Dynamical Systems and Space Mission Design

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Acknowledgements

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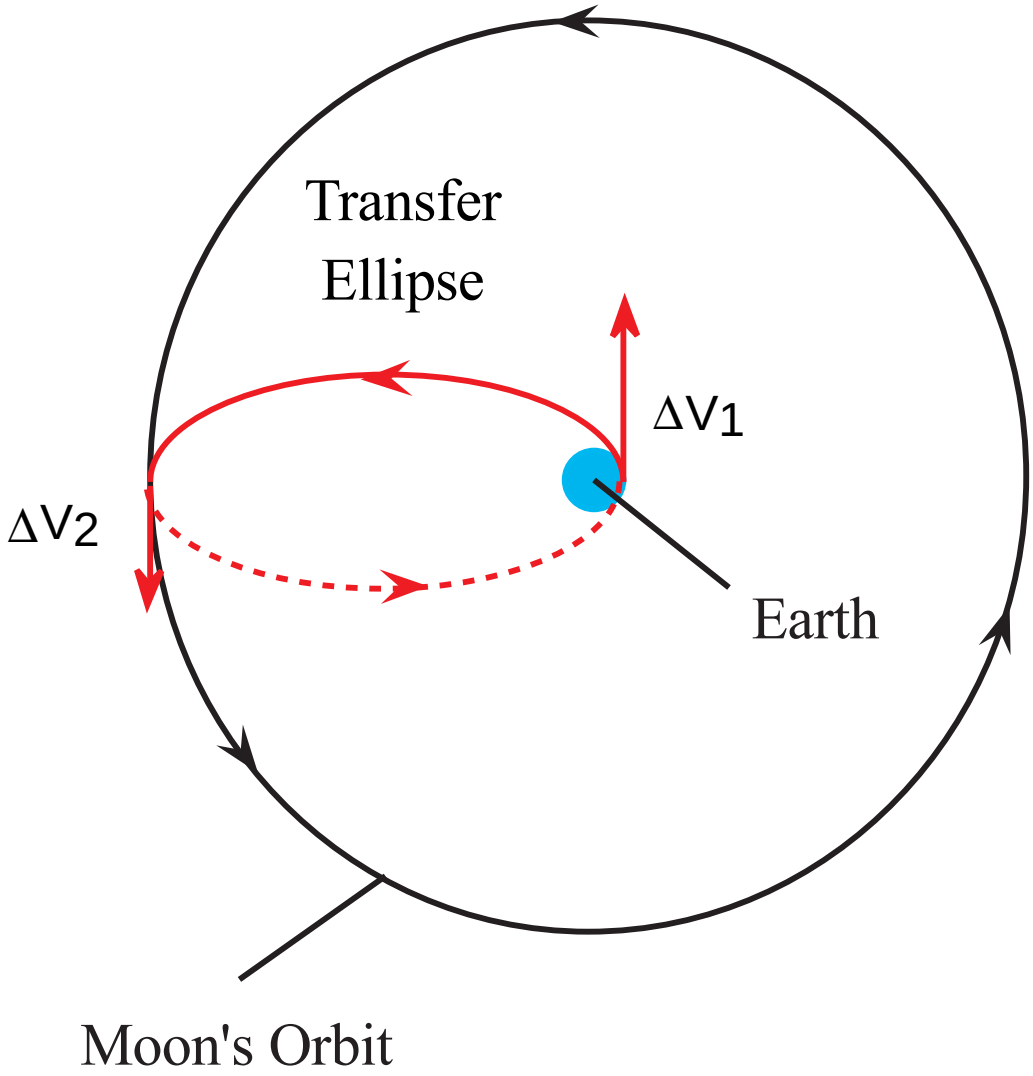
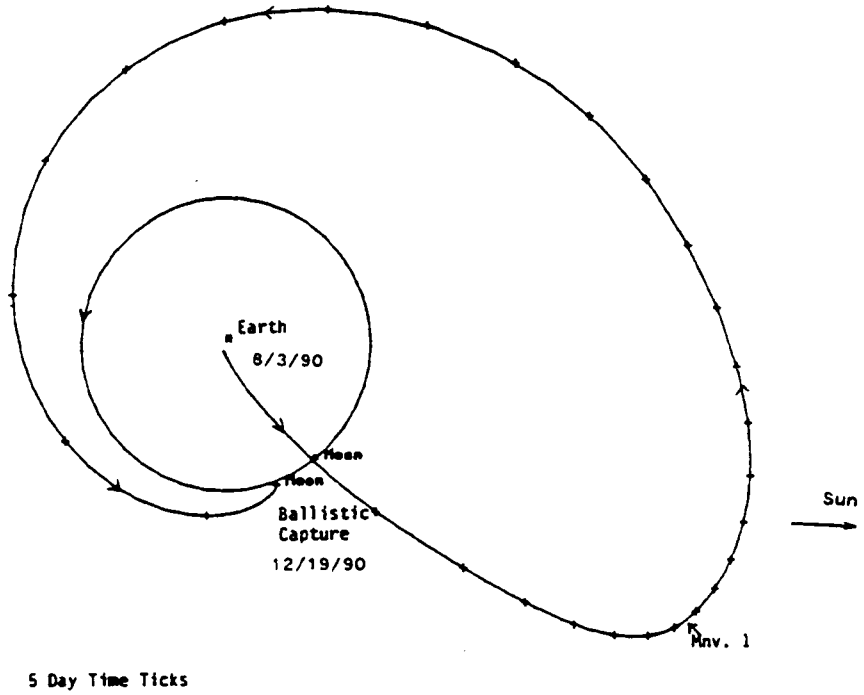
Introduction: Hohmann Transfer

- Traditional transfer from Earth to Moon is by **Hohmann transfer**.
- 2 body Keplerian **ellipse** from Earth to Moon. Need 2 ΔVs.



Sun-Perturbed E-M Transfer with Ballistic Capture

- In 1991, Muses-A did not have enough propellant to reach Moon by **Hohmann transfer**.
- Belbruno/Miller designed a **Sun**-perturbed **Earth-to-Moon transfer** with **ballistic capture** at Moon.
- Similar techniques used by Japanese team to save mission.

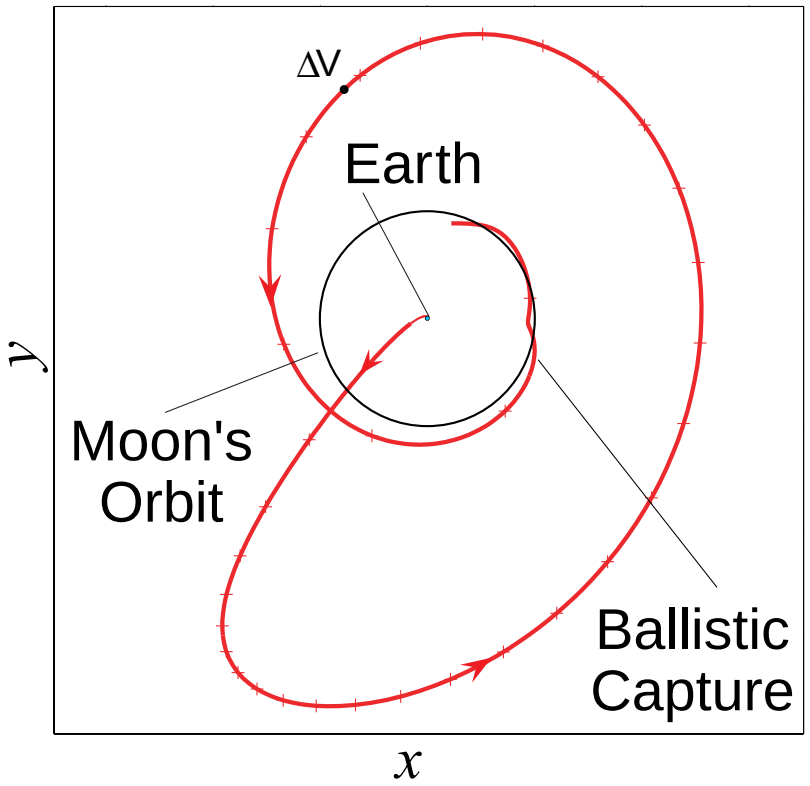


Numerical simulation of a ballistic capture transfer trajectory for the Japanese spacecraft Hiten: ecliptic plane projection, sun's direction indicated at Earth injection. (from Belbruno and Miller [1993])

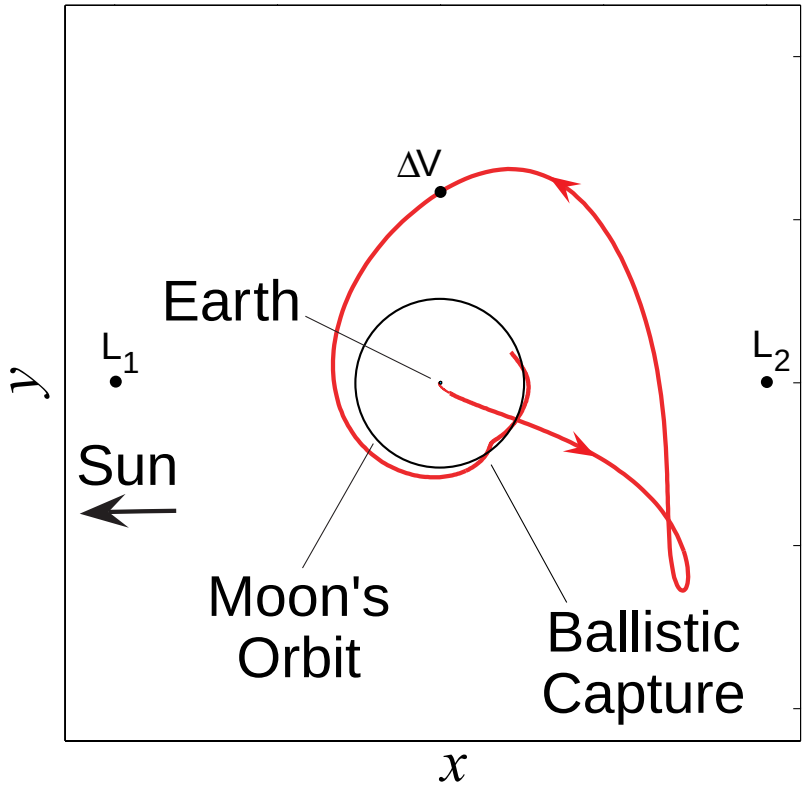
Sun-Perturbed E-M Transfer with Ballistic Capture

- We provide a theoretical basis and a numerical procedure for constructing such ballistic capture transfer.
- By considering **Sun-Earth-Moon-SC 4-body system** as **2 coupled 3-body systems**.
- Better seen in Sun-Earth **rotating frame**.

Inertial Frame

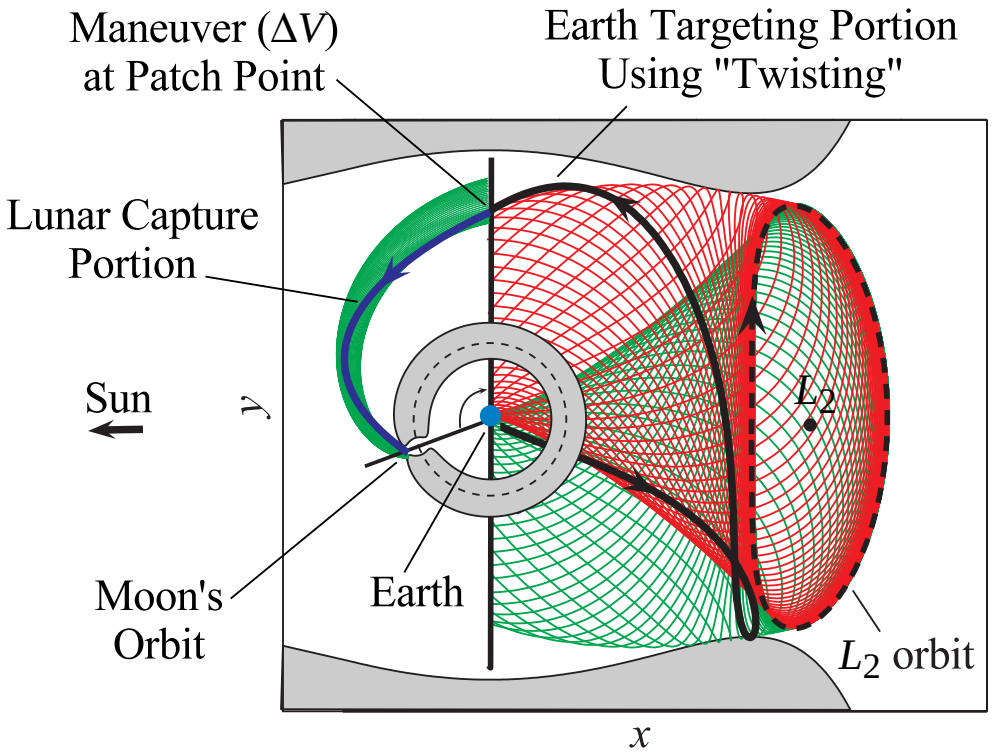


Sun-Earth Rotating Frame



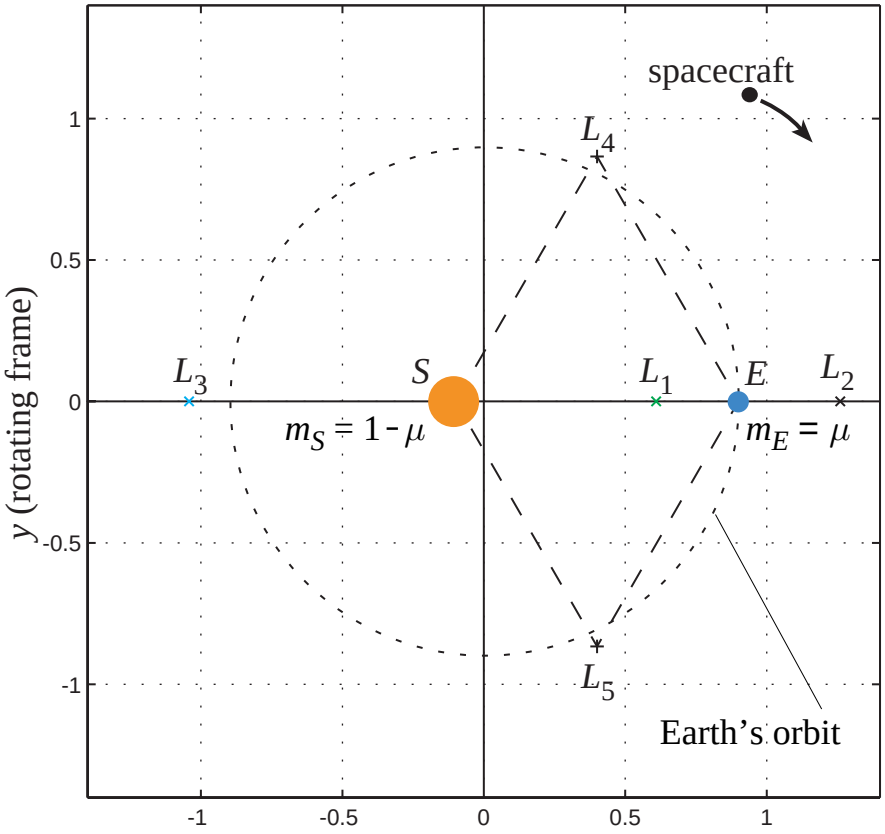
Introduction: Coupled 3-Body Systems

- Find **position/velocity** for spacecraft
  - integrating **forward**, SC guided by **Earth-Moon manifold** and get ballistically captured at Moon;
  - integrating **backward**, SC hugs **Sun-Earth manifolds** with a twist and return to Earth.
- Based on **Koon, Lo, Marsden and Ross** [2000].



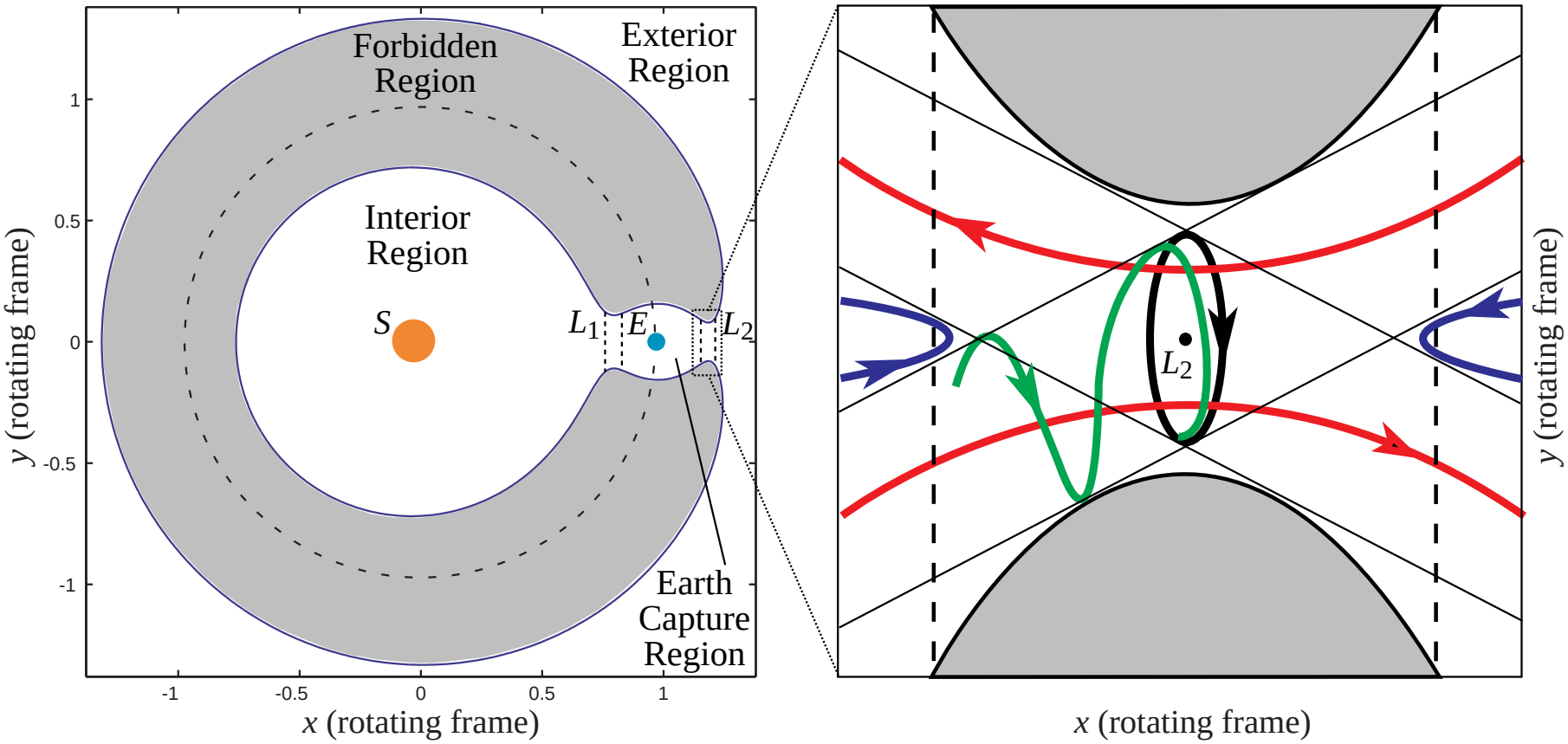
Planar Circular Restricted Three-Body Problem

- Sun and Earth **total mass** normalized to 1;
  rotate about center of mass, with **angular velocity** equal to 1.
  3rd body has infinitesimal mass.
- **Rotating coordinate system** with origin at center of mass,
  S and E fixed at $(-\mu, 0)$ and $(1-\mu, 0)$.
- Has 3 unstable **equilibrium points** on S-E line.



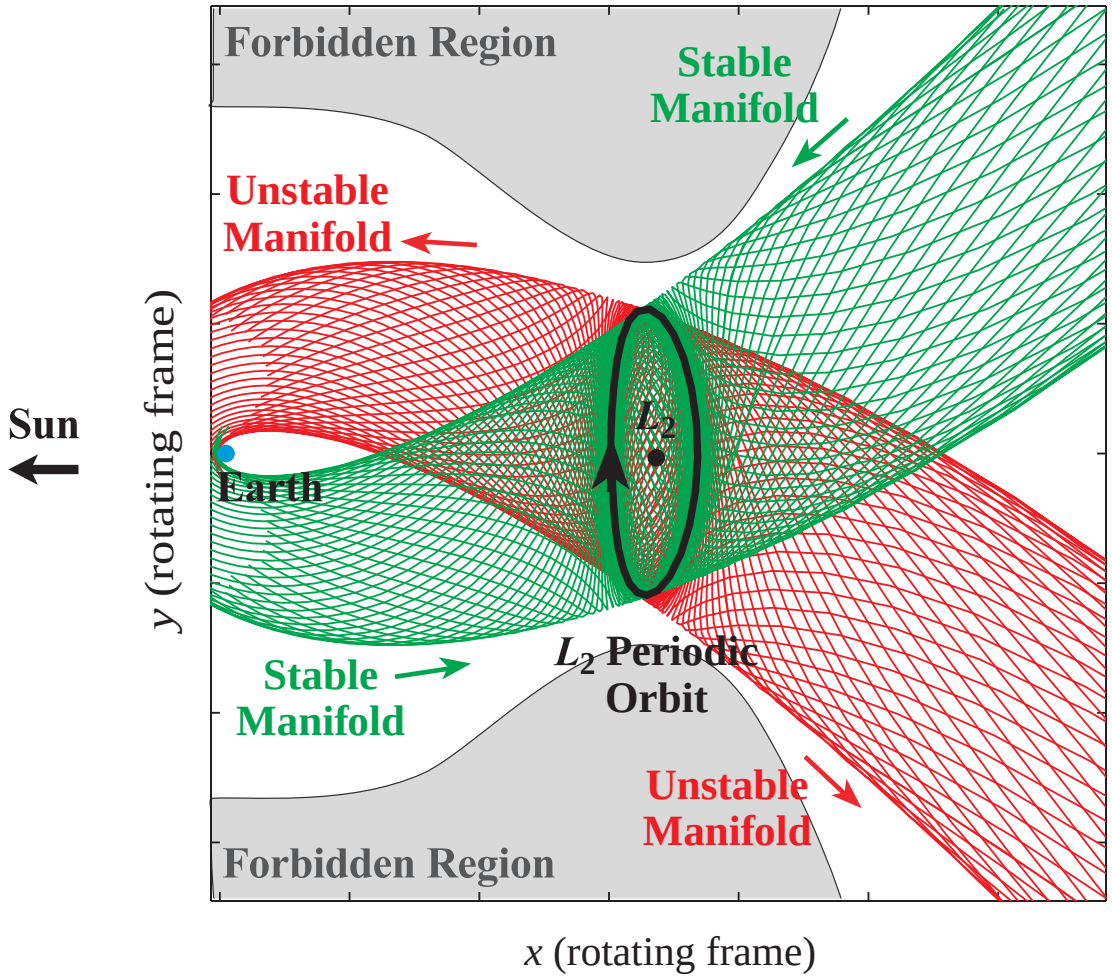
PCR3BP: The Flow near L_1 and L_2

- For **energy value** just above that of L_2,
  Hill's region contains a **"neck"** about L_1 & L_2.
- SC can make **transition** through these equilibrium regions.
- 4 types of orbits:
  periodic, **asymptotic**. **transit** & **nontransit**.



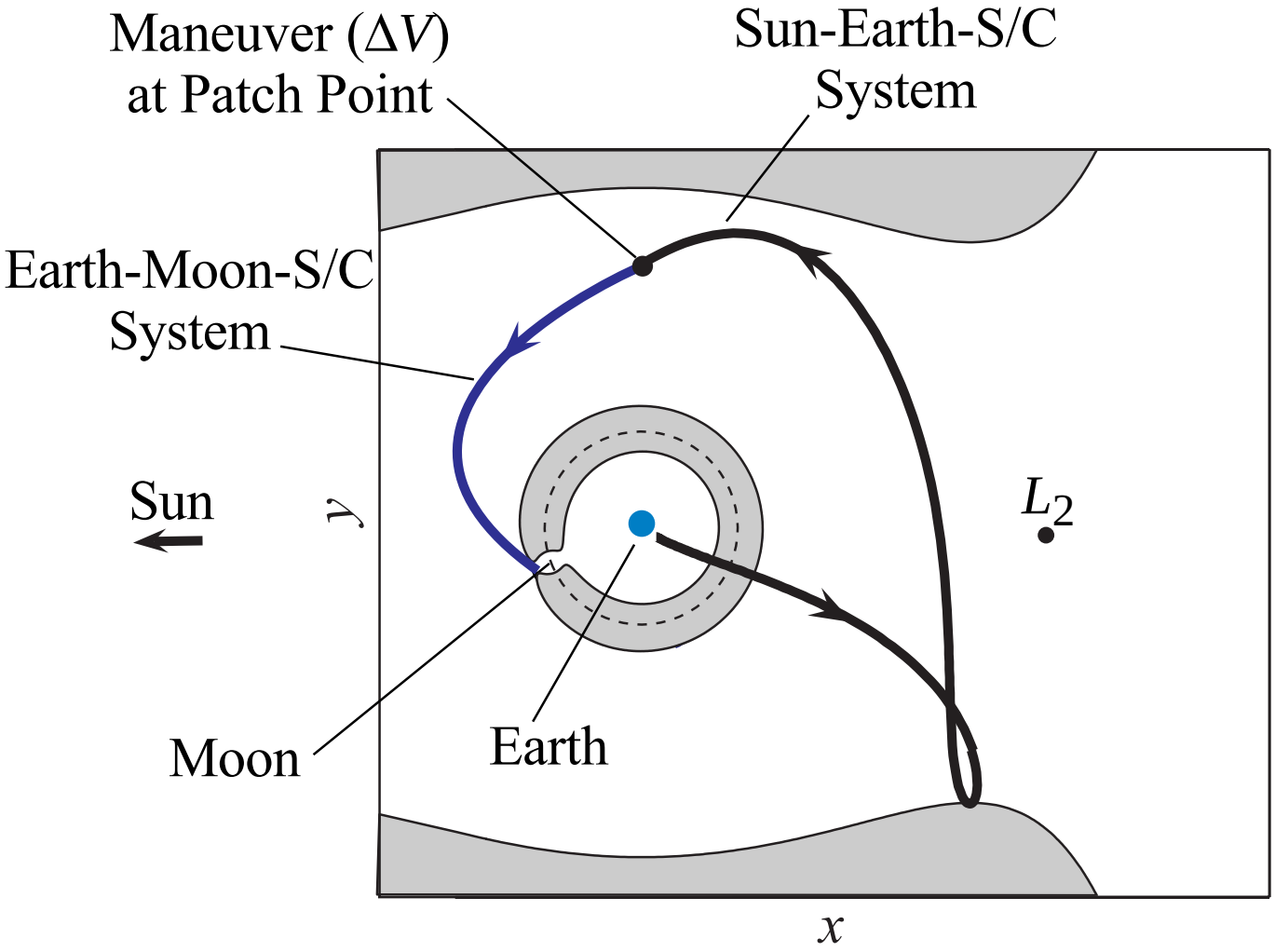
PCR3BP: Invariant Manifold as Separatrix

- **Invariant manifold tubes** act as **separatrices** for the flow in equilibrium regions.
  - Those inside the tubes are **transit** orbits.
  - Those outside the tubes are **non-transit** orbits.



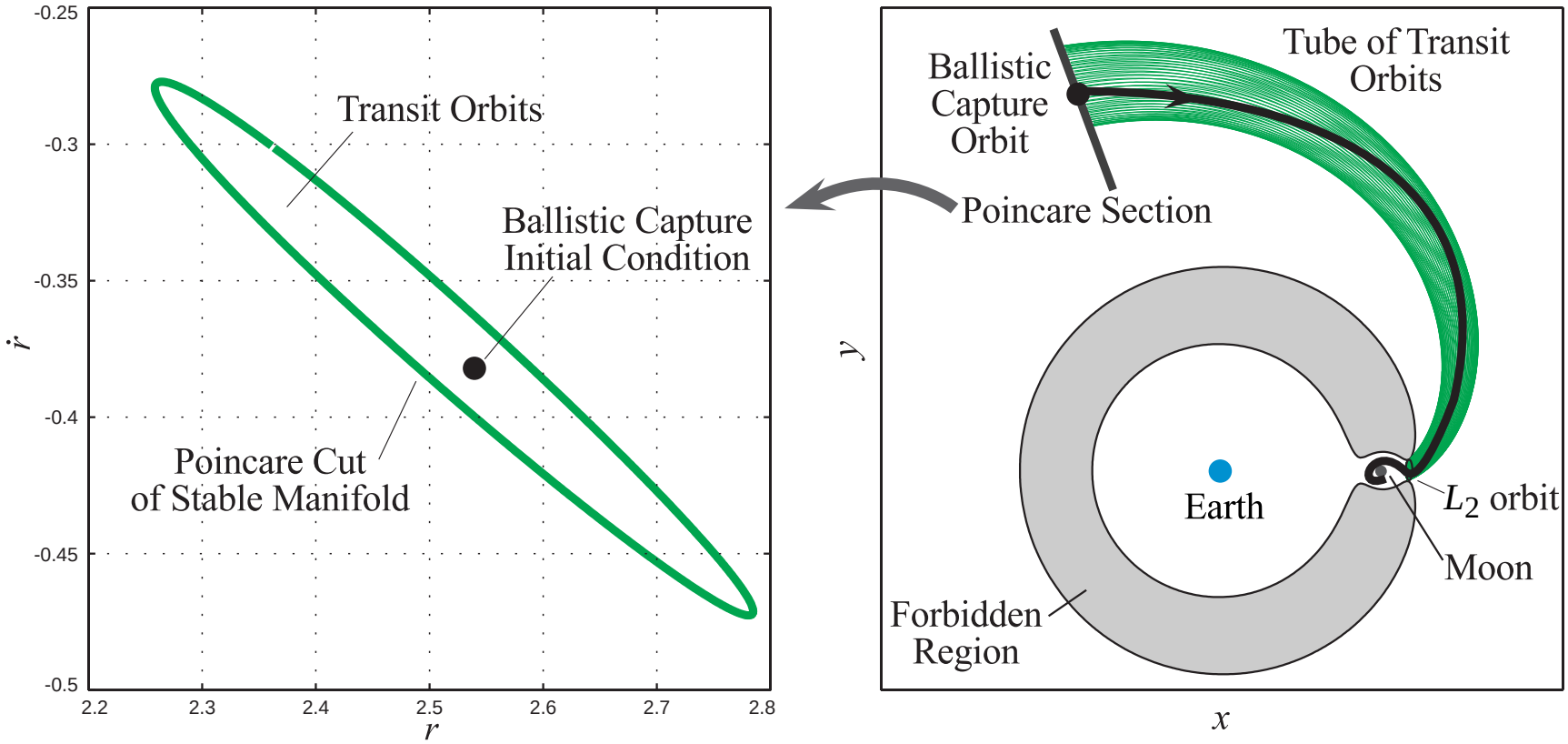
Schematic of "Shoot the Moon" Trajectory,

- 2 portions of the trajectory in **Sun-Earth rotating frame**:
  - **Sun-Earth libration point portion**.
  - **Lunar ballistic capture portion**.



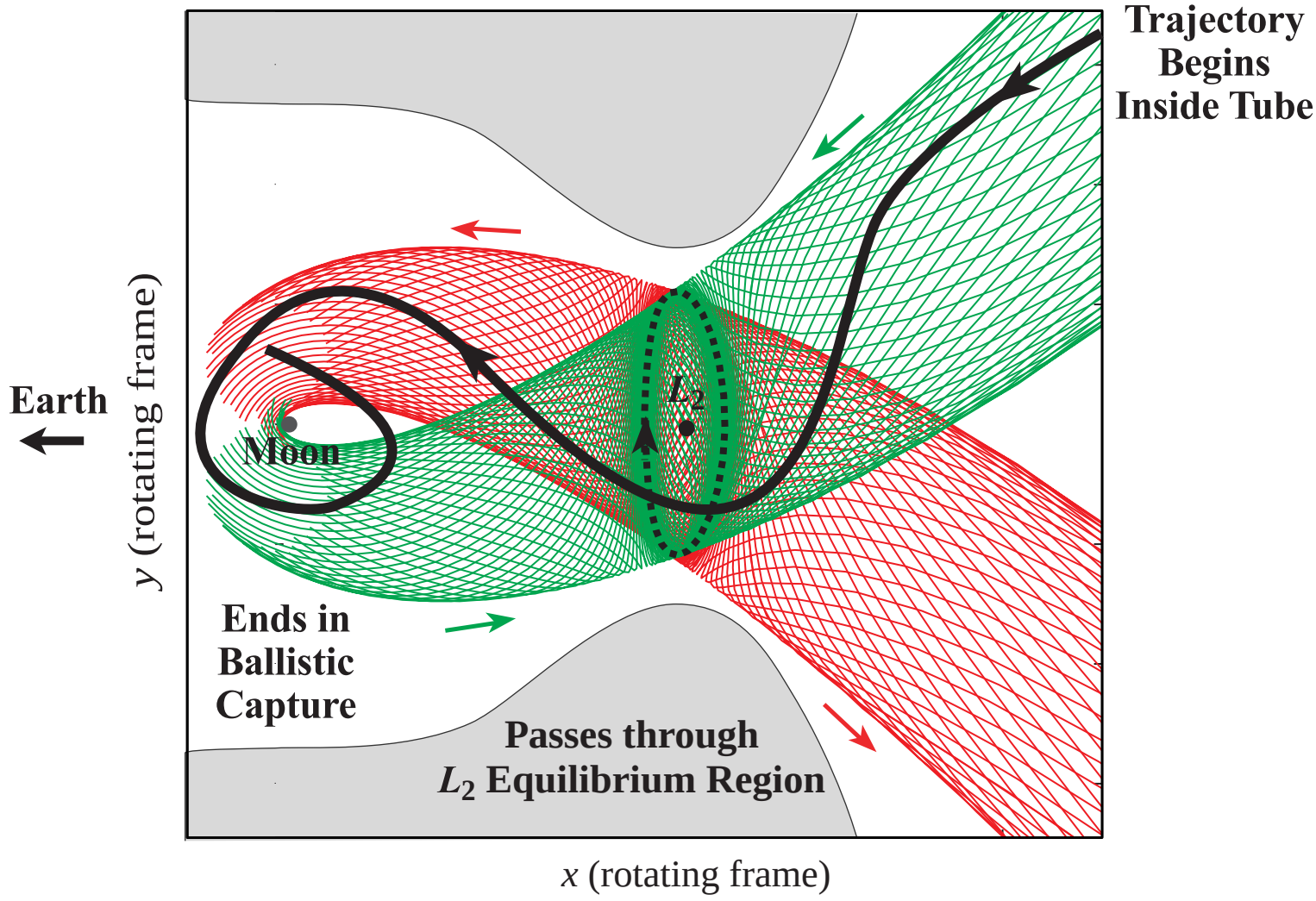
Lunar Ballistic Capture Portion

- **Stable manifold tube** provides **temporary capture** mechanism by second primary.
- **Stable manifold tube** of periodic orbit around L_2 guides spacecraft towards **ballistic capture** by Moon.



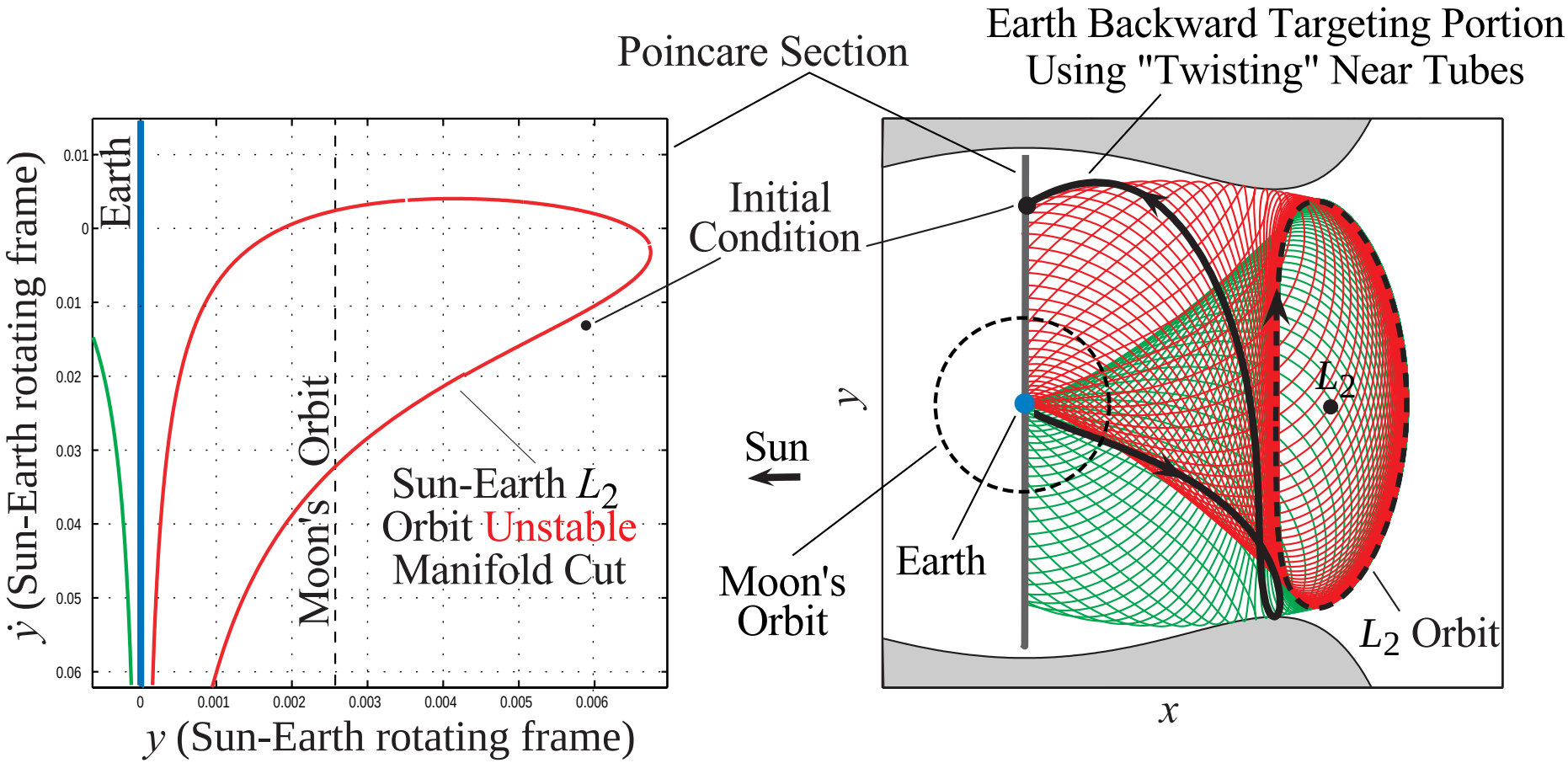
Lunar Ballistic Capture Portion

- By saving (on-board) fuel for lunar **ballistic capture** portion, this design uses **less** fuel than Earth-to-Moon **Hohmann transfer**.



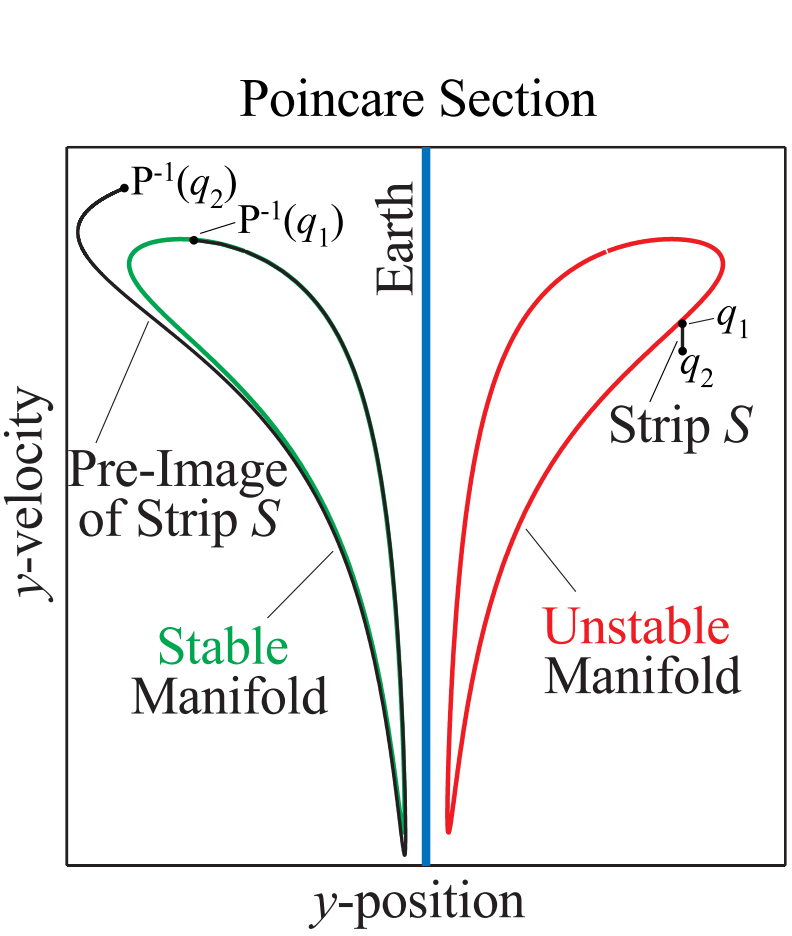
Sun-Earth Libration Point Portion

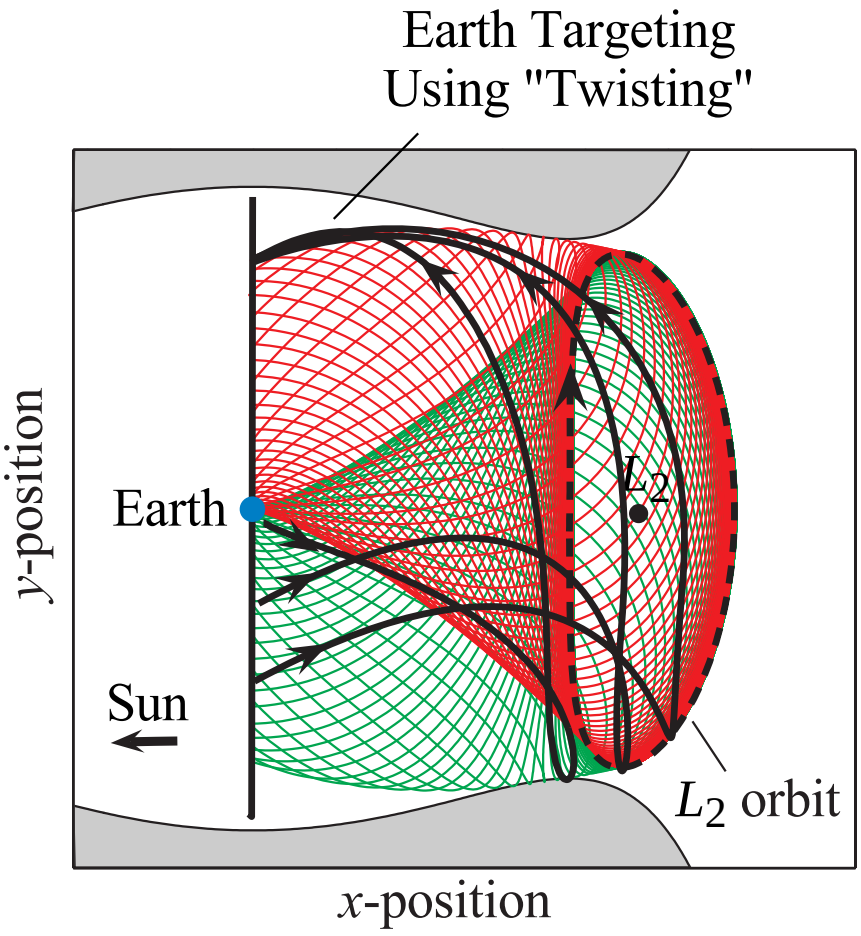
- Pick **initial condition** outside **Poincaré cut**, backward integrate to produce a trajectory:
  - hugs **unstable** manifold back to L_2 region with a **twist**,
  - hugs **stable** manifold back towards Earth.



Sun-Earth Libration Point Portion

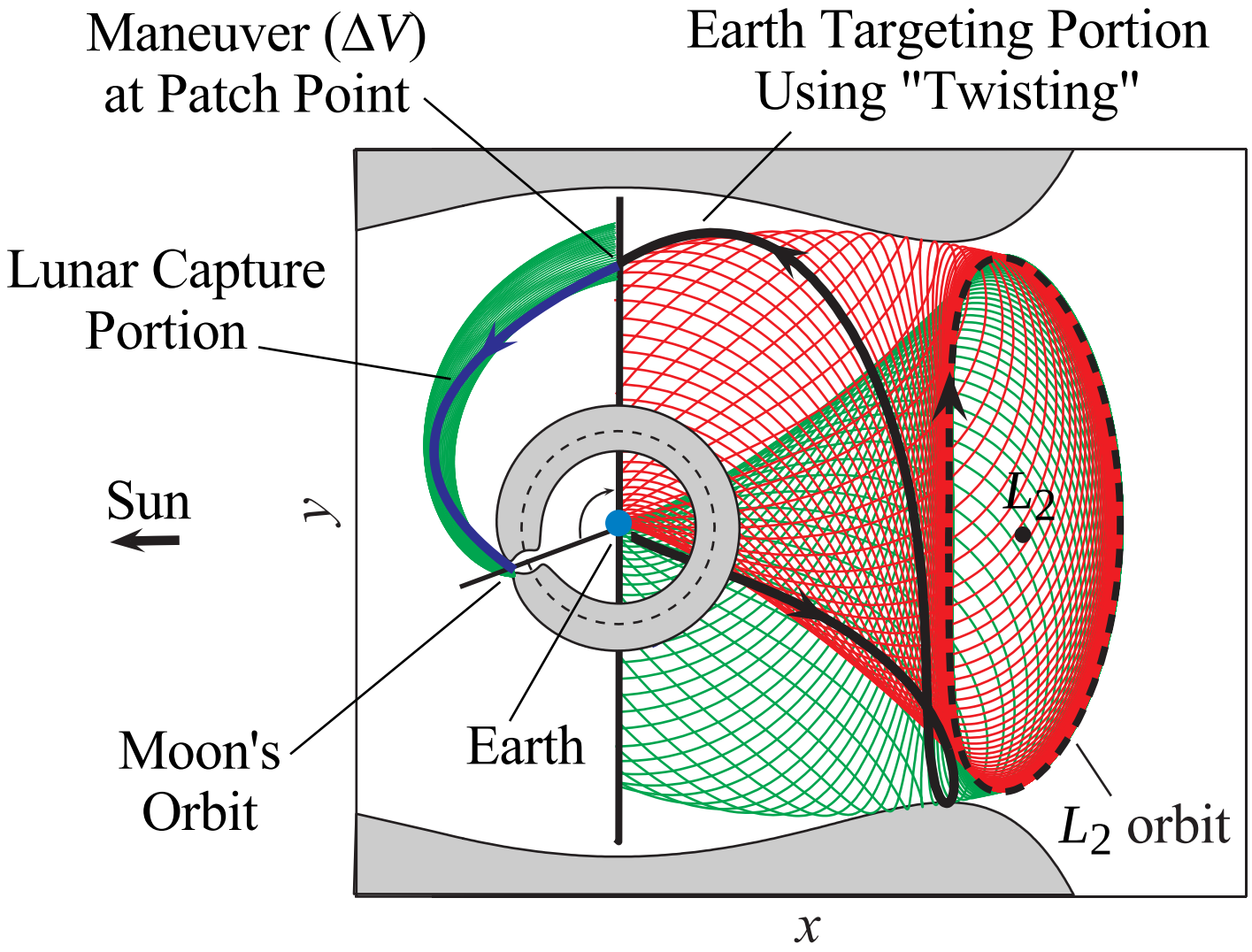
- Amount of **twist** depends sensitively on **distance** from manifold, can change dramatically with small ΔV.
- With small ΔV,
  can target back to (200 km) Earth **parking orbit**.





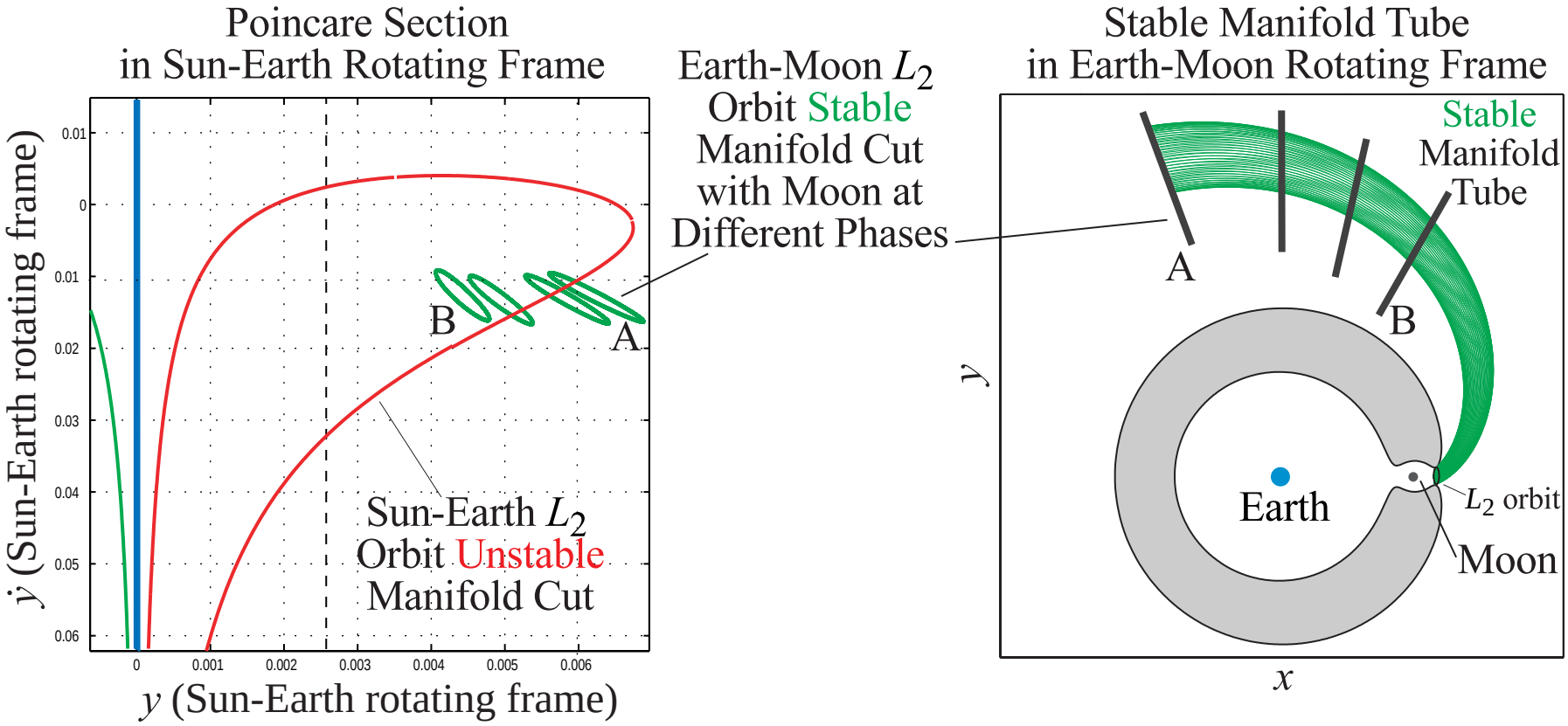
Connecting Two Portions

- Recall: Sun-Earth-Moon-SC **4-body system** as **2 coupled 3-body systems**.



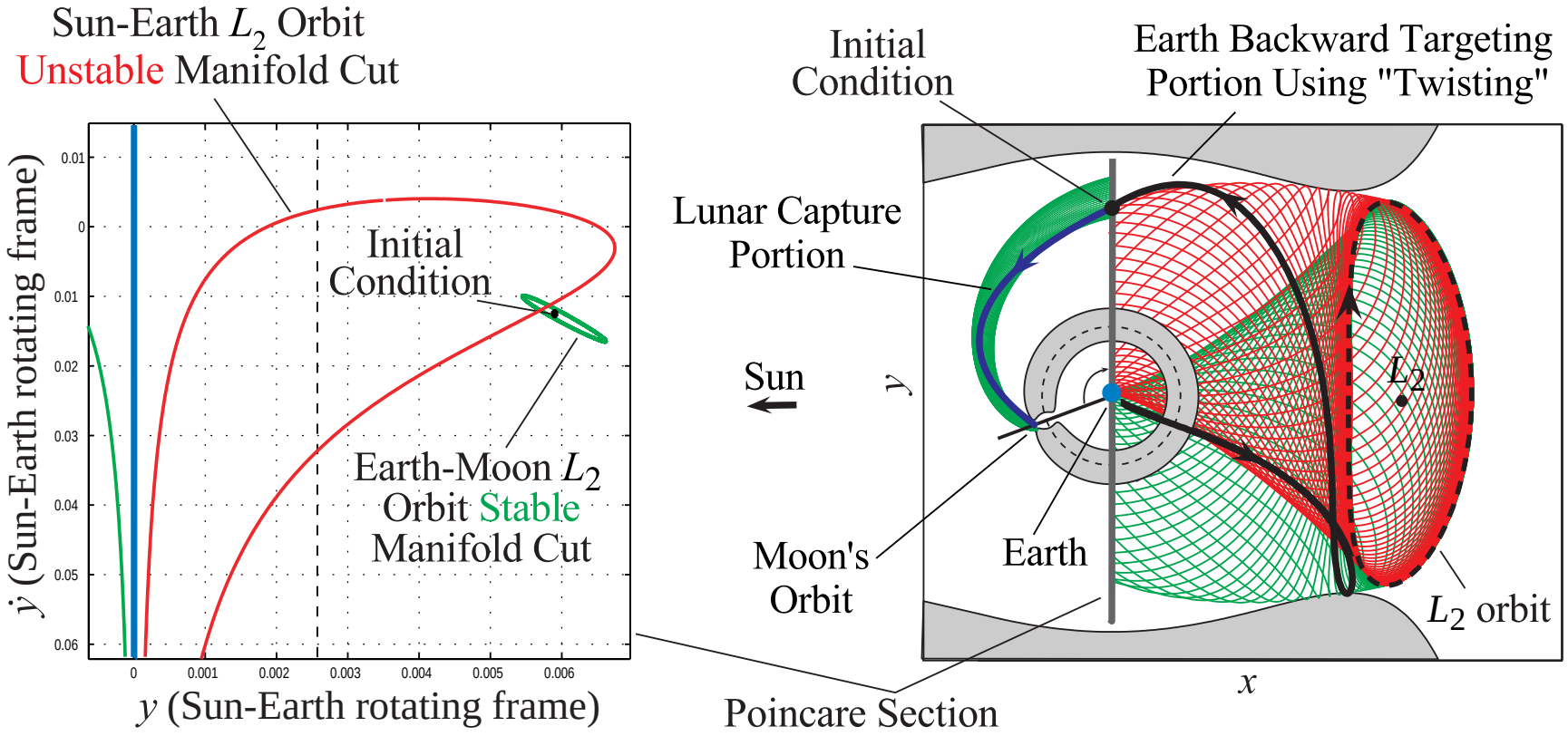
Connecting Two Portions

- Vary phase of Moon until **Earth-Moon L_2 manifold cut** intersects **Sun-Earth L_2 manifold curve**.



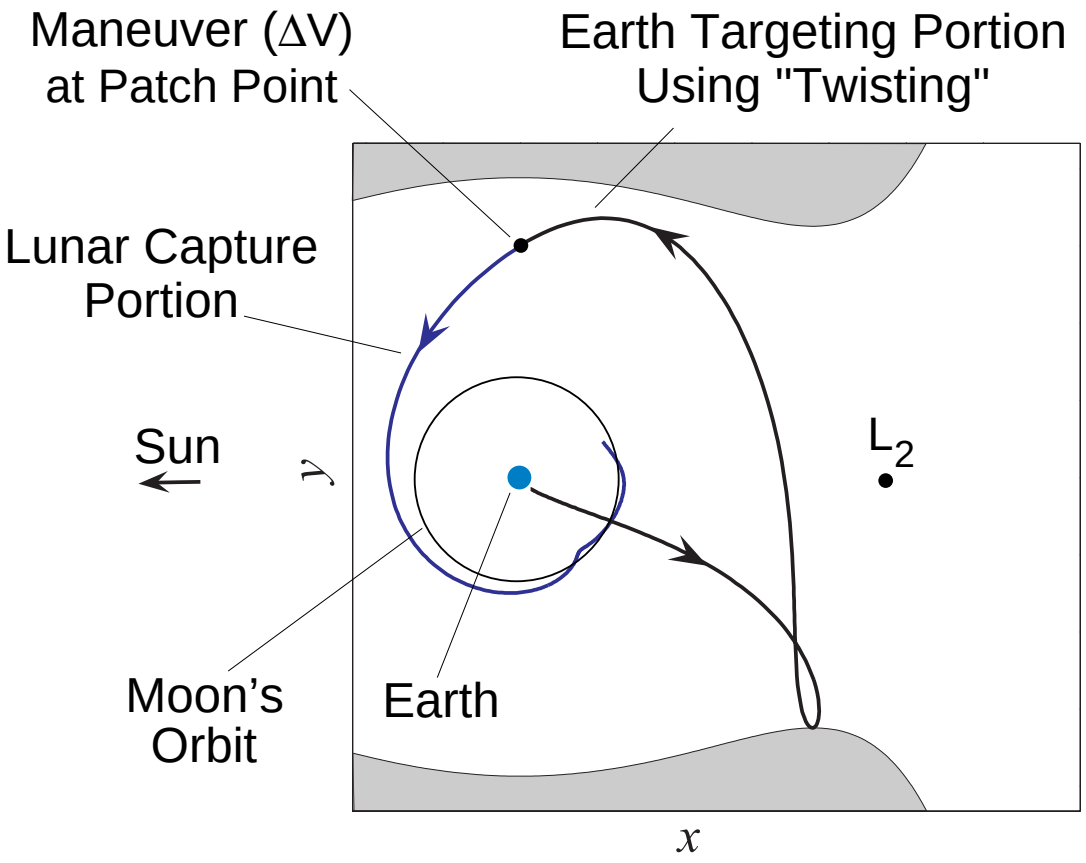
Connecting Two Portions

- Pick **initial condition** in region
  - in interior of **green curve**
  - but in exterior of **red curves**.



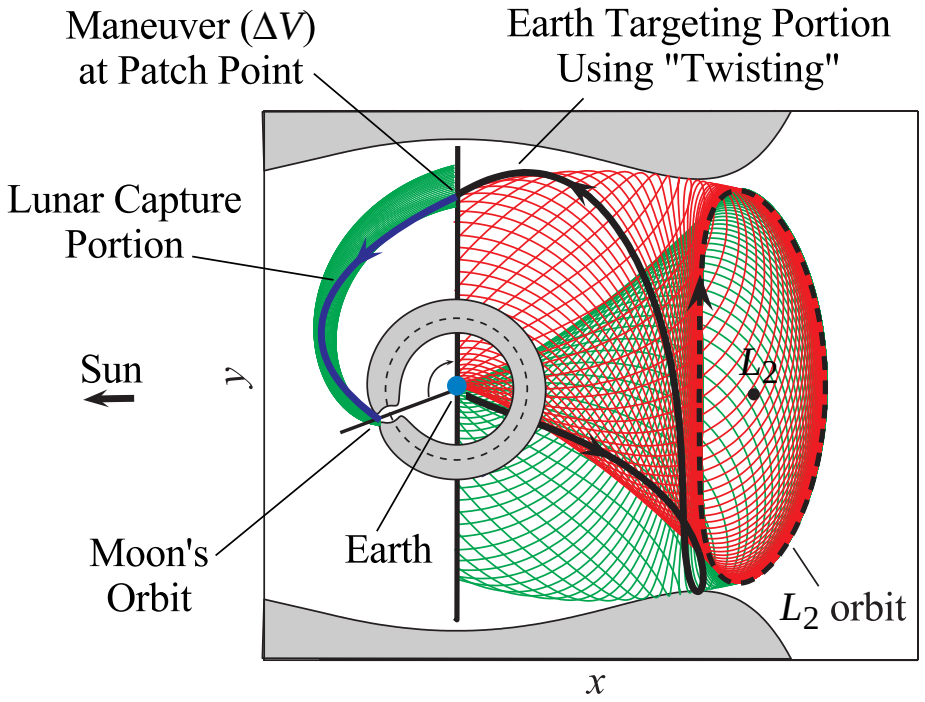
Connecting Two Portions

- With slight modification (a 34 m/s ΔV at patch point), this produces a solution in bicircular 4-body problem.
- Since capture at Moon is **natural** (zero ΔV), amount of on-board fuel needed is lowered (by about 20%).



Arguments for Coupled 3-Body Model

- Outside **Moon's sphere of influence** (20,000 km),
  - can neglect **Moon's perturbation** on S-E-SC 3-body system,
  - can use **Sun-Earth invariant manifold structure**.
- Midcourse ΔV is performed as SC is entering **Earth's sphere of influence** (900,000 km),
  - can neglect **Sun's perturbation** on E-M-SC 3-body system,
  - can use **Earth-Moon manifold structuere** for capture.



Future Work

- Using **differential correction**,
  can utilize this trajectory as initial guess
  - to find **3-dimensional** "Shoot the Moon" trajectory,
  - with full solar system model.
- **Optimize trajectory**
  - by applying optimal control (e.g., COOPT),
  - with continuous (low) thrust.
- Develop procedure for **coupling multiple 3-body systems**,
  which will aid
  - in design of innovative **space missions**
  - in understanding
    subtle non-Keplerian **transport** throughout solar system.

References and Other Informations

- **Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross**
  Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics,
  Chaos, vol. 10, (2000) pp. 427-469.
  - **http://www.cds.caltech.edu/~marsden/**
  - Clip on "current issue" of
    http://ojps.aip.org/chaos/

- **Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross**
  "Shoot the Moon."

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