CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

D. G. MacMynowski
Fall 2010

Problem Set #4

Issued: 18 Oct 10
Due: 25 Oct 10

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Consider the normalized, linearized inverted pendulum which is described by

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u = Ax + Bu, \qquad y = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = Cx$$

Determine a state feedback and reference gain $u = -Kx + k_r r$ that gives a closed loop system with unit static gain (steady-state output $y = r$) and with the characteristic polynomial $s^2 + 2\zeta_0\omega_0 s + \omega_0^2$.

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CDS 110a

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   Determine a state feedback and reference gain $u = -Kx + k_r r$ that gives a closed loop system with unit static gain (steady-state output $y = r$) and with the characteristic polynomial $s^2 + 2\zeta_0\omega_0 s + \omega_0^2$.

2. Åström and Murray, Exercise 6.3

3. Åström and Murray, Exercise 6.10

   Assume that the A matrix is diagonalizable (the theorem is valid but hard to prove with a non-trivial Jordan form).

4. Åström and Murray, Exercise 6.12

   Download the file `bike_linmod.m` from the course web-site, which contains the parameters for the bicycle and generates the matrices M, C, K_0 and K_2 in Eq. (3.7) of the text.

   Find the controller gains corresponding to choosing the final pair of complex poles at $-1 \pm i$ as stated in the text, and also with these poles at $-2 \pm 2i$ and $-5 \pm 5i$. In addition to calculating the state feedback gains, solve for the reference gain k_r as well! When simulating the response to a step change in the steering reference of 0.002 rad, plot both the steering angle output δ and the torque command.