1. Åström and Murray, Exercise 1.4

2. Åström and Murray, Exercise 2.6, parts (a) and (b)
1. Åström and Murray, Exercise 1.4
2. Åström and Murray, Exercise 2.1
3. Consider a damped spring–mass system with dynamics
   \[ m\ddot{q} + c\dot{q} + kq = F. \]
   Let \( \omega_0 = \sqrt{k/m} \) be the natural frequency and \( \zeta = c/(2\sqrt{km}) \) be the damping ratio.
   (a) Show that by rescaling the equations, we can write the dynamics in the form
   \[
   \ddot{q} + 2\zeta\omega_0\dot{q} + \omega_0^2 q = \omega_0^2 u,
   \]
   where \( u = F/k \). This form of the dynamics is that of a linear oscillator with natural frequency \( \omega_0 \) and damping ratio \( \zeta \).
   (b) Show that the system can be further normalized and written in the form
   \[
   \frac{dz_1}{d\tau} = z_2, \quad \frac{dz_2}{d\tau} = -z_1 - 2\zeta z_2 + v.
   \]
   The essential dynamics of the system are governed by a single damping parameter \( \zeta \). The \( Q \)-value defined as \( Q = 1/2\zeta \) is sometimes used instead of \( \zeta \).
   (c) Show that the solution for the unforced system (\( v = 0 \)) with no damping (\( \zeta = 0 \)) is given by
   \[
   z_1(\tau) = z_1(0) \cos \tau + z_2(0) \sin \tau, \quad z_2(\tau) = -z_1(0) \sin \tau + z_2(0) \cos \tau.
   \]
   Invert the scaling relations to find the form of the solution \( q(t) \) in terms of \( q(0) \), \( \dot{q}(0) \) and \( \omega_0 \).
   (d) Consider the case where \( \zeta = 0 \) and \( u(t) = \sin \omega t, \omega > \omega_0 \). Solve for \( z_1(\tau) \), the normalized output of the oscillator, with initial conditions \( z_1(0) = z_2(0) = 0 \) and use this result to find the solution for \( q(t) \).
4. Read the queuing system described in Example 2.10. The long delays created by temporary overloads can be reduced by rejecting requests when the queue gets large. This allows requests that are accepted to be serviced quickly and requests that cannot be accommodated to receive a rejection quickly so that they can try another server.
   As in Example 2.10, the dynamics are given by:
   \[
   \frac{dx}{dt} = \lambda u - \mu_{\text{max}} \frac{x}{x + 1},
   \]
Consider an admission control strategy described by

\[ u = \text{sat}_{(0,1)}(k(r - x)), \]  

(S1.4)

where the controller is a simple proportional control with saturation (\(\text{sat}_{(a,b)}\) is defined by equation (3.9)) and \(r\) is the desired (reference) maximum queue length. Use a simulation to show that this controller reduces the rush-hour effect and explain how the choice of \(r\) affects the system dynamics.

You should choose the parameters of your simulation to match those in Example 2.10: \(\mu_{\text{max}} = 1, \lambda = 0.5\) at time 0, increasing to \(\lambda = 4\) at time 20 and returning to \(\lambda = 0.5\) at time 25. Test your controller using reference queue lengths of \(r = 2\) and \(r = 5\) and explore several different values for \(k\). (It is useful for intuition in choosing \(k\) to sketch the control \(u\) as a function of the queue length \(x\), noting that \(u = 0\) means all requests are rejected and \(u = 1\) means all requests are accepted.) Compare the queue length with the uncontrolled system in Figure 2.19b. Your solution should include the MATLAB code that you used plus plots for the final value of \(k\) you chose (and the two values of \(r\)). Make sure to label your plots and describe how your controller reduces the rush hour effect.

Notice that this server control problem is not a conventional regulation problem where we wish to hold the output (here the queue length) constant. The problem is instead to make sure that the queue length does not become too large when there are many service requests.