Comets, Nonlinear Dynamics and Space Mission Design

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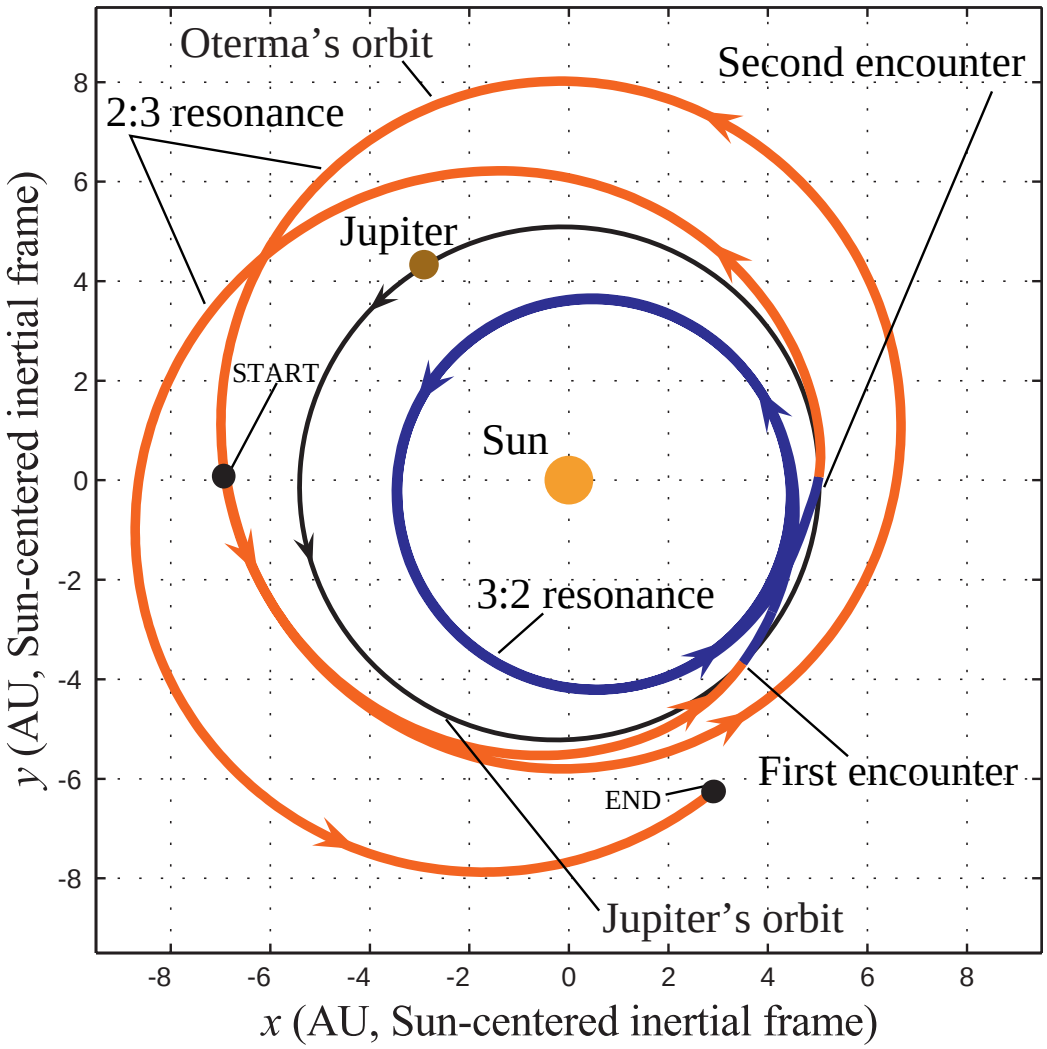
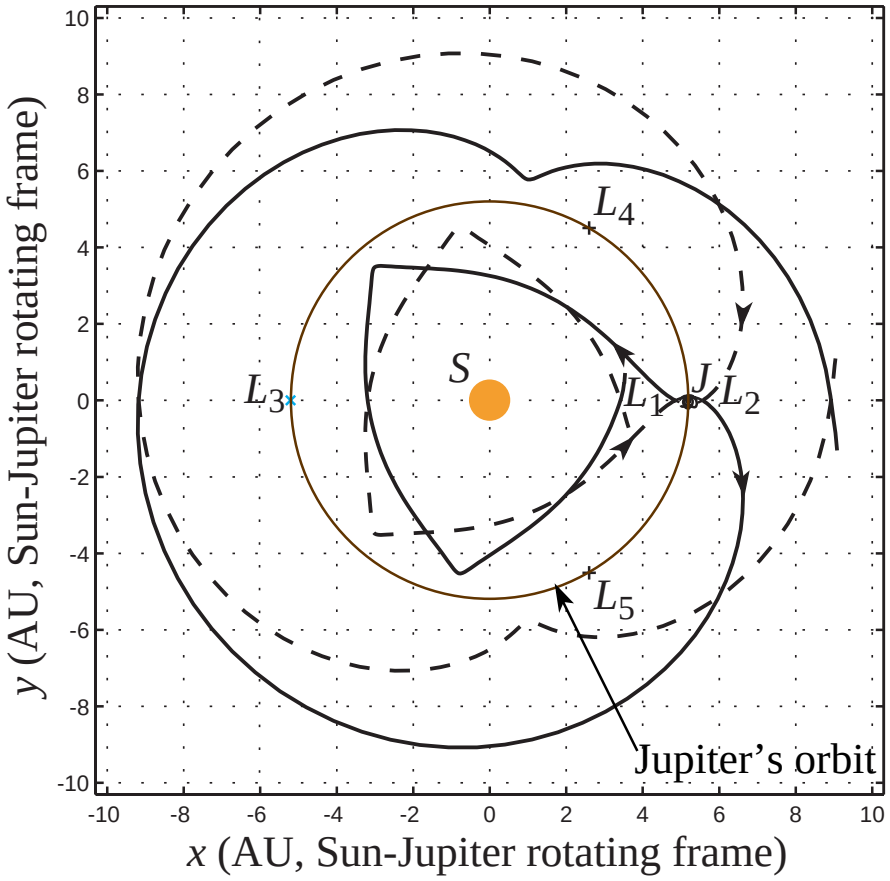
Jupiter Comets

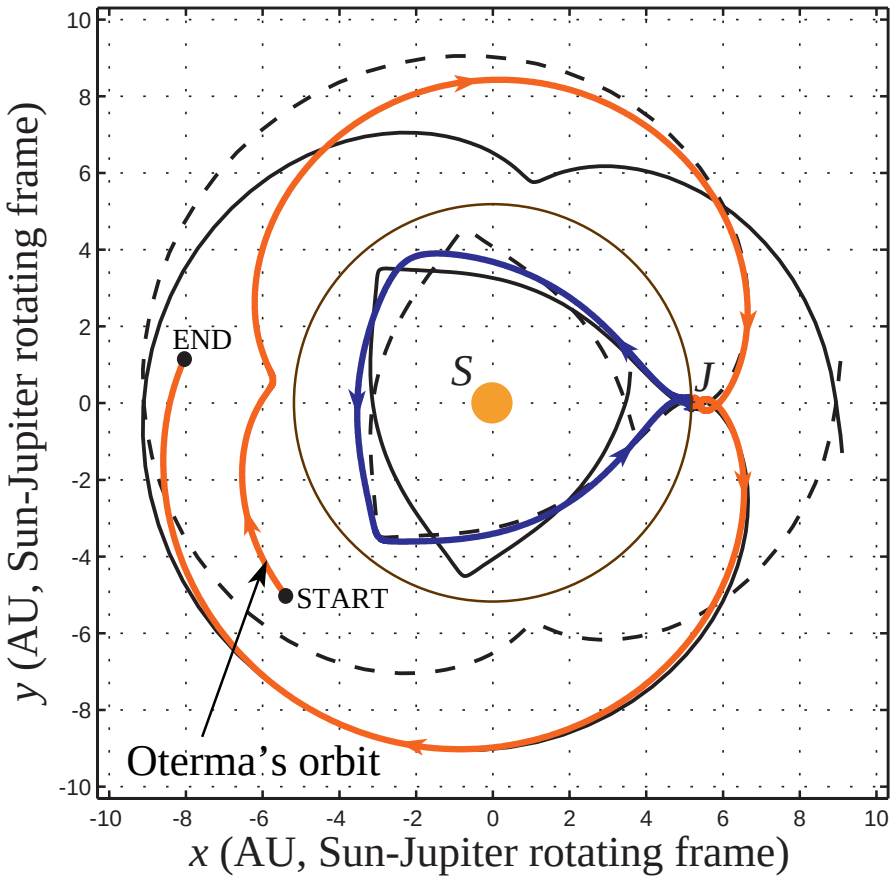
- **Rapid transition** from **outside** to **inside** Jupiter's orbit.
- **Captured temporarily** by Jupiter during transition.
- **Exterior** (**2:3 resonance**). **Interior** (**3:2 resonance**).



Previous Works

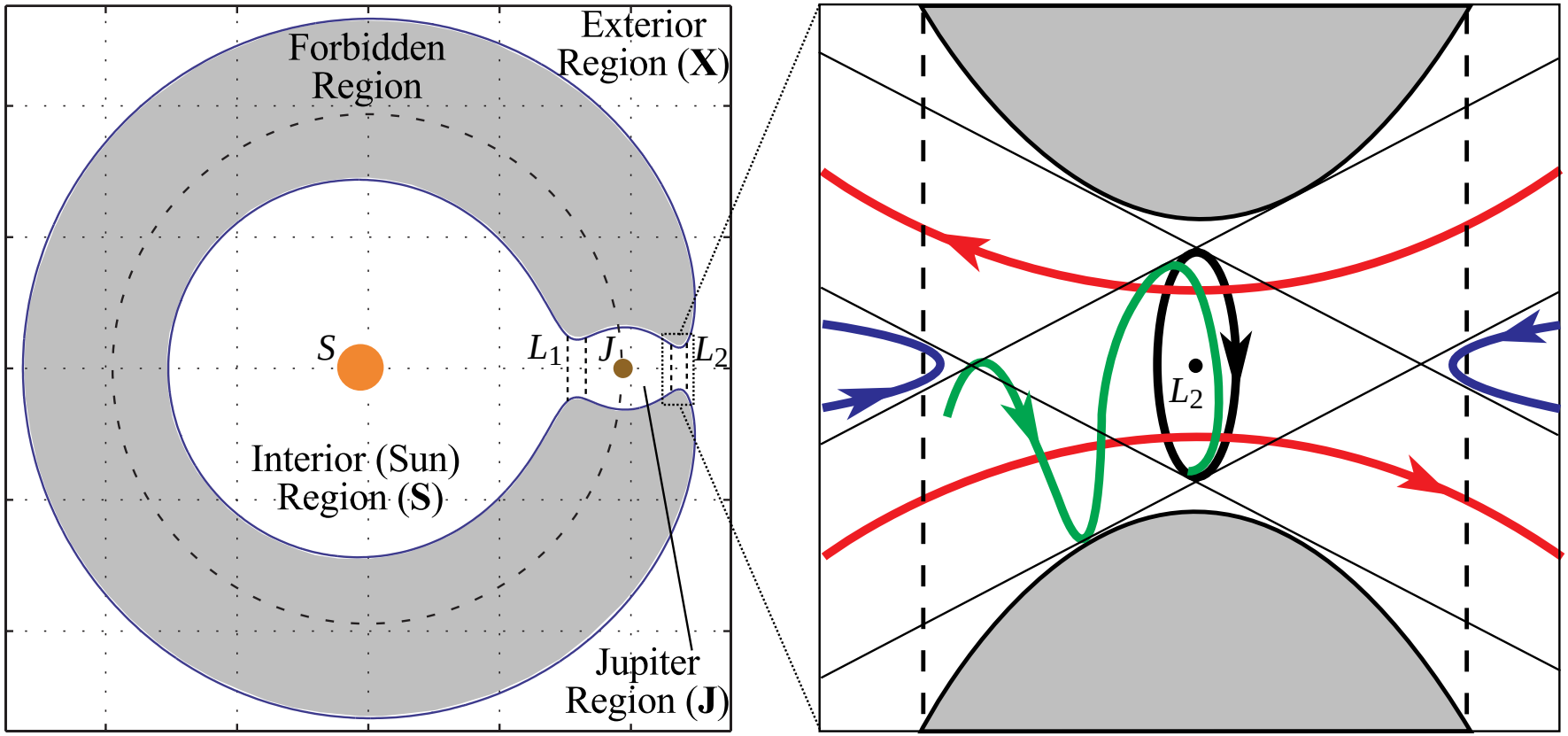
- Belbuno/B. Marsden [1997]
- Lo/Ross [1997] :
  - Comet in **rotating frame** follows **invariant manifolds**.
- Moser/Conley/McGehee. Libre/Martinez/Simó [1985].





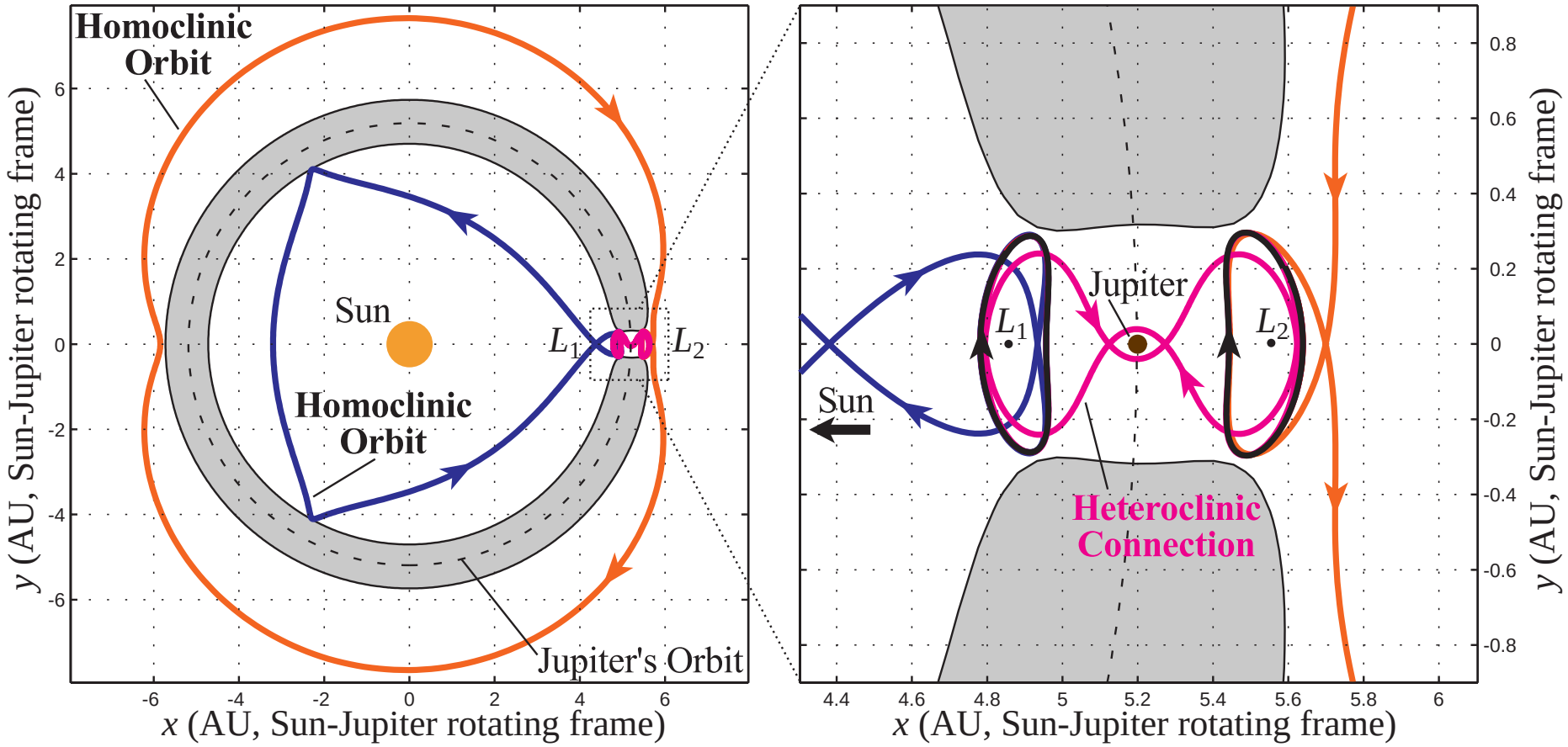
The Flow near L_1 and L_2

- For **energy value** just above that of L_2, **Hill's region** contains a **"neck"** about L_1 & L_2.
- Comet can make **transition** through these equilibrium regions.
- Four types of orbits: periodic, asymptotic, transit & nontransit.



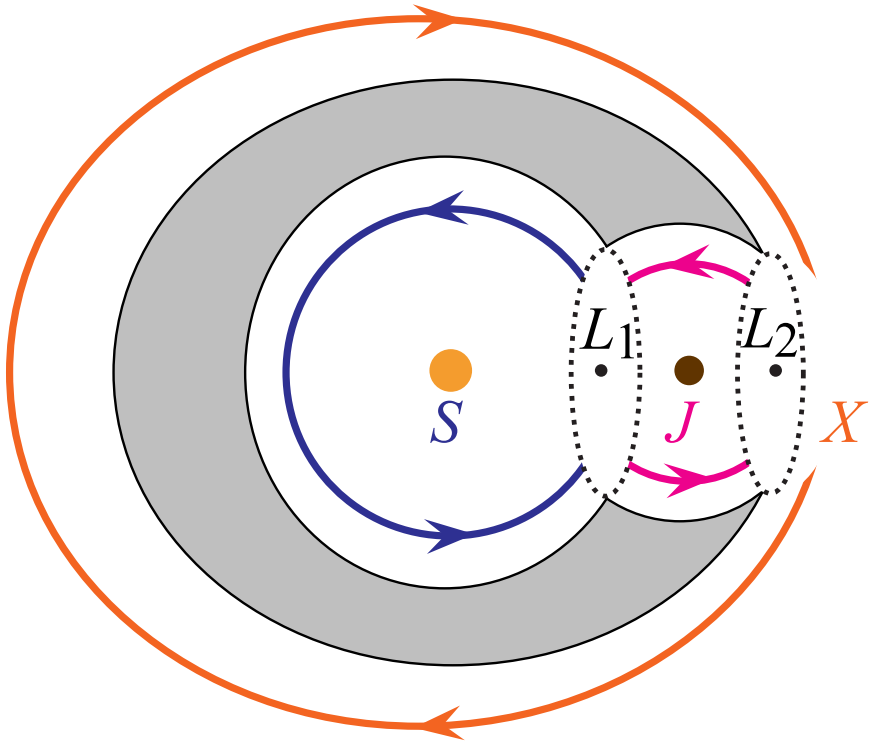
Major Result (A): Heteroclinic Connection

- Found **heteroclinic connection** between pair of periodic orbits.
- Found a large class of **orbits** near this (homo/heteroclinic) ***chain***.
- Comet can follow these ***channels*** in rapid transition.



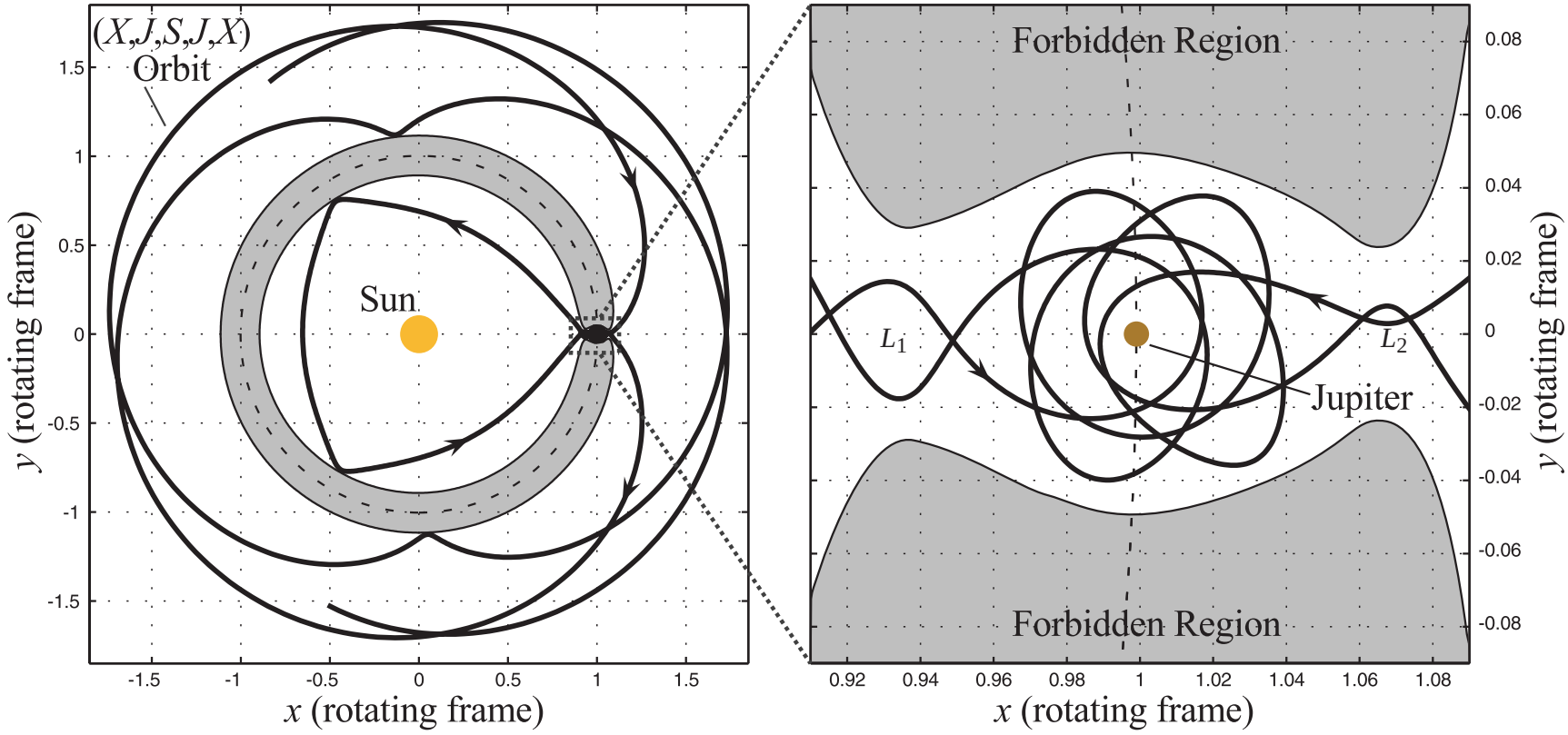
Major Result (B): Existence of Transitional Orbits

- **Symbolic sequence** used to label itinerary of each comet orbit.
- ***Main Theorem***: For any admissible **itinerary**, e.g., $(\ldots, \mathbf{X}, \mathbf{J}, \mathbf{S}, \mathbf{J}, \mathbf{X}, \ldots)$, there exists an orbit whose **whereabouts** matches this **itinerary**.
- Can even specify **number of revolutions** the comet makes around Sun & Jupiter (plus L_1 & L_2).



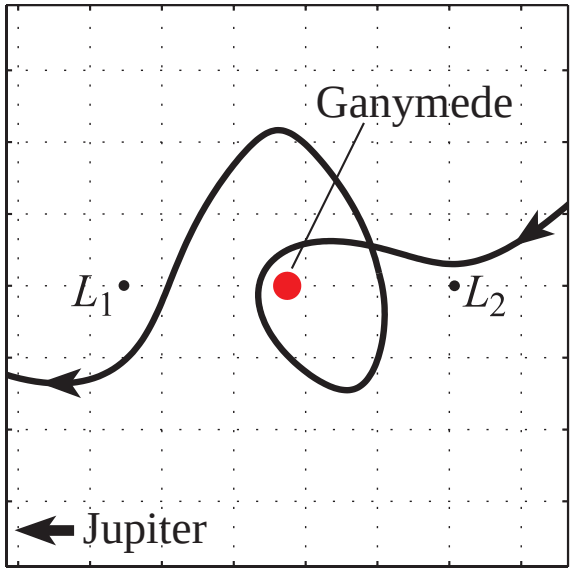
Major Result (C): Numerical Construction of Orbits

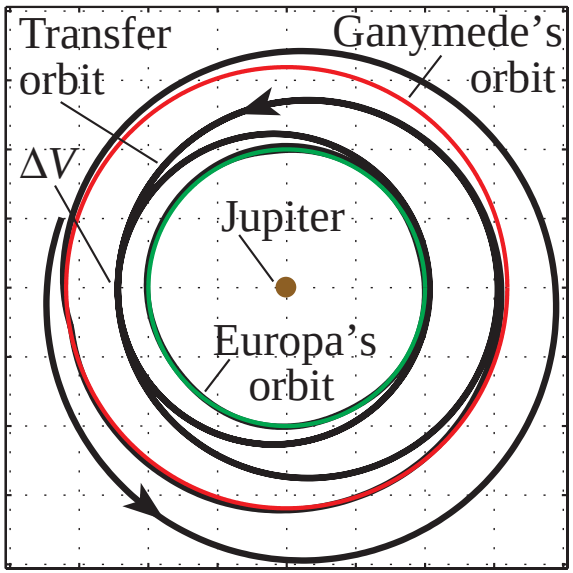
- Developed procedure to construct orbit with **prescribed itinerary**.
- Example: An orbit with itinerary $(\mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X})$.

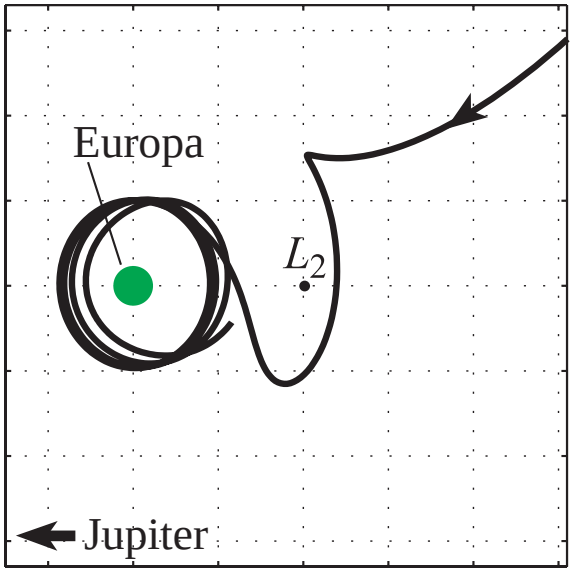


Application: "Petit Grand Tour"

- **"Petit Grand Tour"** of Jupiter's moons.
- **Preliminary example**: 1 orbit around **Ganymede**.
  4 orbits around **Europa**, etc.

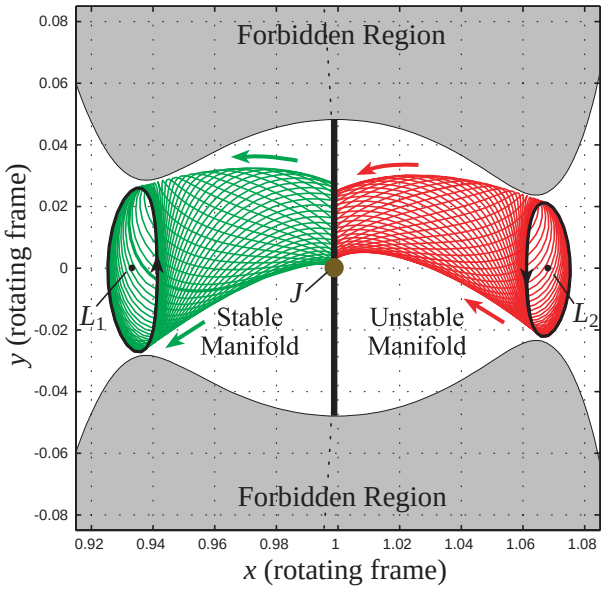


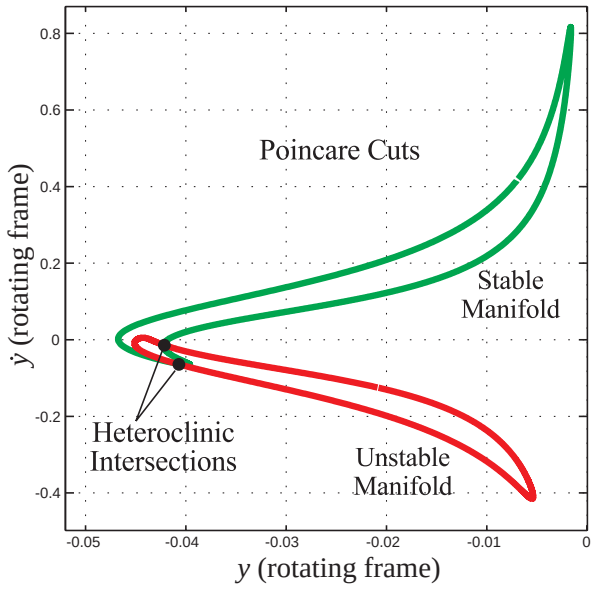


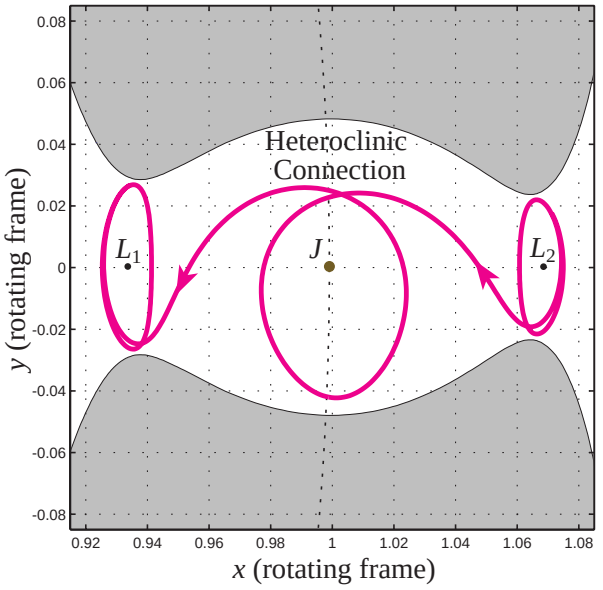


Technical Details (A): Heteroclinic Connection

- Find an **intersection** of **stable**/**unstable** manifolds.
- Locate the intersections using Poincaré **cuts** of stable/unstable manifolds.
- Point of intersection **integrated** to produce heteroclinic conn.

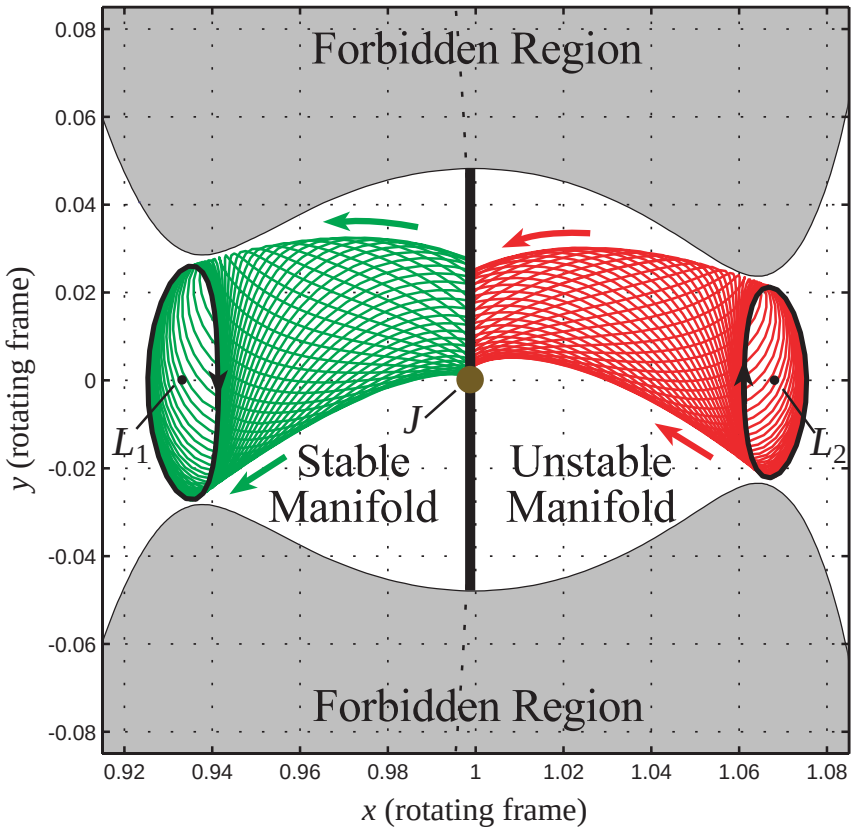


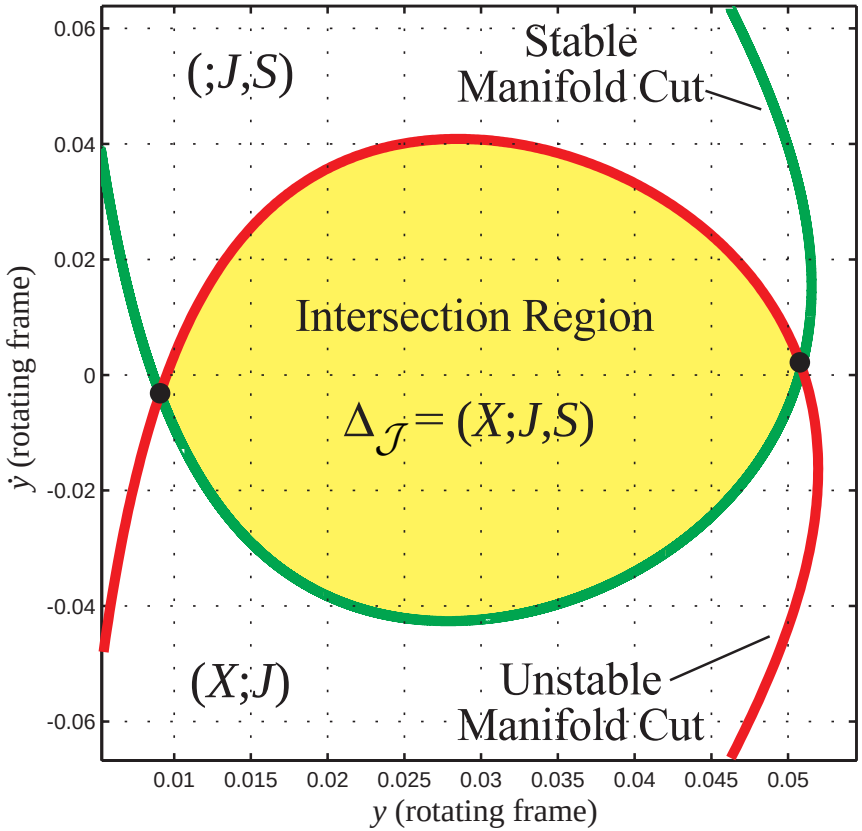




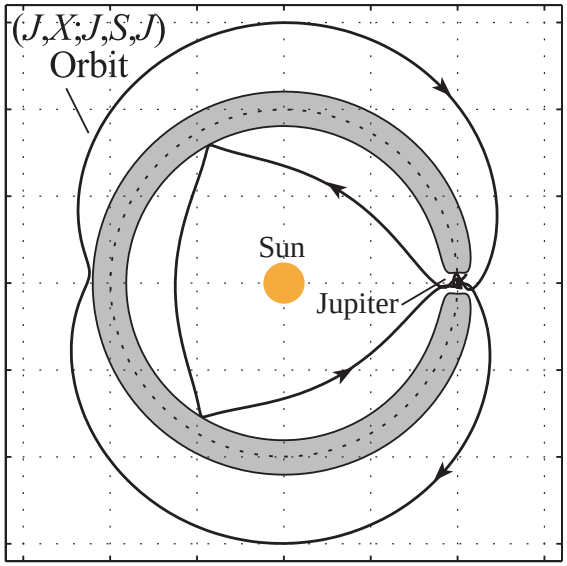
Details (B): Construction of $(\mathbf{J}, \mathbf{X}; \mathbf{J}, \mathbf{S}, \mathbf{J})$ Orbits

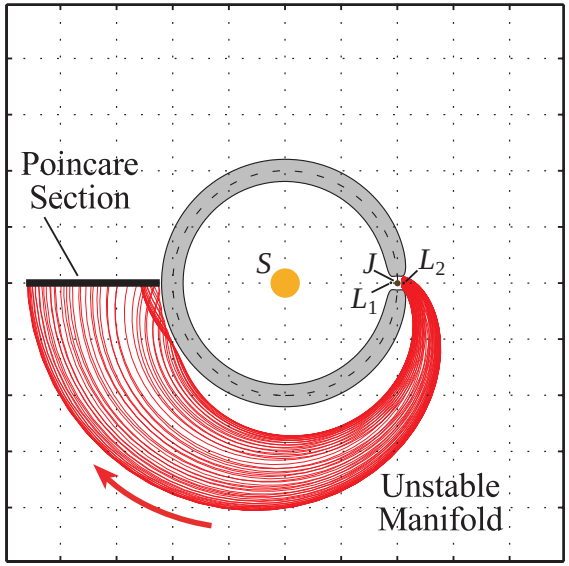
- Invariant manifold **tubes** separate transit from nontransit orbits.
- **Green curve** (Poincaré cut of L_1 stable manifold).
  Red curve (cut of L_2 unstable manifold).
- Any point inside the intersection region Δ_J is a $(\mathbf{X}; \mathbf{J}, \mathbf{S})$ orbit.

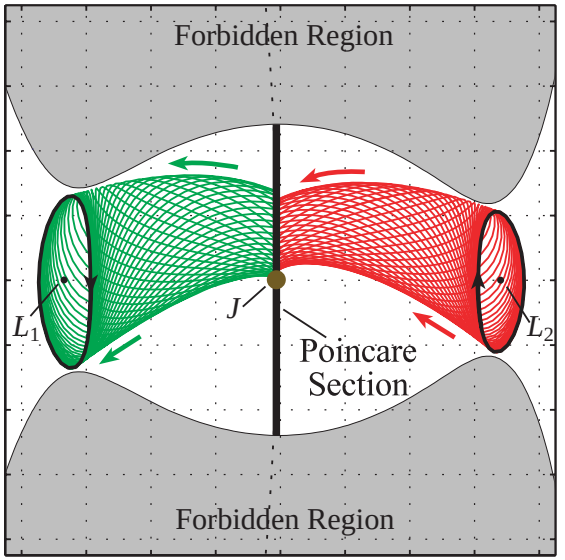




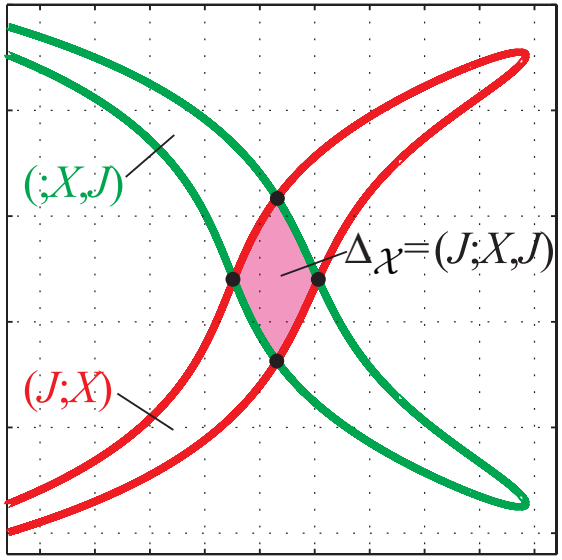
Details (B) cont'd: Construction of (J, X; J, S, J) Orbits





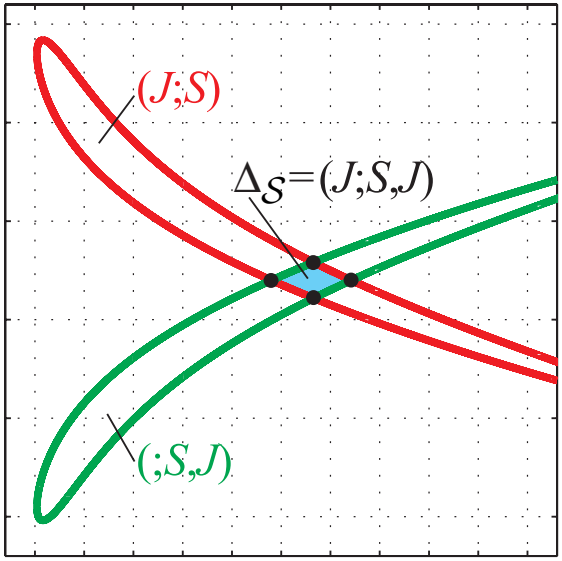


$\Delta_{\mathcal{X}}=(J;X,J)$



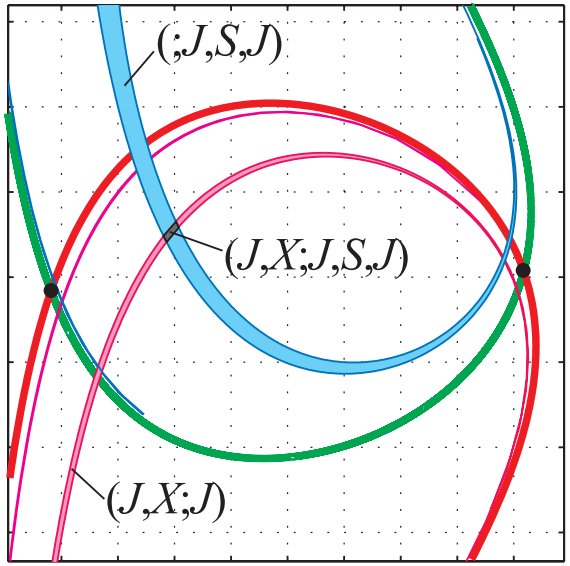
Exterior Region

$\Delta_{\mathcal{S}}=(J;S,J)$



Interior Region

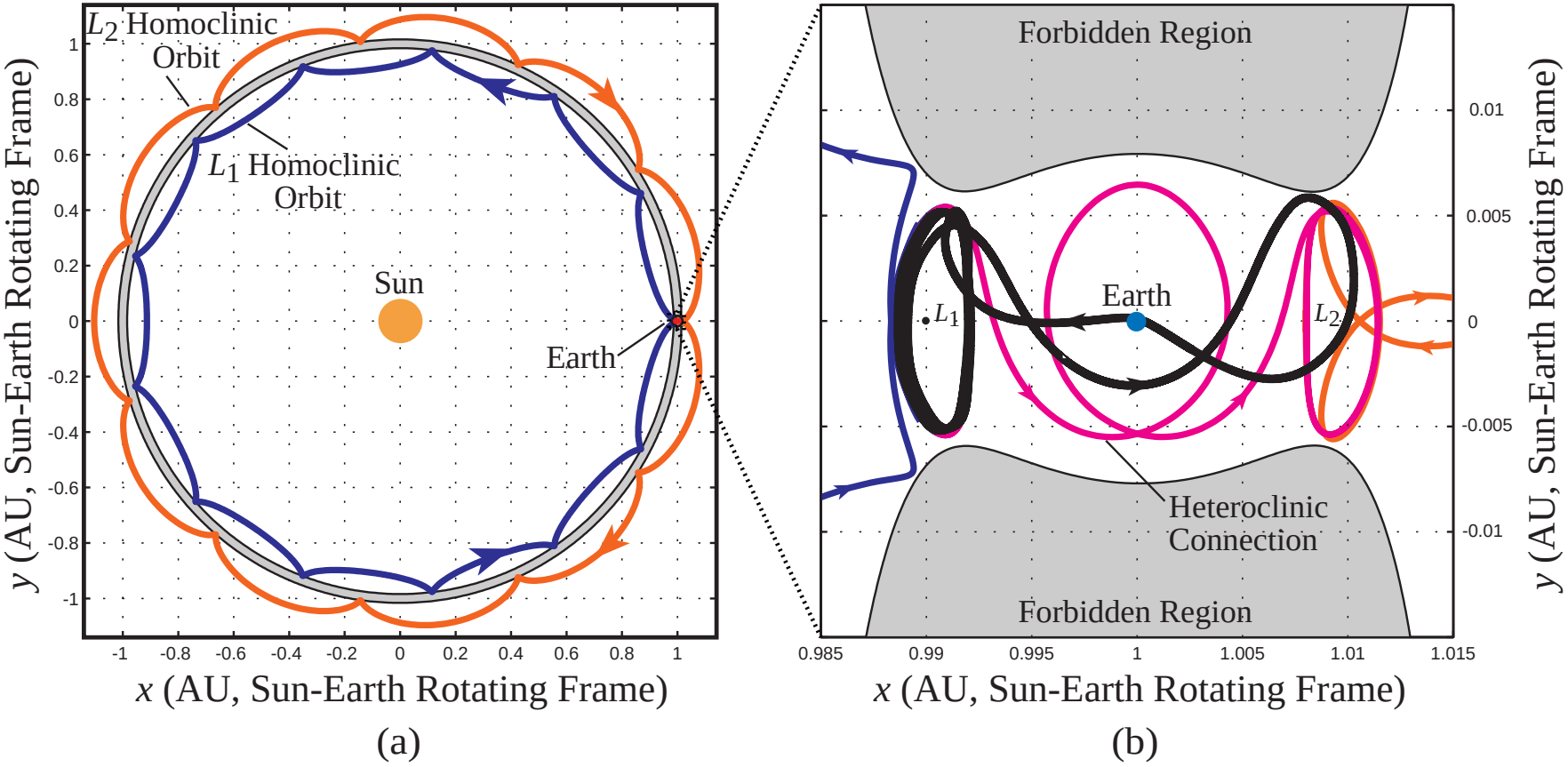
$(J,X;J,S,J)$



Jupiter Region

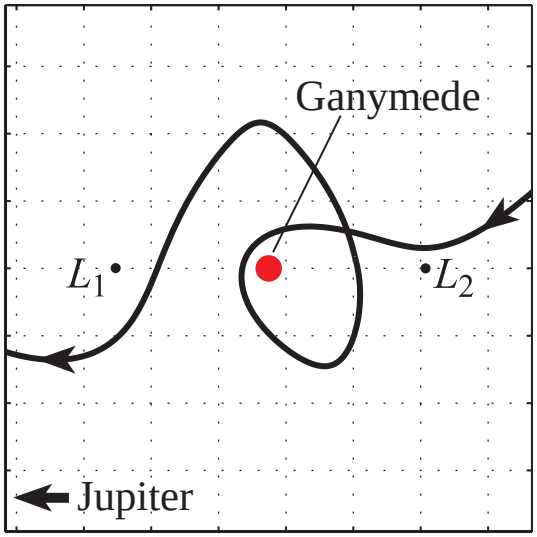
Future Work: Extension to 3 Dimensions

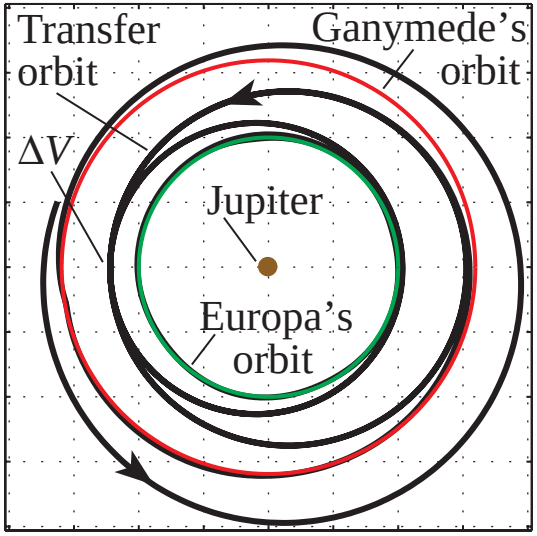
- Find **chains/dynamical channels** for 3D periodic (halo) orbits, and use them for **low fuel** deployment of spacecraft.
- Obtain better understanding of phase space geometry near L_1 & L_2; use it to design/control **constellations** of spacecraft.

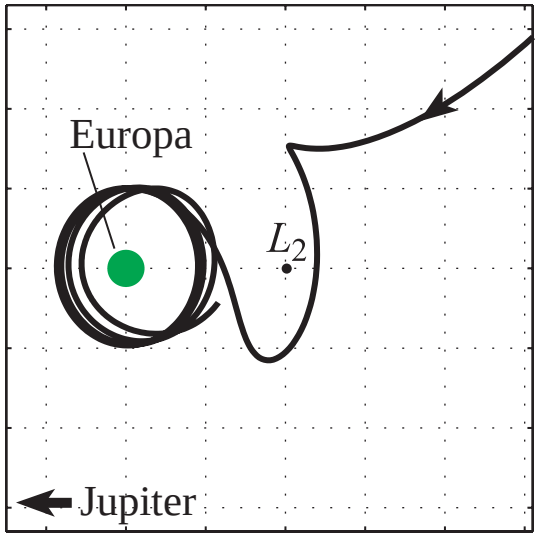


Future Work: Coupling Two 3-Body Systems

- We could apply our method to understand the dynamics governing the **transport** between adjacent planets.
- Preliminary result on a **"Petit Grand Tour"** of Jupiter's moons.

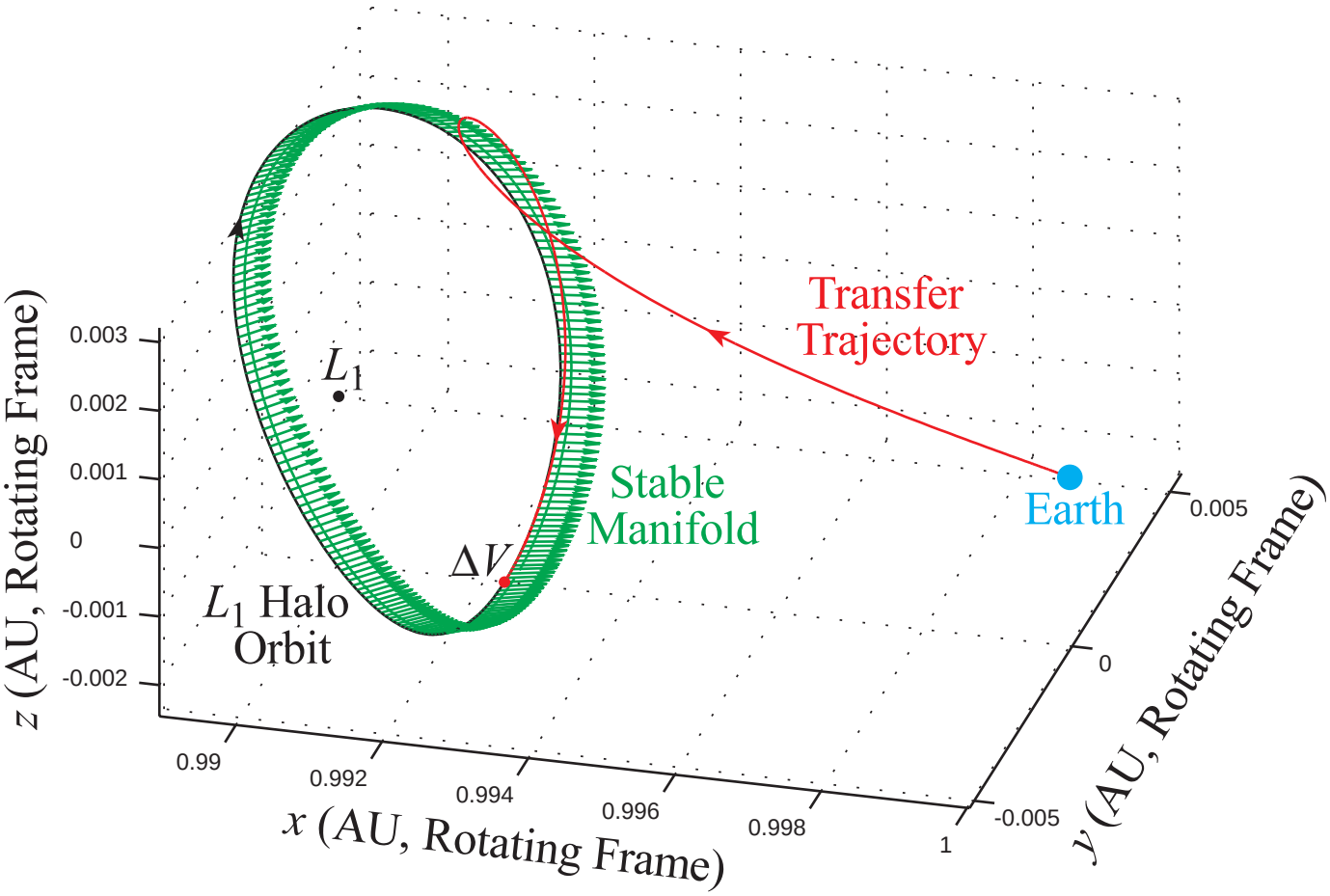






Future Work: Optimal Control/Nonlinear Dynamics

- Develop optimal control algorithm with natural dynamics built in.
- Started using optimal control to target **stable manifold** of L_1 halo orbit.



Future Work: 4 or More Body Problems

- **Interplanetary transport** and distribution of material.
- Dynamics of larger class of comets and near-Earth asteroids.