Halo Orbit Mission Correction Maneuvers Using Optimal Control

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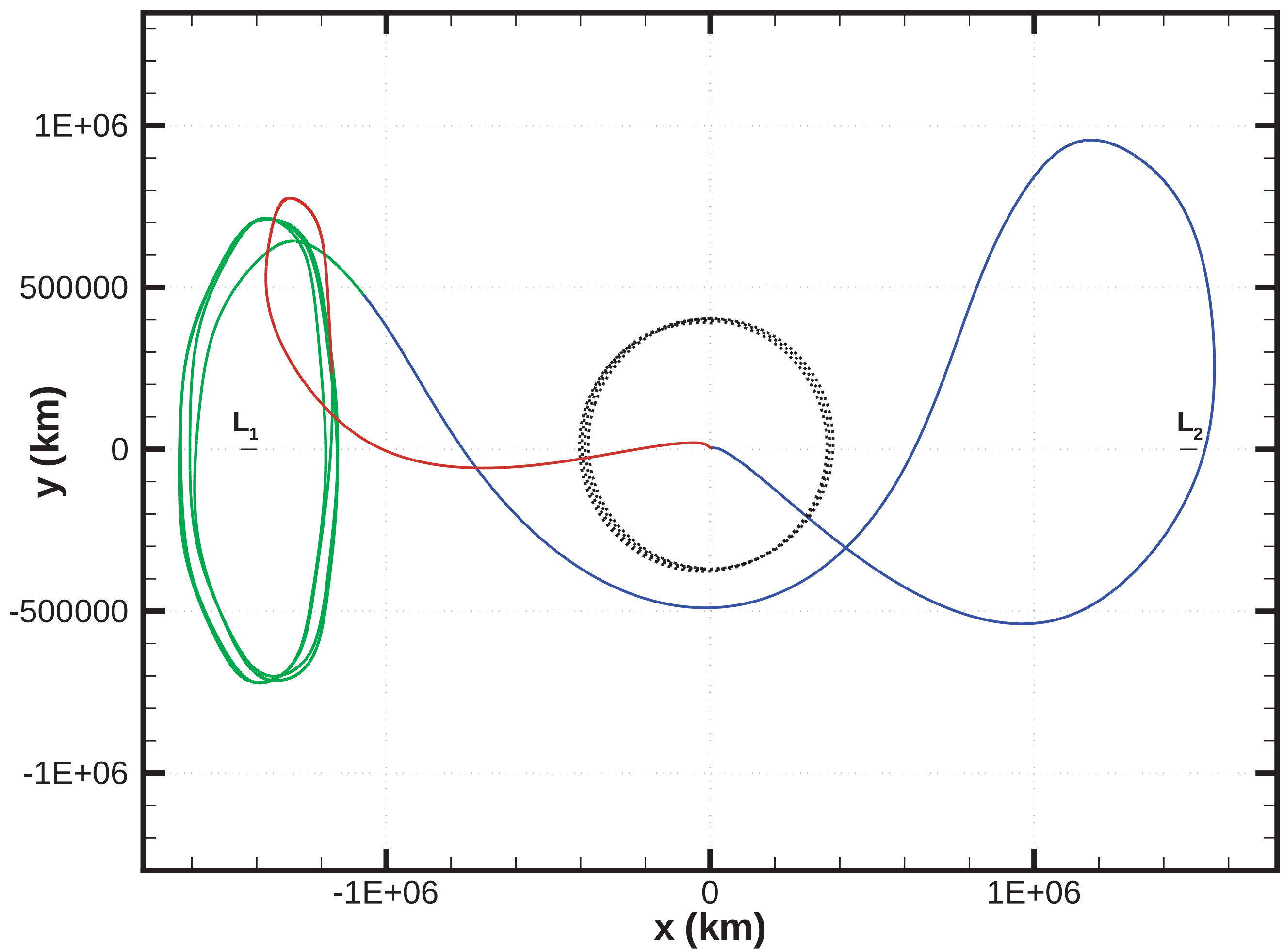
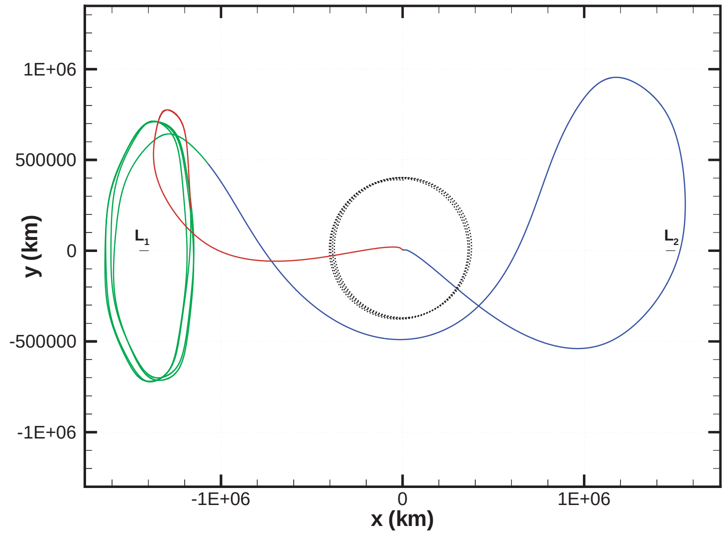
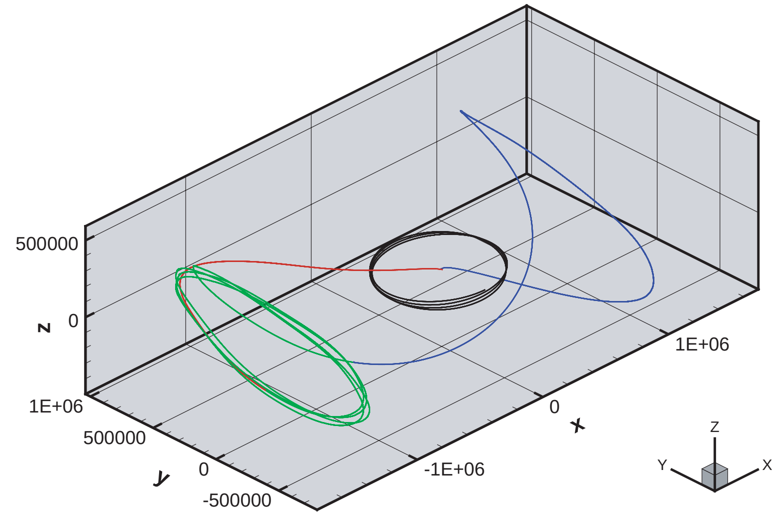
Genesis Discovery Mission

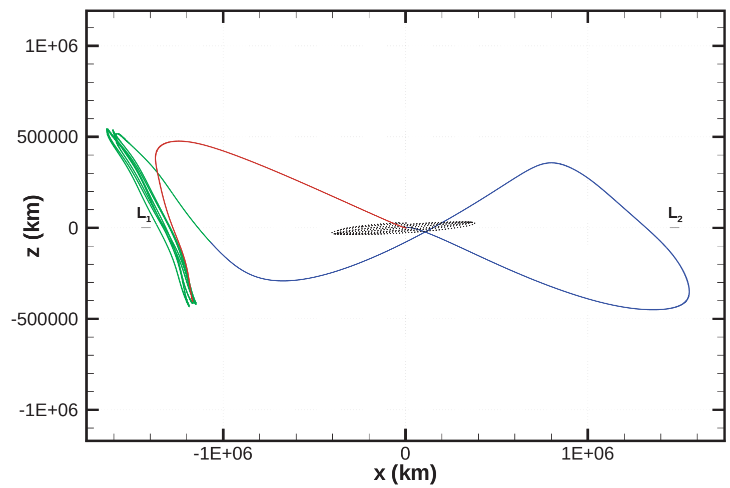
- Solar wind sample return mission.
- Show Genesis **halo orbit**, the **transfer** and **return** trajectories in a rotating frame.

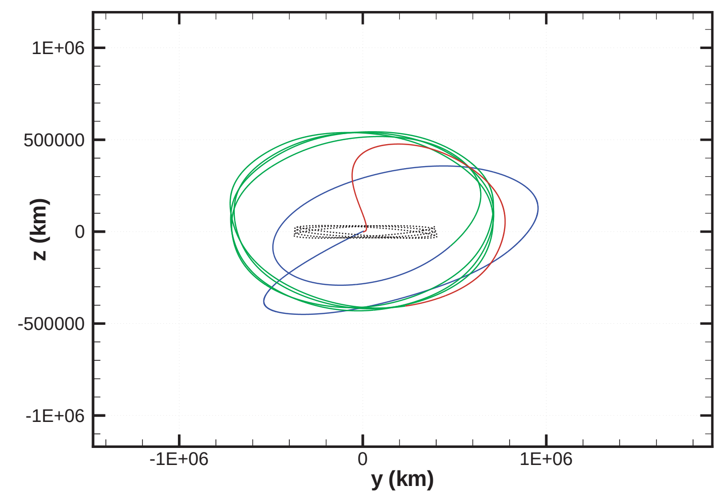


Genesis Discovery Mission



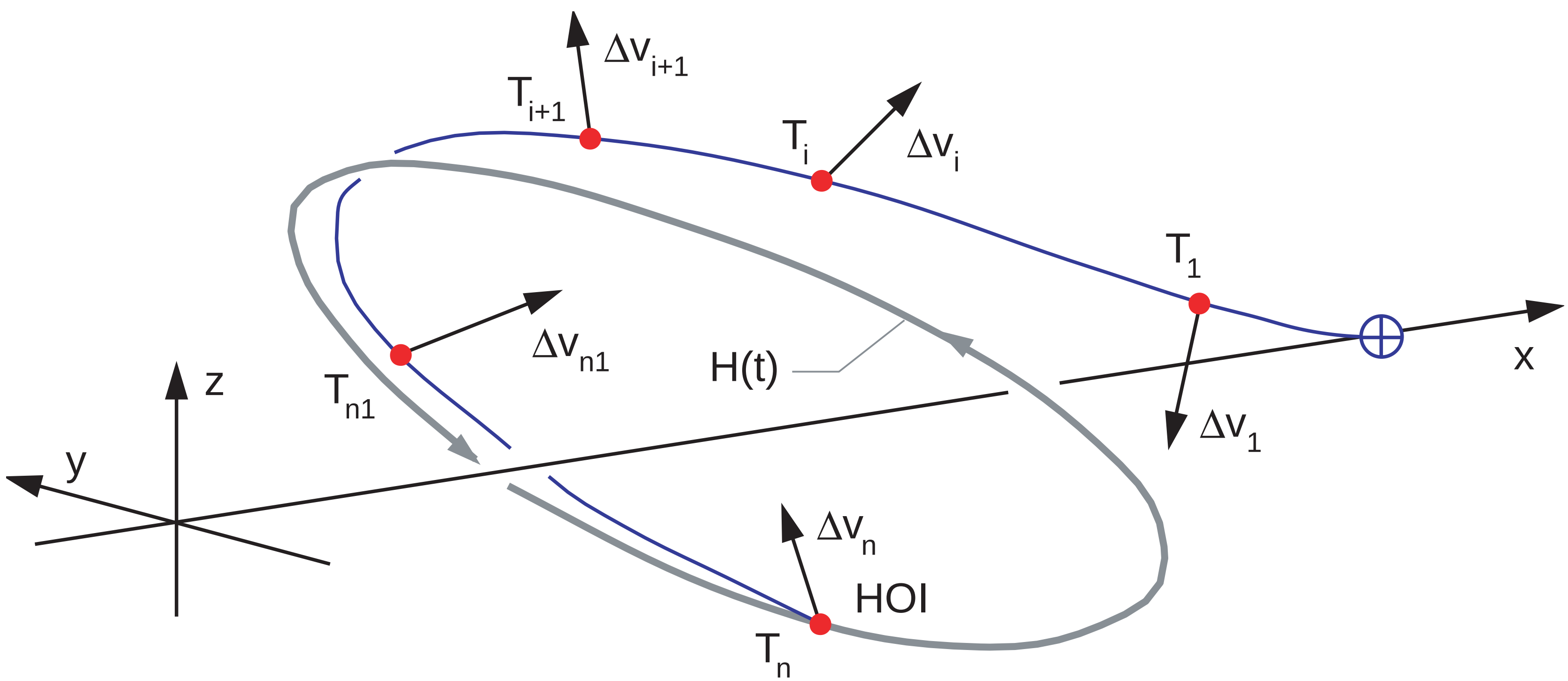






Trajectory Correction Maneuvers Problem (TCM)

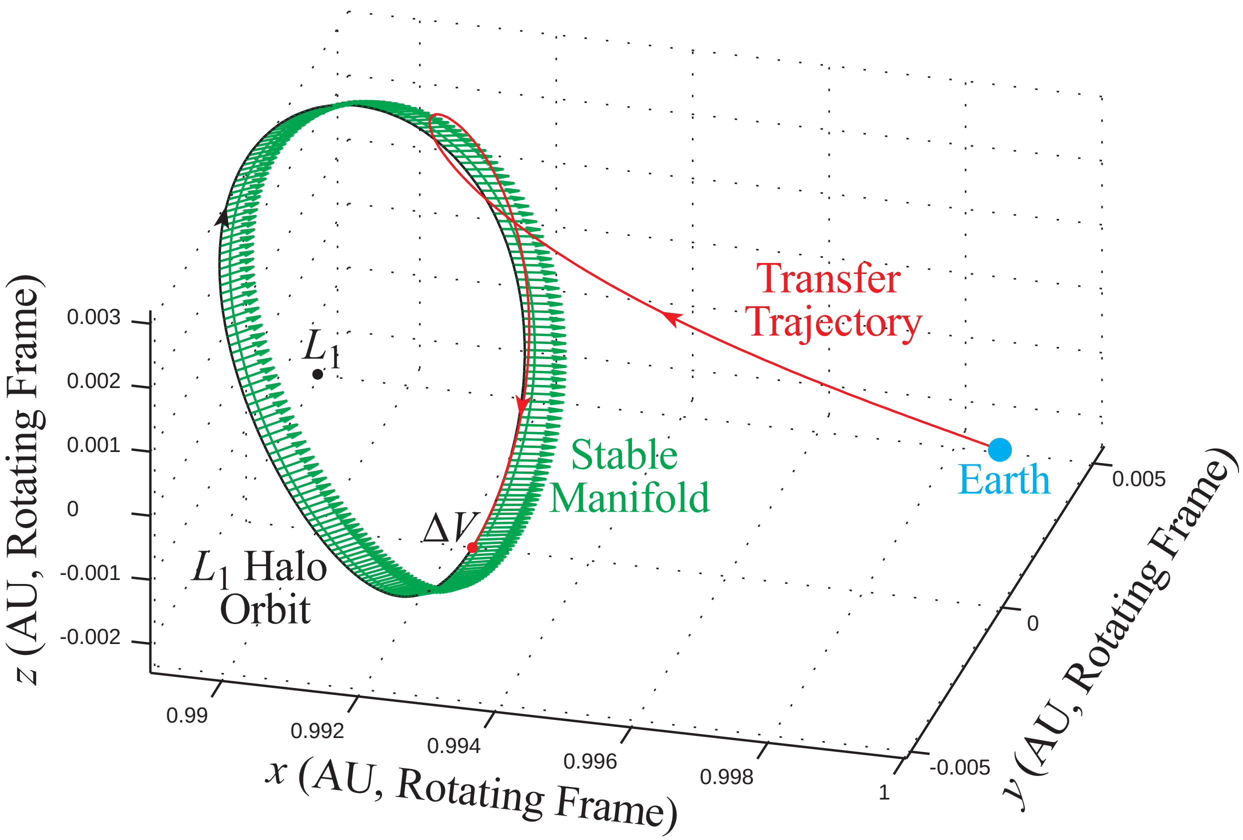
- Before checkout completed, TCM1 is difficult and risky. Genesis prefers TCM1 at **2-7 days** after launch.
- Beyond **1 day**, correction ΔV based on traditional **linear analysis** can become prohibitively high.
- The desire to delay TCM1 but to stay within ΔV budget drives us to use **nonlinear** approach.



Merge Optimal Control with Dynamical Systems

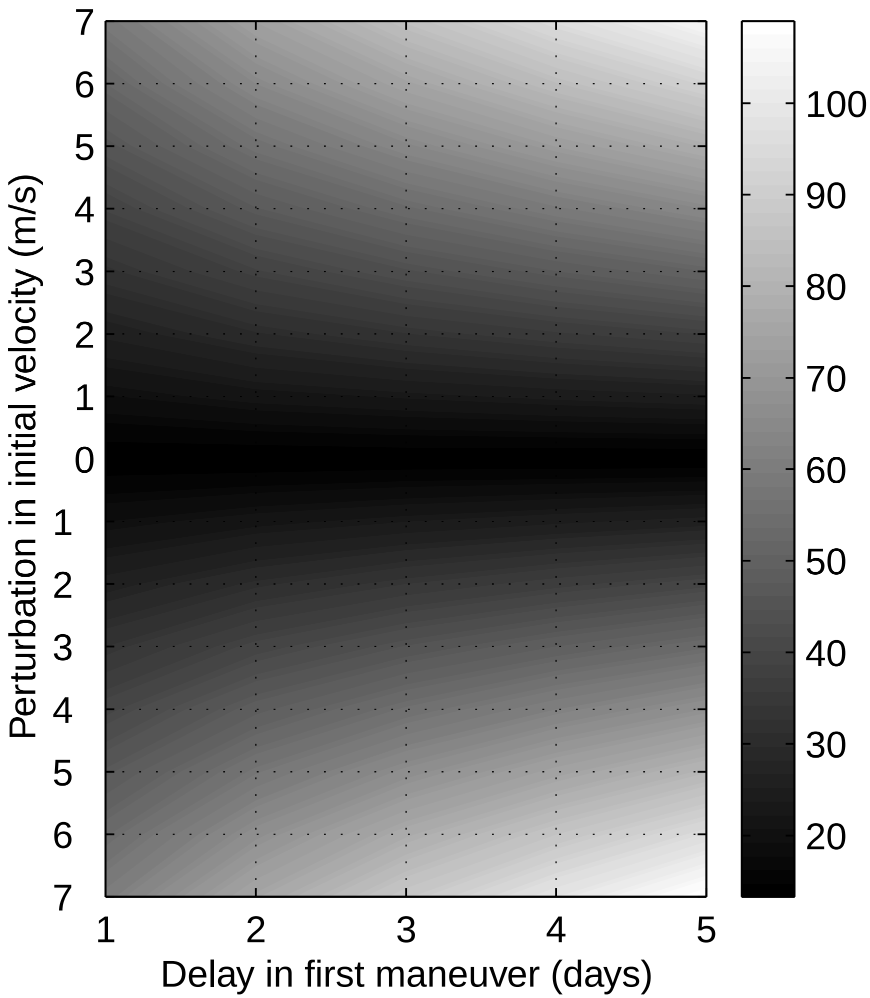
▶ 2 similar but different approaches were explored, based on merging **optimal control/dynamical systems**.

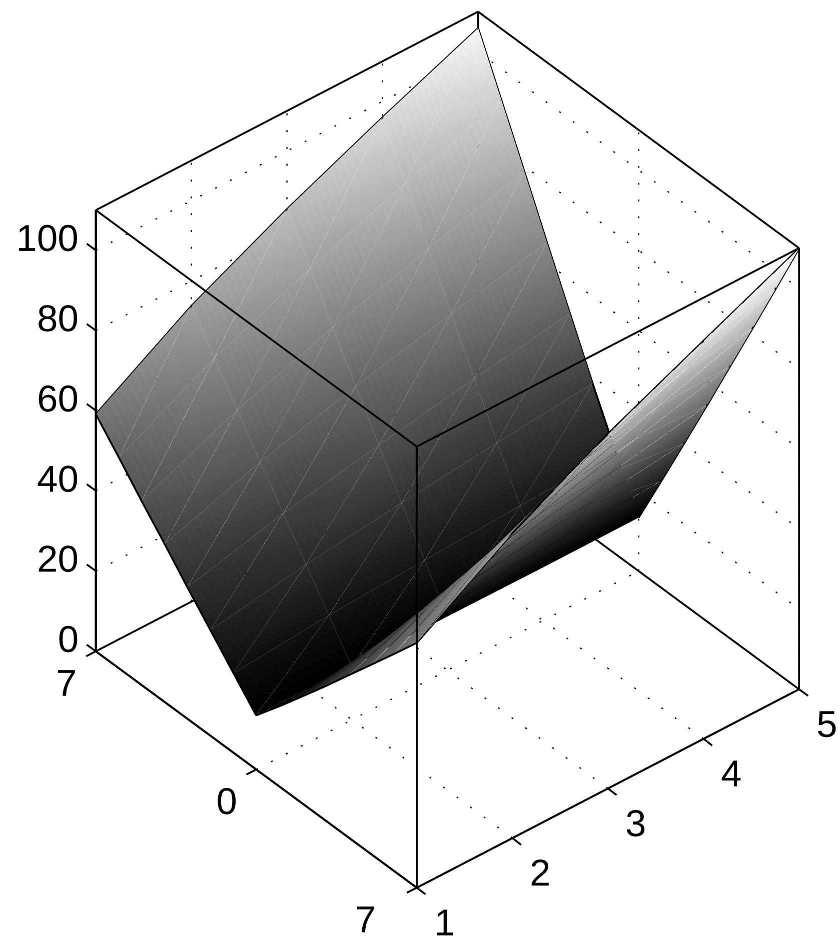
- Halo Orbit Insertion: re-target halo orbit with original nominal trajectory as **first guess**.
- Stable Manifold Insertion: target **stable manifold**.



Within Genesis ΔV Budget

- Obtain in both cases an optimal maneuver strategy, within Genesis ΔV budget of 150 m/s.





Optimal Maneuver Trajectory for HOI

Two Main Ideas

- Theoretically, **optimal control** is a favorite approach in **trajectory generation**.

$$\max \int C, \quad \text{with} \quad \dot{x} = f(x, u).$$

  - Resulting trajectories can be optimal in fuel consumption.
- But **numerically**, there exist many difficulties.
  - Existing numerical algorithms would not converge whenever underlying dynamics is sensitive.
- Tackle these from 2 fronts:
  - Explore "**direct method**" optimal control algorithms.
  - **Merge** optimal control with **dynamical systems**.

Direct vs. Indirect Method

- **Indirect Method**: equations derived by Calculus of Variations or Pontryagin Maximum Principle.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q} \qquad \text{where} \quad L = C + \lambda(\dot{x} - f(x,u)).$$

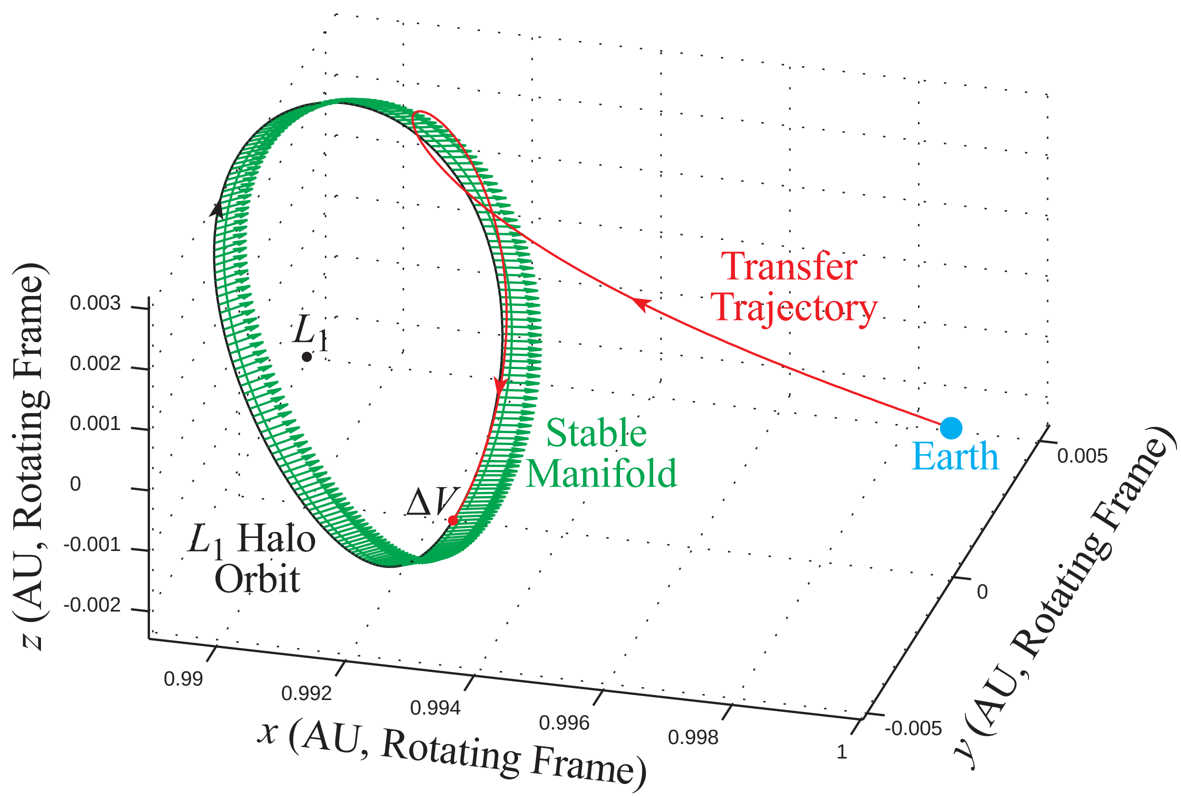
- Main drawbacks:
  - 2 point boundary value problem (numerically sensitive).
  - Need a good first guess. But it is difficult to guess λ.
- **Direct Method**:

$$\max \int C, \qquad \text{with} \quad \dot{x} = f(x,u)$$

  - approximated by a **discrete optimization** problem
  - solved by **SQP** (sequential quadratic programming) software.
- Resulting algorithm avoids many difficulties and is very robust.

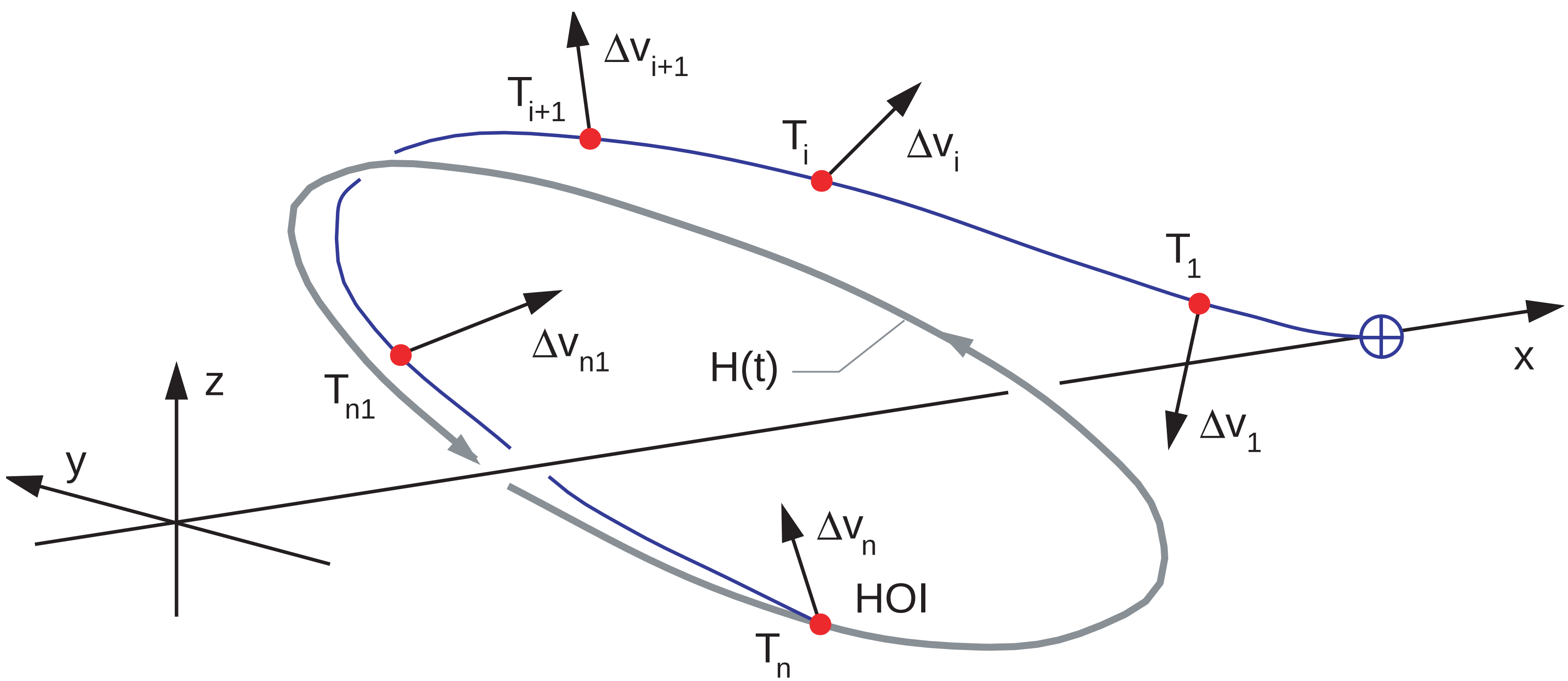
Merge Optimal Control with Dynamical Systems

- **Optimal control** techniques need to work together with **dynamical systems** tools.
  - Can help in constructing a superior **first guess**.
  - Suggest better formulation based on **geometry** of phase space.
  - Exploit **mechanical** nature of the problem.



Technical Details: HOI and MOI Techniques

- Both are **similar** once cast as optimal control problem.
  - **HOI**: final maneuver allowed at halo orbit at $T_{HOI} = t_{max}$.
  - **MOI**: final maneuver on stable manifold at $T_{MOI} < t_{max}$.
- Find **maneuver times** and **sizes** for an optimal trajectory starting near Earth and ending on the specified halo orbit such that **TCM1** is delayed by at least a prescribed amount.



Technical Details: Halo Orbit Insertion

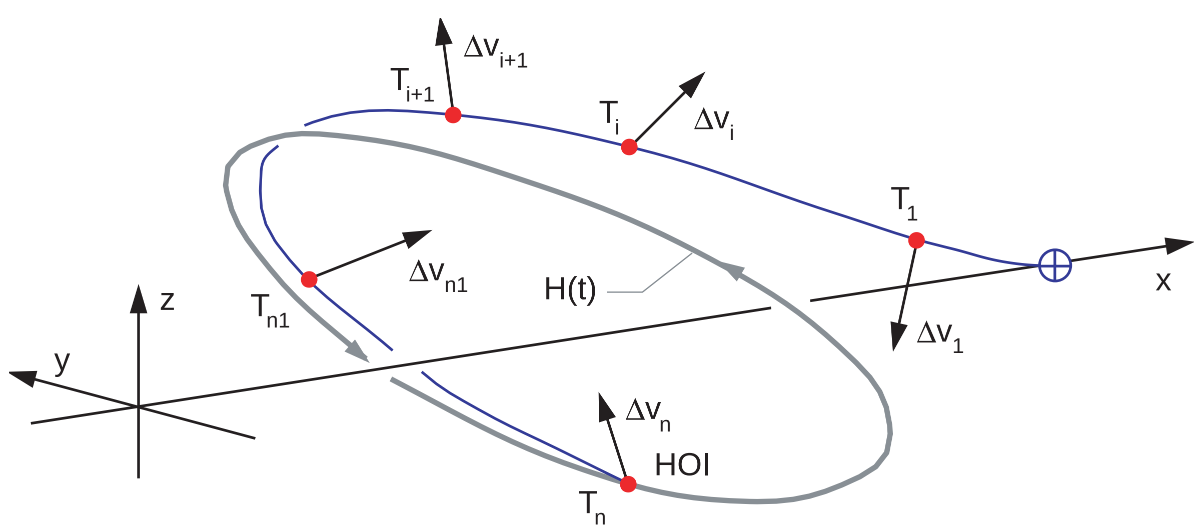
- Use Circular Restricted Three Body Problem as model

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}).$$

- To deal with **discontinuous** impulsive controls, equations solved simultaneously between 2 maneuvers.
  - Position **continuity** constraints at each maneuver,

$$\mathbf{x}_i^p(T_i) = \mathbf{x}_{i+1}^p(T_i), \qquad i = 1, 2, ..., n-1.$$

  - Final position is on halo orbit, $\mathbf{x}_n^p(T_n) = \mathbf{x}_H^p(T_n)$.



Technical Details: Halo Orbit Insertion

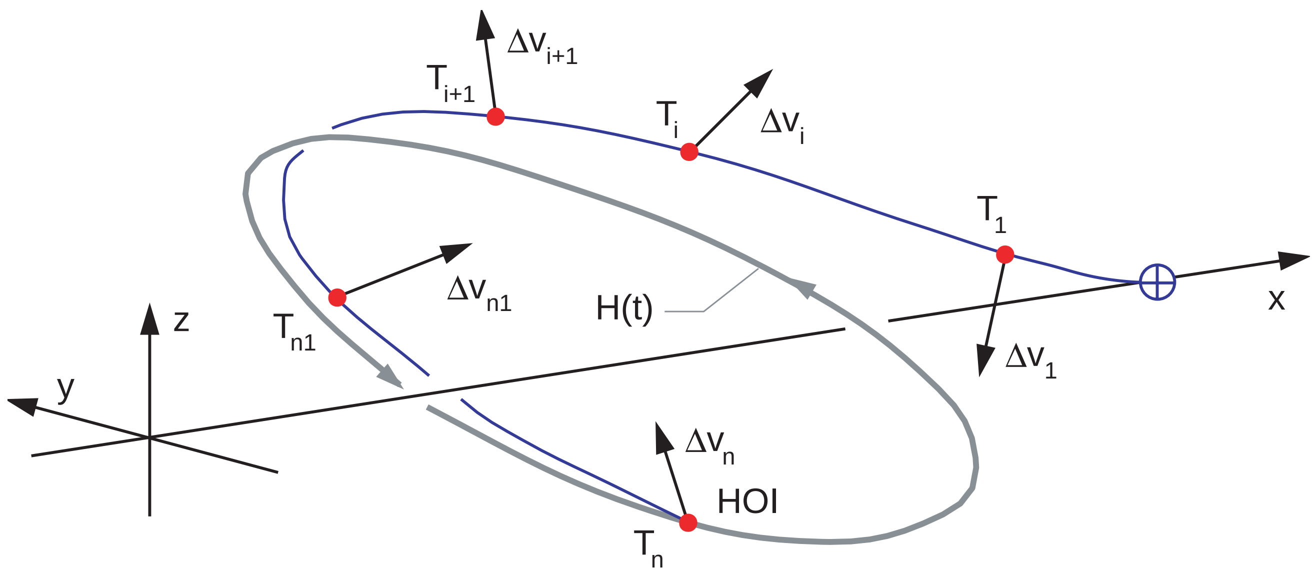
- TCM1 delayed by at least a prescribed amount: $T_1 \geq TCM1_{min}$.
- With cost function as some measure of control $\Delta V's$

$$\Delta \mathbf{v}_i = \mathbf{x}^v_{i+1}(T_i) - \mathbf{x}^v_i(T_i), \qquad \Delta \mathbf{v}_n = \mathbf{x}^v_H(T_n) - \mathbf{x}^v_n(T_n),$$

optimization problem becomes

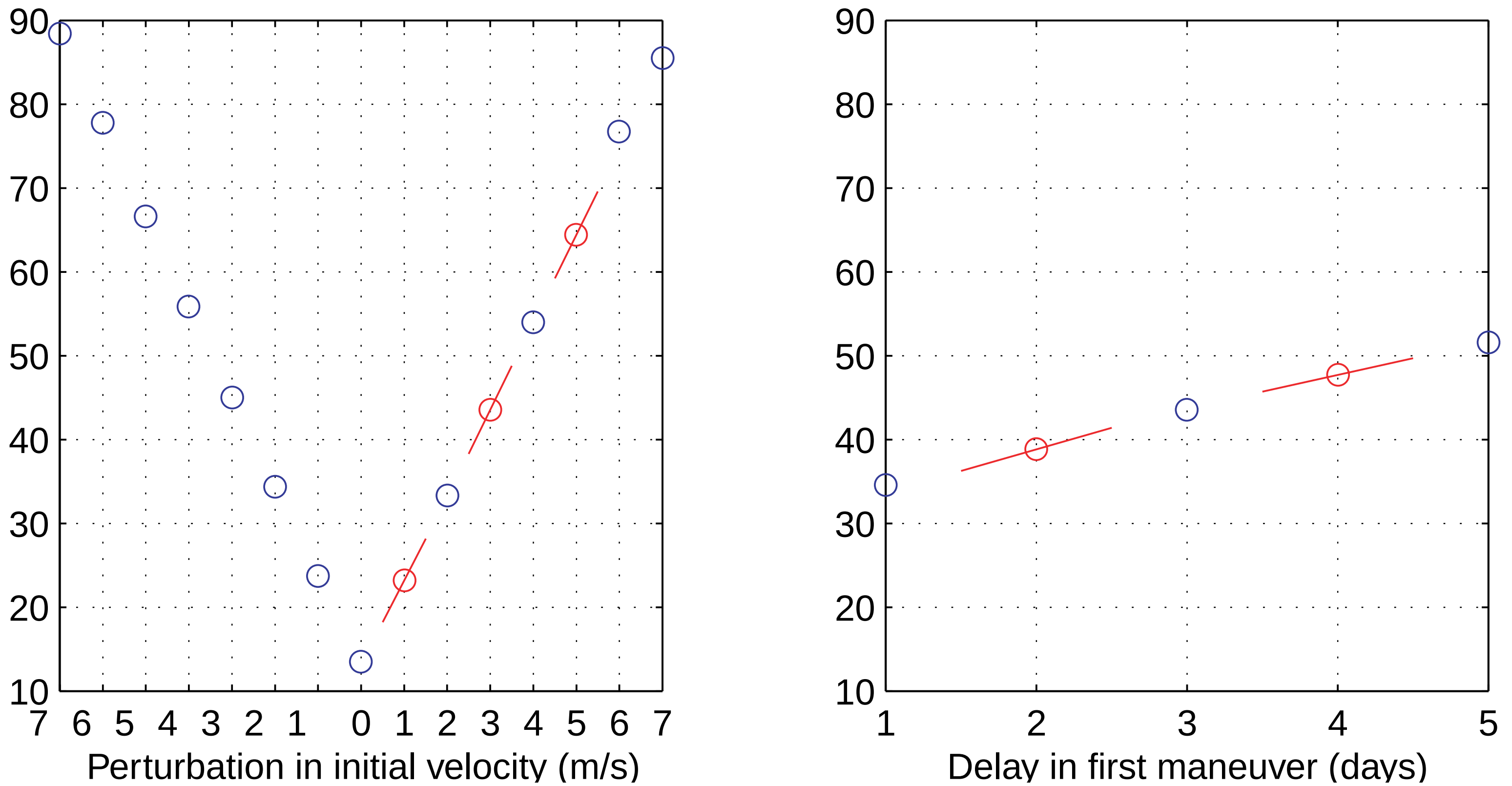
$$\min_{T_i, \mathbf{x}_i, \Delta \mathbf{v}_i} C(\Delta \mathbf{v}_i),$$

subject to constraints given above.



Technical Details: Software COOPT

- COOPT based on **direct method**.
  - provides **optimal solution** with nominal trajectory as first guess.
  - provides estimations of how different **launch velocity errors** and **delays in TCM1** affect the changes in **control** ΔV.



Conclusions and Future Work

- Have used optimal control for halo orbit correction maneuvers.
- COOPT or similar software (**direct method**) and method of **optimal control/dynamical systems** can be used for many future missions.
  - Petit Grand Tour and Shoot the Moon.
  - Formation flight near halo orbit or for earthbound satellites.