

CALIFORNIA INSTITUTE OF TECHNOLOGY  
Control and Dynamical Systems

CDS 101

D. G. MacMynowski  
Fall 2010

Problem Set #8

Issued: 22 Nov 10  
Due: 1 Dec 10\*

**\*NOTE: This already includes an automatic free grace period; we cannot accomodate additional grace period beyond this and grade homework in time.**

**Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).**

1. For the control systems below, design a P, PI, PD or PID control law that stabilizes the system, gives less than 1% error at zero frequency and gives at least 30° phase margin. You may use any method (loop shaping, Ziegler–Nichols, eigenvalue assignment, etc) and you only need to design one type of controller (as long as it meets the specification), but be sure to explain why you chose your controller, and include appropriate plots or calculations showing that all specifications are met. For the closed loop system, determine the steady-state error in response to a step input and the maximum frequency for which the closed loop system can track with less than 25% error.

- (a) Disk drive read head positioning system:

$$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}$$

- (b) Drug administration/compartment model (AM08, Section 3.6):

$$P(s) = \frac{1.5s + 0.75}{s^2 + 0.7s + 0.05}$$

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2. Consider a first-order system with a PI controller given by

$$P(s) = \frac{b}{s + a} \quad C(s) = k_p \left( 1 + \frac{1}{T_i s} \right).$$

In this problem we will explore how varying the gains  $k_p$  and  $T_i$  affect the closed loop dynamics.

- (a) Suppose we want the closed loop system to have the characteristic polynomial

$$s^2 + 2\zeta\omega_0 s + \omega_0^2.$$

Derive a formula for  $k_p$  and  $T_i$  in terms of the parameters  $a$ ,  $b$ ,  $\zeta$  and  $\omega_0$ .

- (b) Suppose that we choose  $a = 1$ ,  $b = 1$  and choose  $\zeta$  and  $\omega_0$  such that the closed loop poles of the system are at  $\lambda = \{-20 \pm 10j\}$ . Compute the resulting controller parameters  $k_p$  and  $T_i$  and plot the step and frequency responses for the system.
- (c) Using the process parameters from part (b) and holding  $T_i$  fixed, let  $k_p$  vary from 0 to  $\infty$  (or something very large). Plot the location of the closed loop poles of the system as the gain varies. You should plot your results in two different ways:

- A pair of plots showing the real and imaginary parts of the poles as a function of the gain  $k_p$ , similar to Figure 4.18a in the text.
- A parametric plot, showing the location of the eigenvalues on the complex plane, as  $k_p$  varies. Label the gains at which any interesting features in this plot occur. (This type of plot is called a *root locus* diagram.)

You may find it convenient to use the `subplot` command in MATLAB so that you can present all of your results in a single figure.