$
- Pros
 - Very easy design process
 - $L(s) = k/s$ often works very well
 - Can be used as a first cut, with additional shaping to tune design
- Cons
 - High order controllers (at least same order as the process you are controlling)
 - Requires "perfect" model of your process (since you are inverting it)
 - *Does not work if you have right half plane poles or zeros (not internal stable)*



$$S = \frac{1}{1 + PC} \quad T = \frac{PC}{1 + PC} \quad PS = \frac{P}{1 + PC} \quad CS = \frac{C}{1 + PC}$$

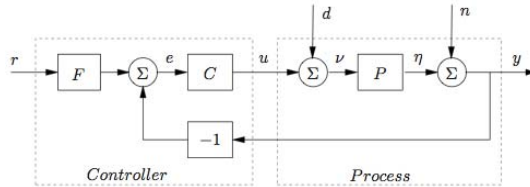
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Aside: Two Degree of Freedom Design and "Gang of four" transfer functions



• For $F=1$:

$$S = \frac{1}{1 + PC} \quad \text{Sensitivity function}$$

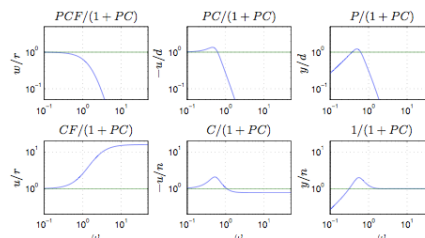
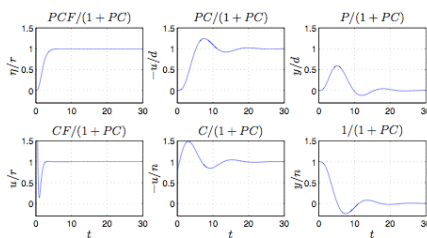
$$T = \frac{PC}{1 + PC} \quad \text{Complementary sensitivity}$$

$$PS = \frac{P}{1 + PC} \quad \text{Load sensitivity}$$

$$CS = \frac{C}{1 + PC} \quad \text{Noise sensitivity}$$

Typical design procedure

- Design C to balance all requirements
- Design F to improve response to reference



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(Better) Loop Shaping Design Tools

- Start with plant, and performance goals
 - If plant is not stable, find starting controller to stabilize; more on Wednesday...
- Increase gain to meet performance...
 - Include an integrator if you want zero steady-state error
- Is it stable? Do you have enough phase margin?
- Tools:
 - Reduce gain until adequate margin
 - **Lead compensator to add phase near crossover:** $K \frac{s+a}{s+b} \quad a < b$
 - Lag compensator to increase low frequency gain: $K \frac{s+a}{s+b} \quad a > b$
 - Proportional + Integral + Differential (PID), next week: $\frac{k_i}{s} + k_p + k_d s$
 - Same basic idea (find appropriate gain and phase to satisfy performance and robustness), different way of describing tools.

- **This is a design; there is no "right" answer!**

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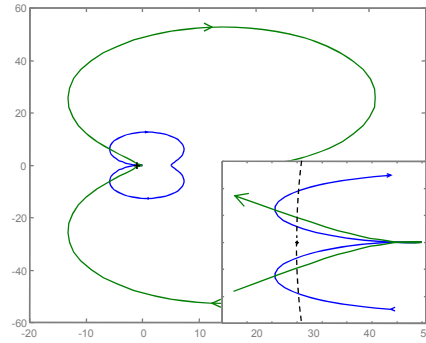
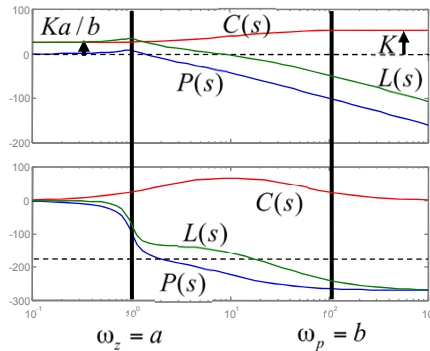
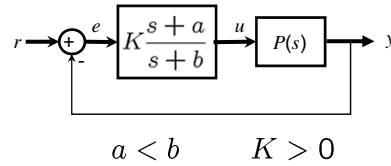
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Lead compensation

- Use to increase phase in frequency band
 - Effect: lifts phase by increasing gain at high frequency
 - Very useful controller; increases PM
 - Bode: add phase between zero and pole
 - Nyquist: increase phase margin



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Example: Control of Vectored Thrust Aircraft



- System description
 - Vector thrust engine attached to wing
 - Inputs: fan thrust, thrust angle (vectored)
 - Outputs: position and orientation
 - States: x, y, θ + derivatives
 - Dynamics: flight aerodynamics

Control approach

- Design “inner loop” control law to regulate pitch (θ) using thrust vectoring
- Second “outer loop” controller regulates the position and altitude by commanding the pitch and thrust
- Basically the same approach as aircraft control laws

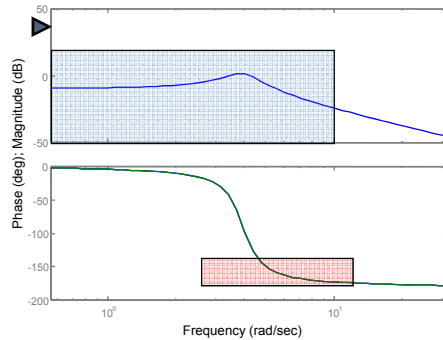
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Performance Specification and Design Approach



$$P(s) = \frac{r}{Js^2 + ds + mg\ell}$$

Performance Specification

- ≤ 1% steady state error
 - Zero frequency gain > 100
- ≤ 10% tracking error up to 10 rad/sec
 - Gain > 10 from 0-10 rad/sec
- ≥ 45° phase margin
 - Gives good relative stability
 - Provides robustness to uncertainty

Design approach

- If choose $C(s)=K$, then poor phase margin
- Add phase lead in 5-50 rad/sec range
- Increase the gain to achieve steady state and tracking performance specs

$$C(s) = K \frac{s+a}{s+b} \quad \begin{array}{l} a = 25 \\ b = 300 \\ K = 15 \times 300 \end{array}$$

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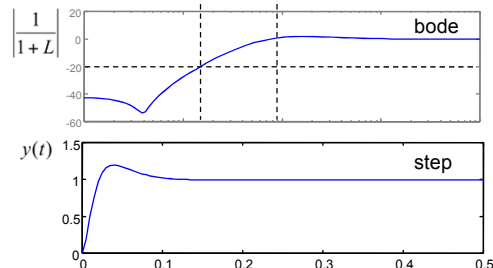
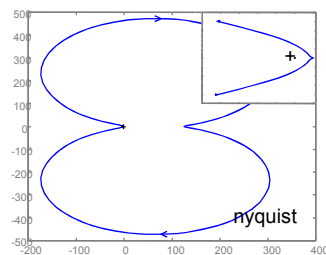
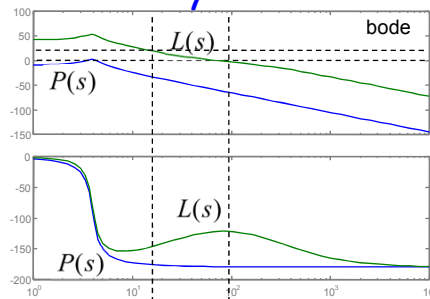
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Control Design and Analysis

- Select parameters to satisfy specs
 - Place phase lead in desired crossover region (given by desired BW)
 - Phase lead peaks at $\omega = \sqrt{ab}$
 - Maximum phase depends on pole/zero ratio:

$$\phi_{\max} = 90^\circ - 2 \tan^{-1} \sqrt{a/b}$$
 - Set gain as needed for tracking + BW
 - Verify controller using Nyquist plot, etc



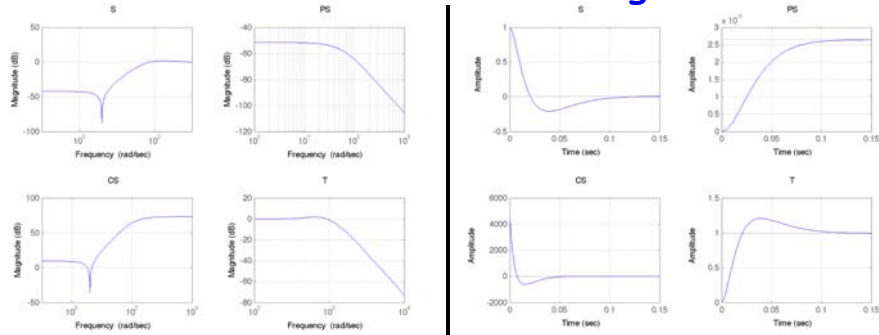
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Control Verification: Gang of 4



- Remarks
 - Check each transfer function to look for peaks, large magnitude, etc
 - Example: Noise sensitivity function (CS) has very high gain; step response verifies poor step response
 - Implication: controller amplifies noise at high frequency \Rightarrow will generate *lots* of motion of control actuators (flaps)
 - Fix: roll off the loop transfer function faster (high frequency pole)

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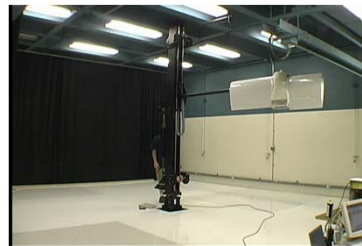
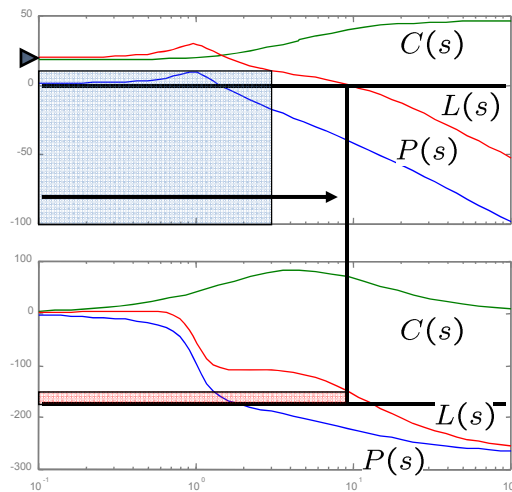


Summary: Loop Shaping

- Loop Shaping for Stability & Performance
 - Steady state error, bandwidth, tracking

Main ideas

- Performance specs give bounds on loop transfer function
- Use controller to shape response
- Gain/phase relationships constrain design approach
- Lead compensator useful to add phase



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