



CDS 101/110a: Lecture 7-1 Loop Analysis of Feedback Systems



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8 November 2010

Goals:

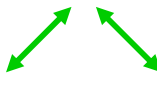
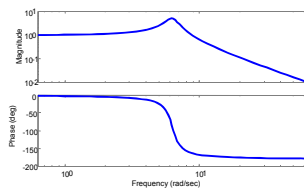
- Show how to compute closed loop stability from open loop properties
- Describe the Nyquist stability criterion for stability of feedback systems
- Define gain and phase margin and determine it from Nyquist and Bode plots

Reading:

- Åström and Murray, *Feedback Systems*, Ch 9.1-9.4
- *Advanced*: Lewis, Chapters 7

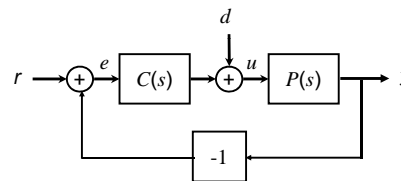
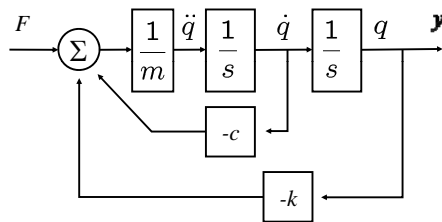
Review From Last Week

$$u = A \sin(\omega t) \rightarrow \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \\ x(0) = 0 \end{cases} \rightarrow y_{ss} = A \cdot |G(i\omega)| \times \sin(\omega t + \arg G(i\omega))$$

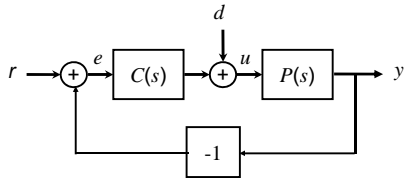


$$G(s) = C(sI - A)^{-1}B + D$$

$$G_{y_2 u_1} = G_{y_2 u_2} G_{y_1 u_1} = \frac{n_1 r_2}{d_1 d_2}$$



Closed Loop Stability



Q: how do open loop dynamics affect the closed loop stability?

- Given open loop transfer function $C(s)P(s)$ determine when system is stable

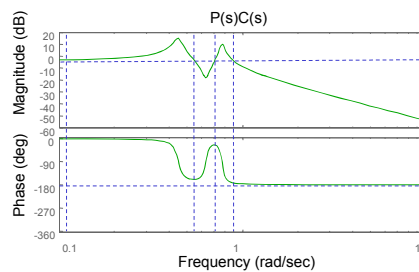
Brute force answer: compute poles of closed loop transfer function

$$H_{yr} = \frac{PC}{1 + PC} = \frac{n_p n_c}{d_p d_c + n_p n_c}$$

- Poles of H_{yr} = zeros of $1 + PC$
- Easy to compute, but not so good for design

Alternative: look for conditions on PC that lead to instability

- Example: if $PC(s) = -1$ for some $s = i\omega$, then system is *not* asymptotically stable
- Condition on PC is much nicer because we can *design* $PC(s)$ by choice of $C(s)$
- However, checking $PC(s) = -1$ is not enough; need more sophisticated check

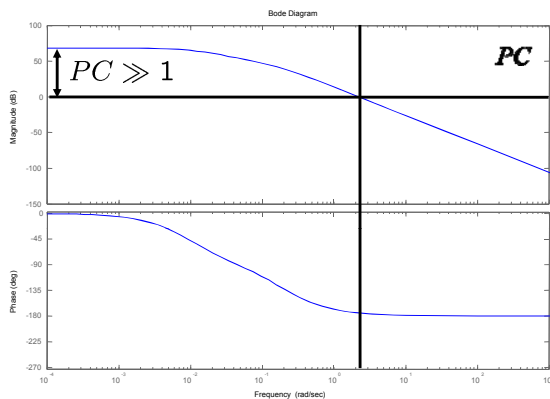


Game Plan: Frequency Domain Design

Goal: figure out how to *design* $C(s)$ so that $1 + C(s)P(s)$ is stable *and* we get good performance

$$H_{yr} = \frac{PC}{1 + PC}$$

- Poles of H_{yr} = zeros of $1 + PC$
- Would also like to “shape” H_{yr} to specify performance at different frequencies



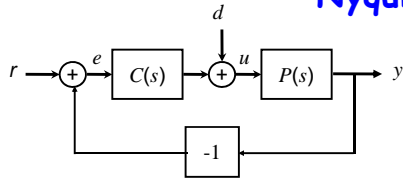
- Low frequency range:

$$PC \gg 1 \Rightarrow \frac{PC}{1 + PC} \simeq 1$$

(good tracking)

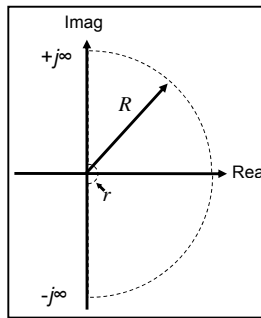
- Bandwidth: frequency at which closed loop response = $1/\sqrt{2}$
 \Rightarrow open loop gain ≈ 1
- Idea: use $C(s)$ to *shape* PC (under certain constraints)
- Need tools to analyze stability and performance for closed loop given PC

Nyquist Criterion



Determine stability from (open) loop transfer function, $L(s) = P(s)C(s)$.

- Use "principle of the argument" from complex variable theory (see reading)



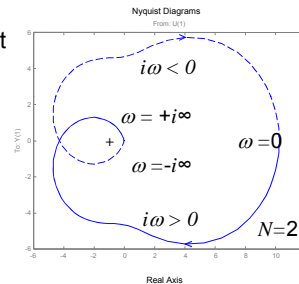
- Nyquist "D" contour
- Take limit as $r \rightarrow 0, R \rightarrow \infty$
- Trace from $-\infty$ to $+\infty$ along imaginary axis

Thm (Nyquist). Consider the Nyquist plot for loop transfer function $L(s)$. Let

- P # RHP poles of $L(s)$
- N # clockwise encirclements of -1
- Z # RHP zeros of $1 + L(s)$

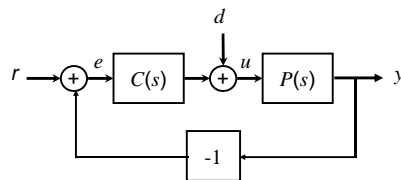
Then

$$Z = N + P$$



- Trace frequency response for $L(s)$ along the Nyquist "D" contour
- Count net # of clockwise encirclements of the -1 point

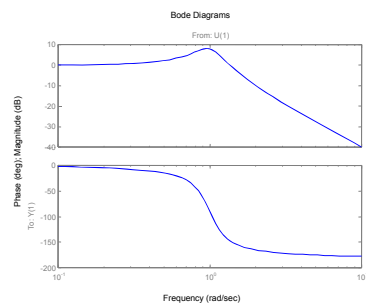
Simple Interpretation of Nyquist



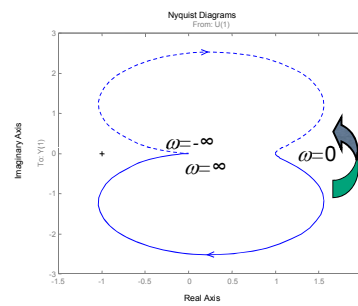
Basic idea: avoid positive feedback

- If $L(s)$ has 180° phase (or greater) and gain greater than 1, then signals are amplified around loop
- Use when phase is monotonic
- General case requires Nyquist

Can generate Nyquist plot from Bode plot + reflection around real axis



ambode(sys) [or bode(sys) in dB]

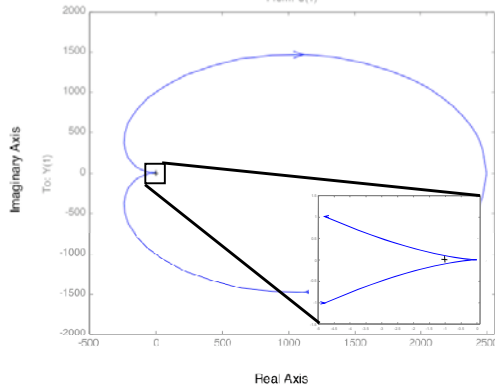


amnyquist(sys)

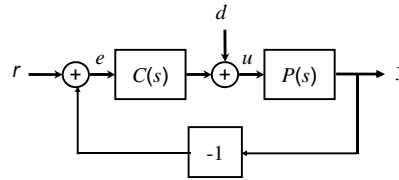
Example: Proportional + Integral* speed controller



Nyquist Diagrams
From: U(1)



* slightly modified; more on the design of this compensator in next week's lecture



$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

$$C(s) = K_p + \frac{K_i}{s + 0.01}$$

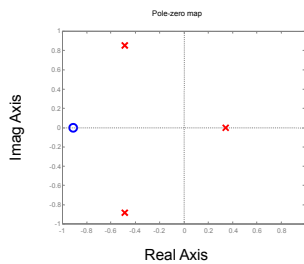
Remarks

- $N = 0, P = 0 \Rightarrow Z = 0$ (stable)
- Need to zoom in to make sure there are no net encirclements
- Note that we don't have to compute closed loop response

More complicated systems

What happens when open loop plant has RHP poles?

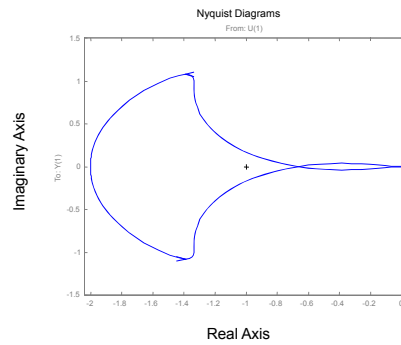
- $1 + PC$ has singularities inside D contour \Rightarrow these must be taken into account



$$L(s) = \frac{s + 1}{s - 0.5} \times \frac{1}{s^2 + s + 1}$$

unstable pole

$$\frac{1}{1 + L} = \frac{s + 1}{(s + 0.35)(s + 0.07 + 1.2i)(s + 0.07 - 1.2i)} \checkmark$$



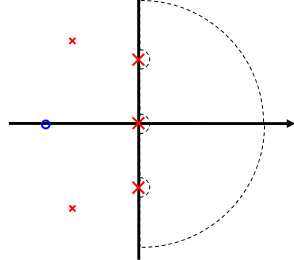
$$N = -1, P = 1 \Rightarrow Z = N + P = 0 \text{ (stable)}$$

Comments and cautions

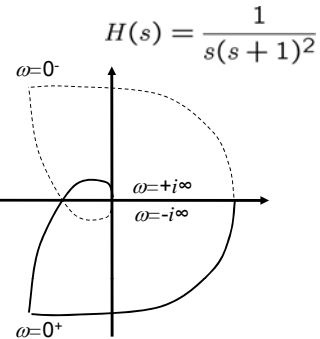
Why is the Nyquist plot *useful*?

- Old answer: easy way to compute stability (before computers and MATLAB)
- Real answer: gives *insight* into stability and robustness; very useful for reasoning about stability

Nyquist plots for systems with poles on the $j\omega$ axis



- chose contour to avoid poles on axis
- need to carefully compute Nyquist plot at these points
- evaluate $H(\epsilon+0i)$ to determine direction



Cautions with using MATLAB

- MATLAB doesn't generate portion of plot for poles on imaginary axis
- These must be drawn in by hand (make sure to get the orientation right!)

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Robust stability: gain and phase margins

Nyquist plot tells us if closed loop is stable, but not *how* stable

Gain margin

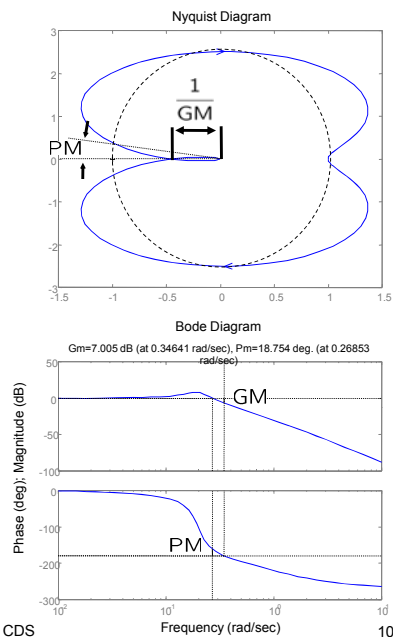
- How much we can modify the *loop gain* and still have the system be stable
- Determined by the location where the loop transfer function crosses 180° phase

Phase margin

- How much we can add "phase delay" and still have the system be stable
- Determined by the phase at which the loop transfer function has unity gain

Bode plot interpretation

- Look for gain = 1, 180° phase crossings
- MATLAB: `margin(sys)`

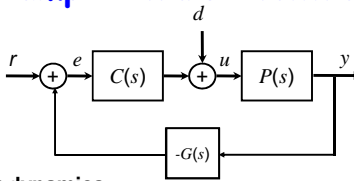


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Example: cruise control



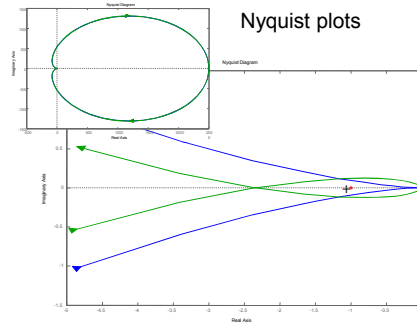
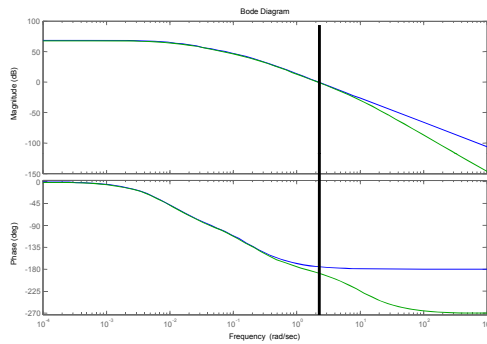
$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

$$C(s) = K_p + \frac{K_i}{s + 0.01}$$

$$G(s) = \frac{10}{s + 10}$$

Effect of additional sensor dynamics

- New speedometer has pole at $s = -10$ (very fast); problems develop in the field
- What's the problem? A: insufficient phase margin in original design (not robust)

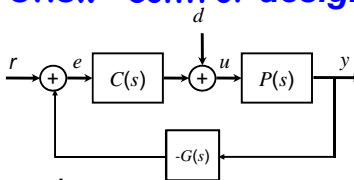


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Preview: control design



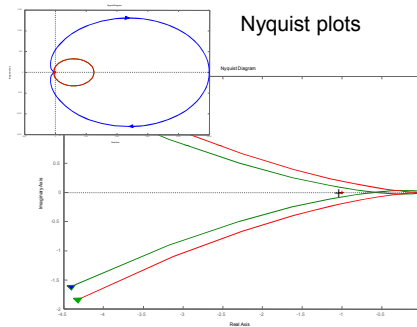
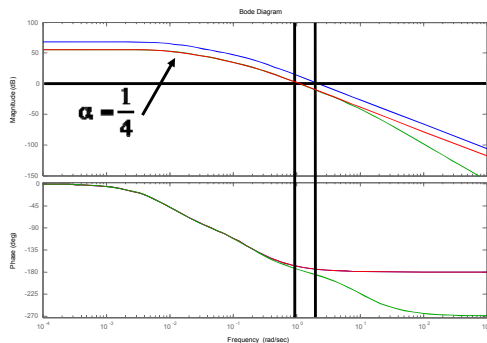
$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

$$C(s) = \alpha \left(K_p + \frac{K_i}{s + 0.01} \right)$$

$$G(s) = \frac{10}{s + 10}$$

Approach: Increase phase margin

- Increase phase margin by reducing gain \Rightarrow can accommodate new sensor dynamics
- Tradeoff: lower gain at low frequencies \Rightarrow less bandwidth, larger steady state error

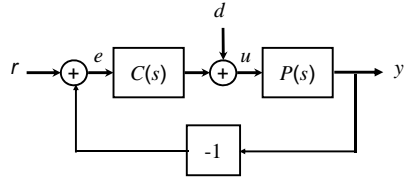


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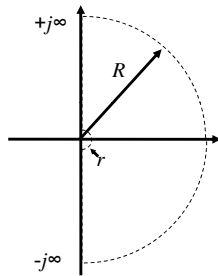
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Summary: Loop Analysis of Feedback Systems



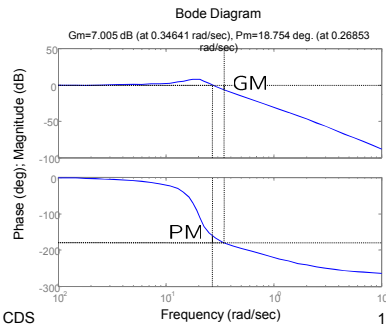
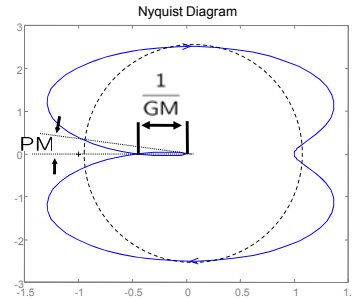
- Nyquist criteria for loop stability
- Gain, phase margin for robustness



Thm (Nyquist).

- P # RHP poles of $L(s)$
 N # CW encirclements
 Z # RHP zeros

$$Z = N + P$$



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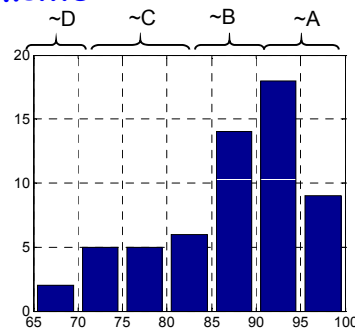
Announcements

Midterm solutions ??

Homework #5 due today at 5 pm

- Remember to put number of hours spent on back of first page

Homework #6 available on the web tonight



Computing your course grade so far:

$$\text{Grade} = \frac{5/9}{4} \times \sum_{i=1}^4 \left(\frac{\text{HW}_i}{\text{max}_i} \right) + (4/9) \times \frac{\text{MT}}{\text{max}_{\text{MT}}}$$

Please pick up copy of lecture notes

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