Dynamical Systems, 3-Body Problem & Low Energy Transfer to the Moon

Koon, Lo, Marsden, and Ross

Wang Sang Koon
Control and Dynamical Systems, Caltech

koon@cds.caltech.edu

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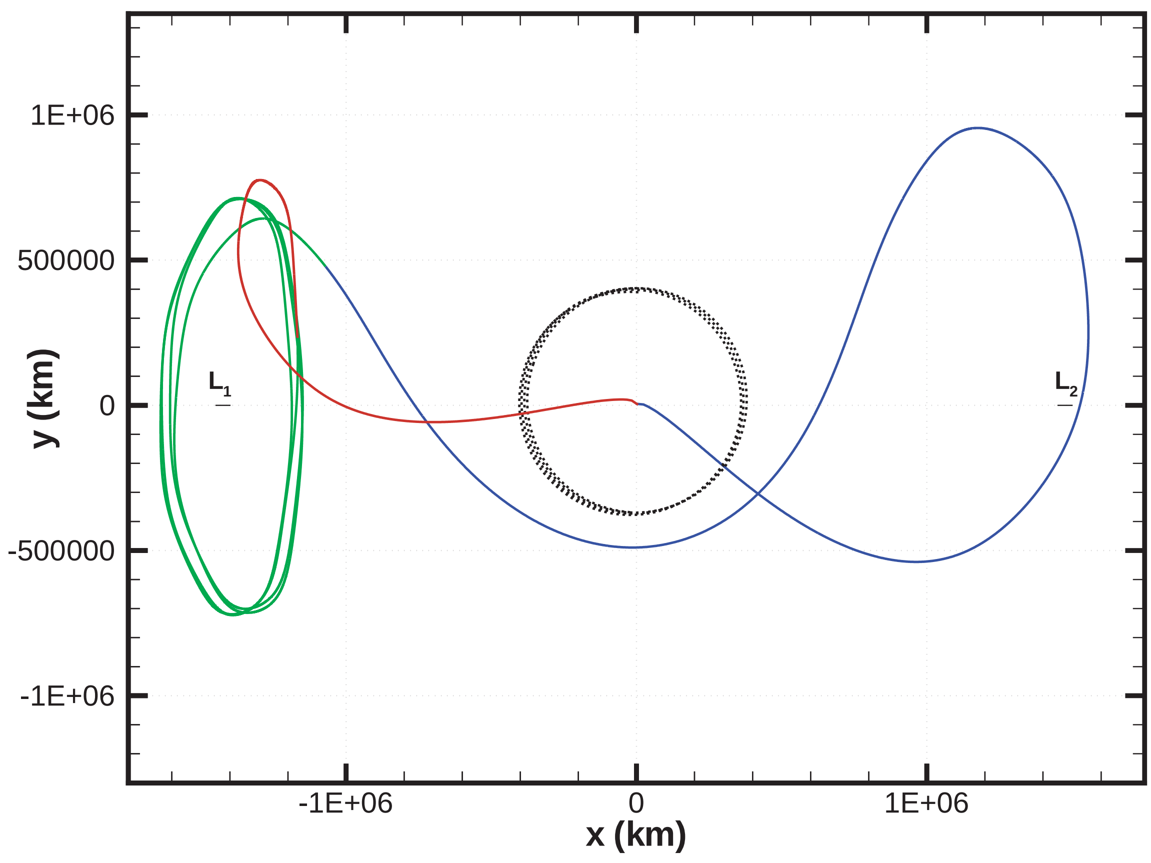
Outline of Presentation

- **Main Theme**
  - how to use dynamical systems theory of 3-body problem in space mission design.
- **Background** and **Motivation**:
  - NASA's Genesis Discovery Mission.
  - Jupiter Comets.
- **Planar Circular Restricted 3-Body Problem**.
- **Major Results** and **Some Technical Details**.
- **Low Energy Transfer to the Moon.**
- **Conclusion and Ongoing Work**.

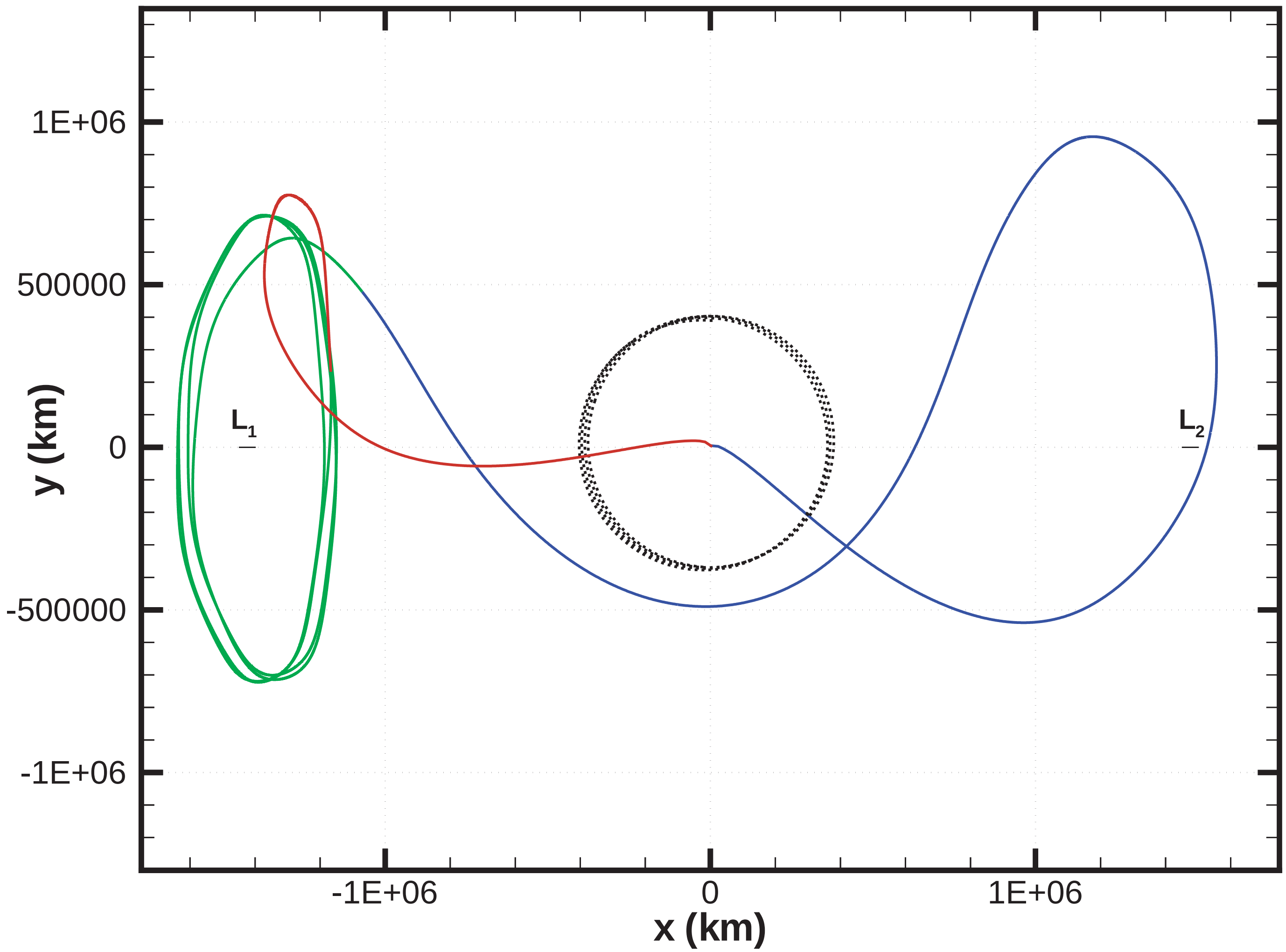
Genesis Discovery Mission

- **Genesis spacecraft** will
  - collect solar wind from a L_1 **halo orbit** for 2 years,
  - **return** those samples to Earth in 2003 for analysis.
- **Halo orbit**, **transfer**/ **return** trajectories in rotating frame.



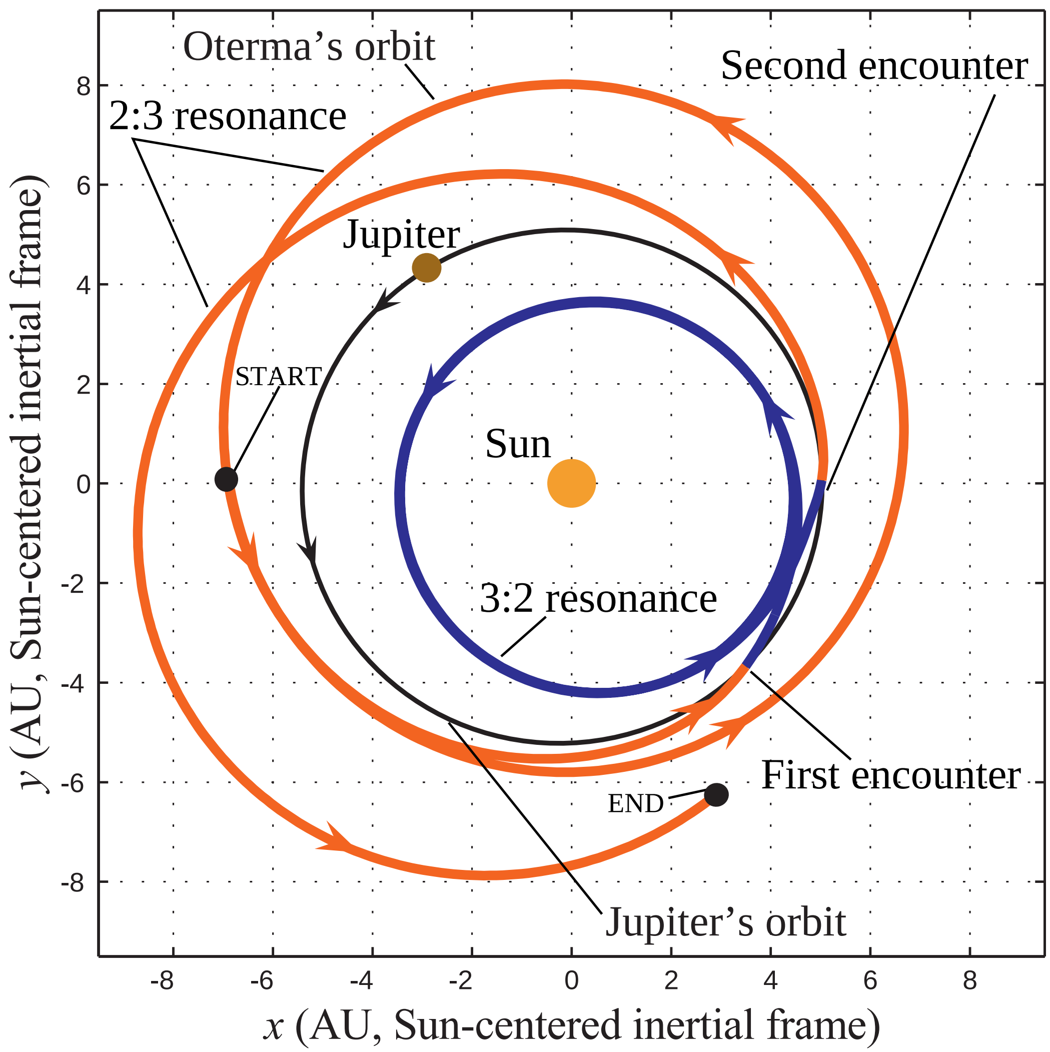
Genesis Discovery Mission

- Must return in Utah during **daytime**.
- **Return-to-Earth portion** utilizes heteroclinic dynamics.



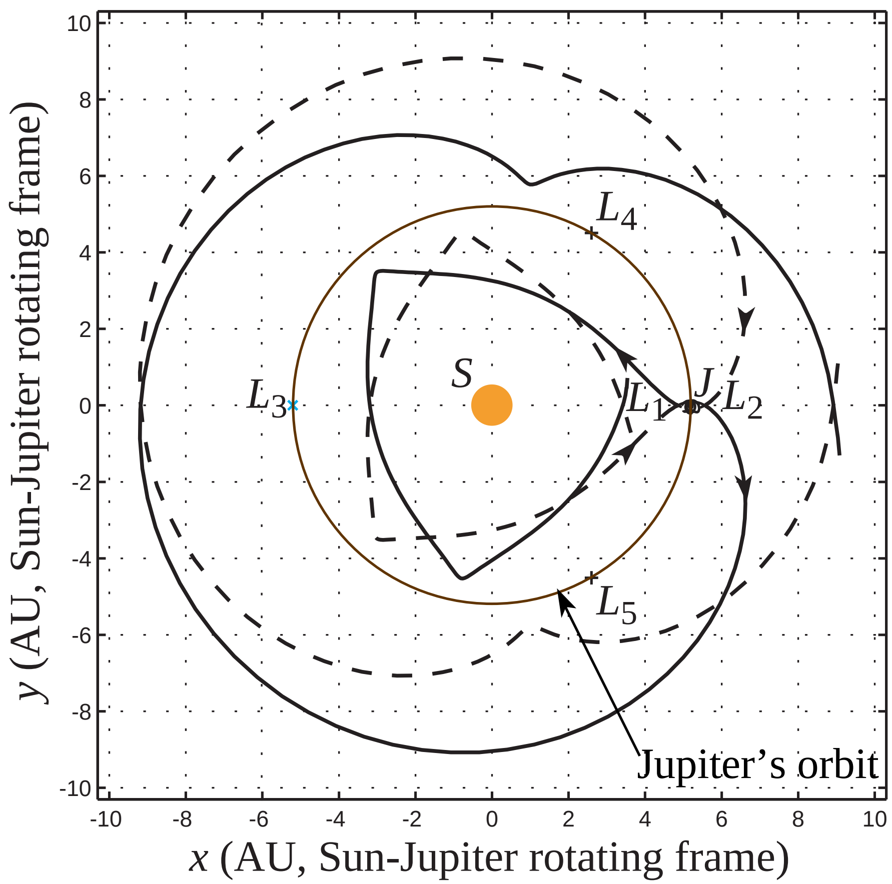
Jupiter Comets

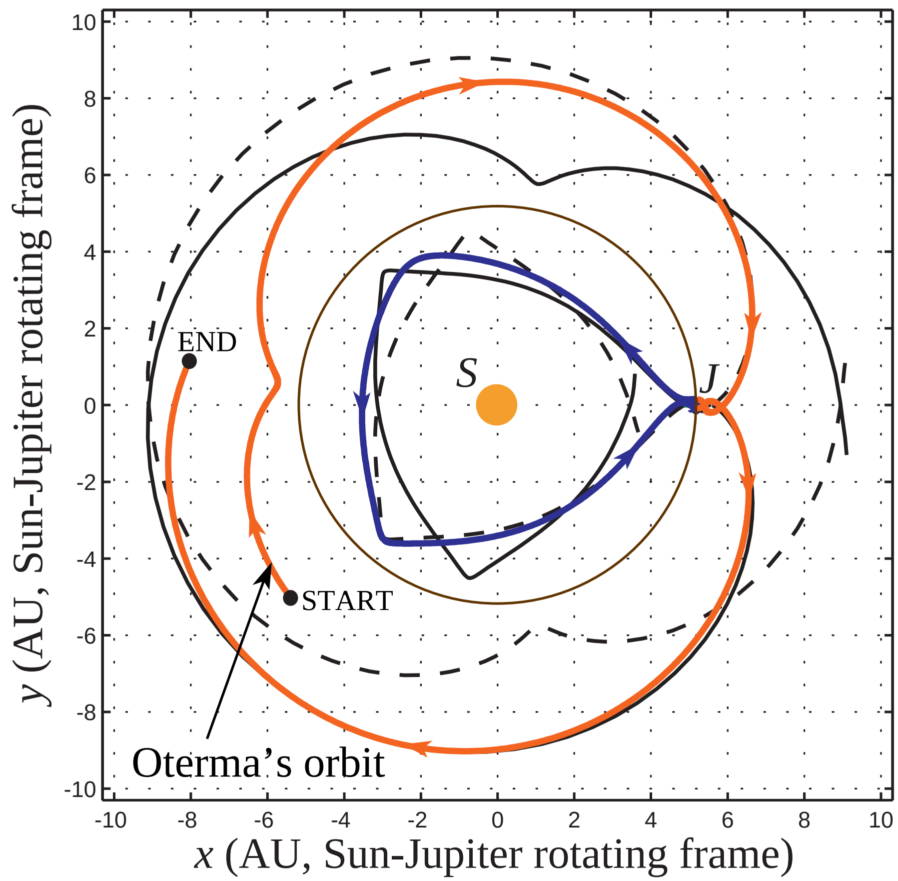
- **Rapid transition** from **outside** to **inside** Jupiter's orbit.
- **Captured temporarily** by Jupiter during transition.
- **Exterior** (**2:3 resonance**). **Interior** (**3:2 resonance**).



Jupiter Comets

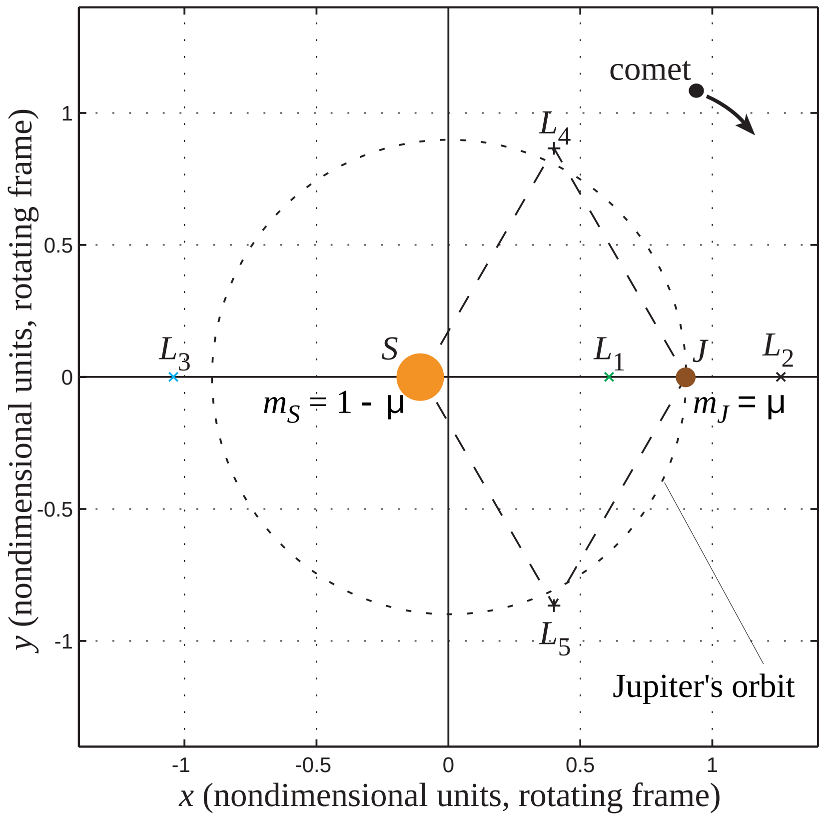
- Belbruno/B. Marsden [1997]
- Lo/Ross [1997] :
  - Comet in **rotating frame** follows **invariant manifolds**.
- Jupiter comets make **resonance transition** near L_1 and L_2.





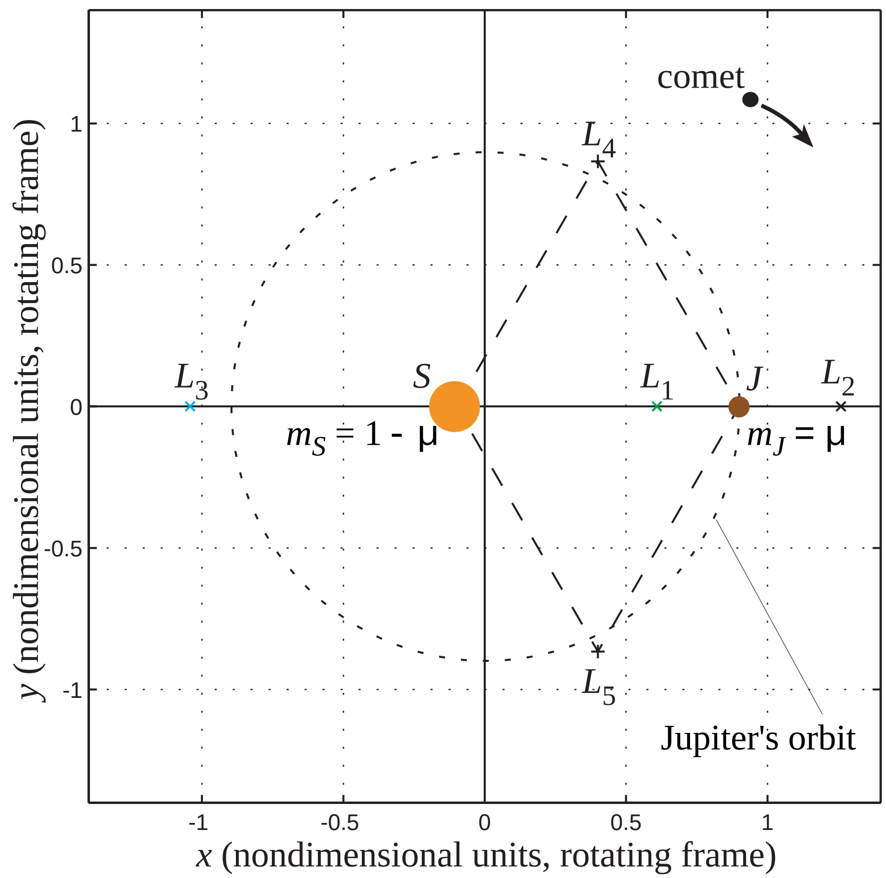
Planar Circular Restricted 3-Body Problem

- **PCR3BP** is a good starting model:
  - Comets mostly **heliocentric**, but their perturbation dominated by **Jupiter's gravitation**.
  - Their motion nearly in Jupiter's **orbital plane**.
  - Jupiter's small **eccentricity** plays little role during transition.



Planar Circular Restricted 3-Body Problem

- 2 main bodies: **Sun** and **Jupiter**.
  - Total mass normalized to 1: $m_J = \mu, \quad m_S = 1 - \mu.$
  - Rotate about center of mass, angular velocity normalized to 1.
- Choose a **rotating** coordinate system with $(0, 0)$ at center of mass, **S** and **J** fixed at $(-\mu, 0)$ and $(1 - \mu, 0)$.

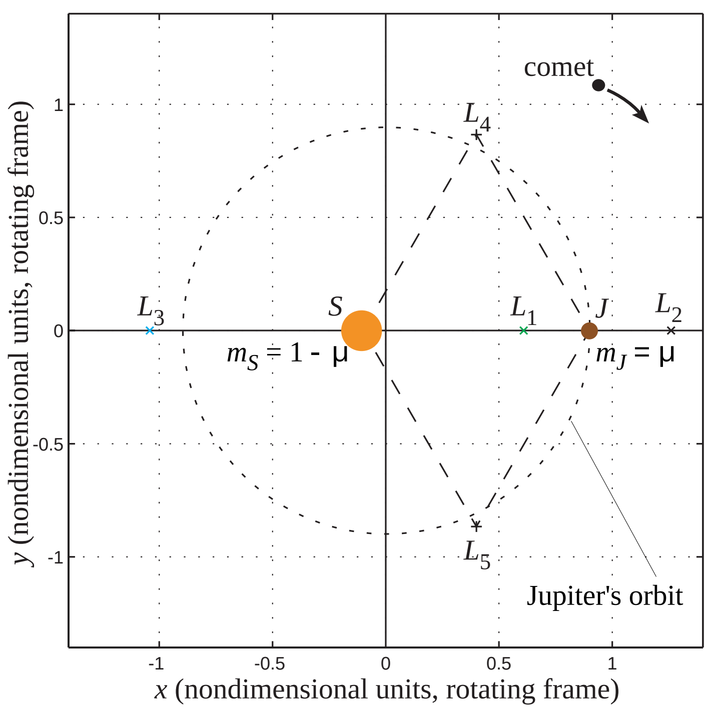


Equilibrium Points (PCR3BP)

- Comet's equations of motion are

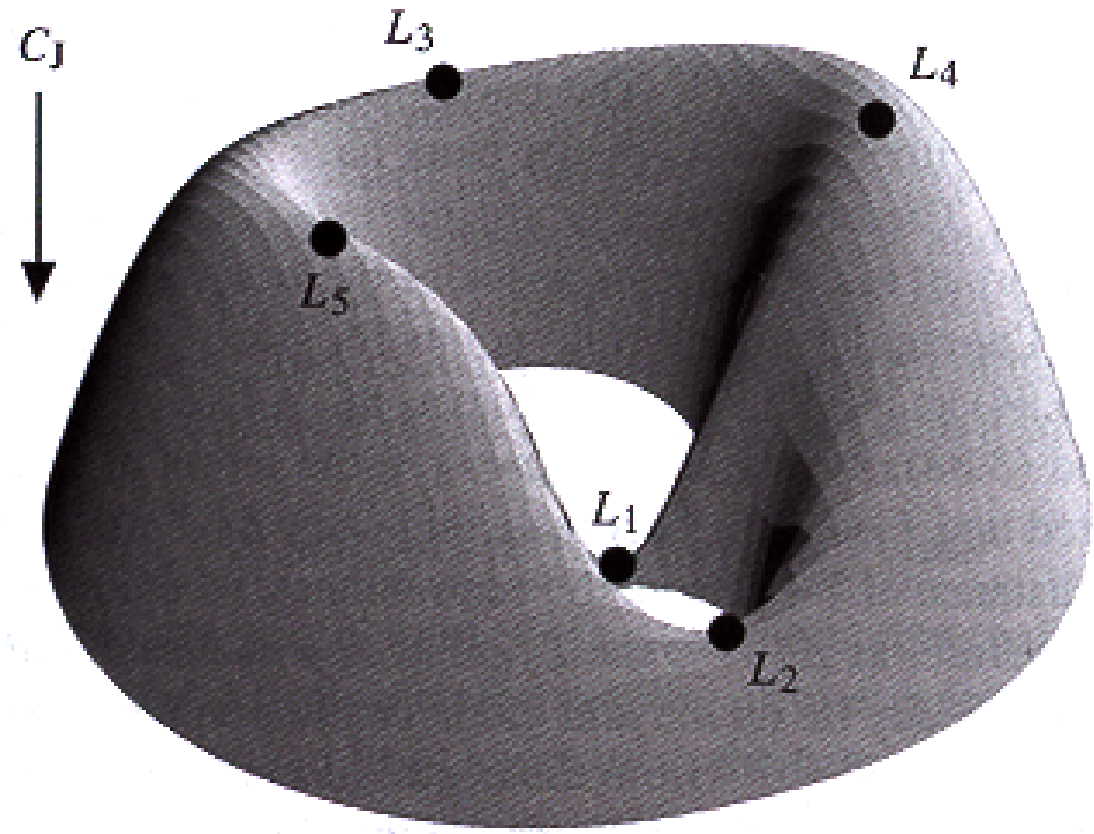
$$\ddot{x} - 2\dot{y} = -\frac{\partial U}{\partial x} \quad \ddot{y} + 2\dot{x} = -\frac{\partial U}{\partial y} \quad U = -\frac{x^2 + y^2}{2} - \frac{1-\mu}{r_s} - \frac{\mu}{r_j}$$

- Five equilibrium points:
  - 3 **unstable** equilibrium points on S-J line, L_1, L_2, L_3.
  - 2 equilateral equilibrium points, L_4, L_5.



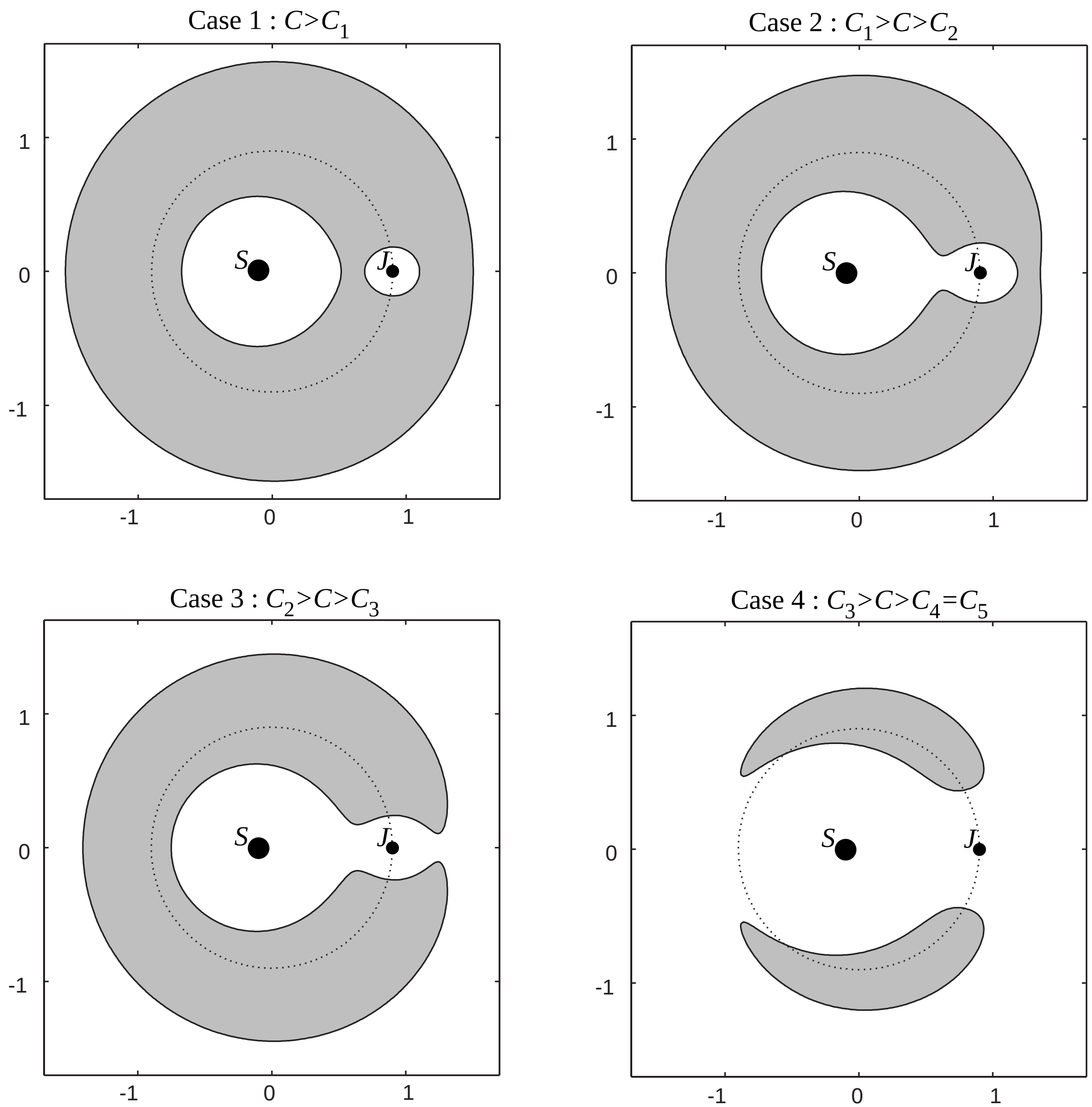
Hill's Region (PCR3BP)

- **Energy integral**: $E(x, y, \dot{x}, \dot{y}) = (\dot{x}^2 + \dot{y}^2)/2 + U(x, y)$.
- E can be used to determine (**Hill's**) **region** in position space where comet is energetically permitted to move.
- **Effective potential**: $U(x, y) = -\frac{x^2+y^2}{2} - \frac{1-\mu}{r_s} - \frac{\mu}{r_j}$.



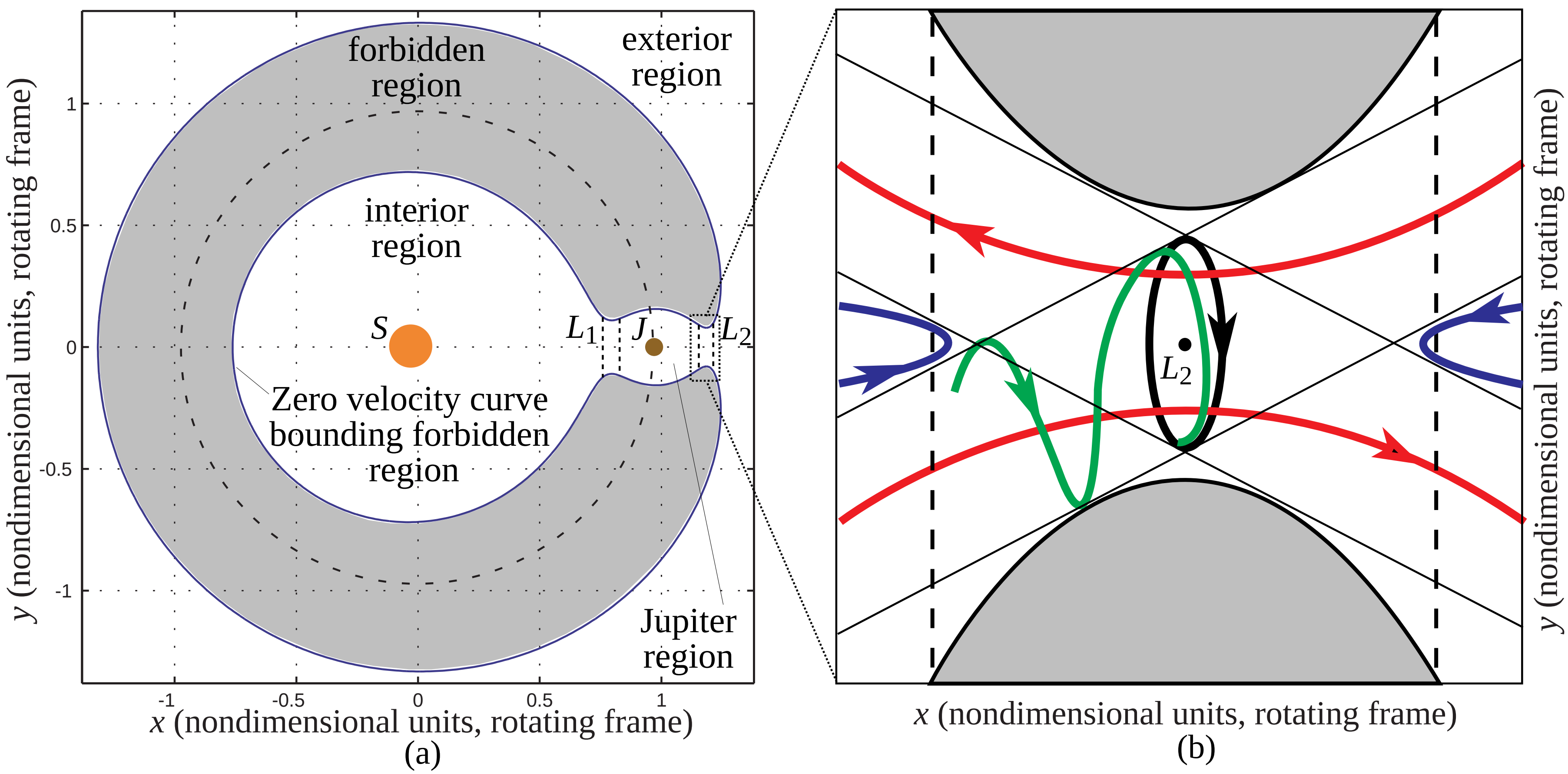
Hill's Region (PCR3BP)

- To fix **energy value** E is to fix **height** of plot of $U(x, y)$. **Contour plots** give **5** cases of Hill's region.



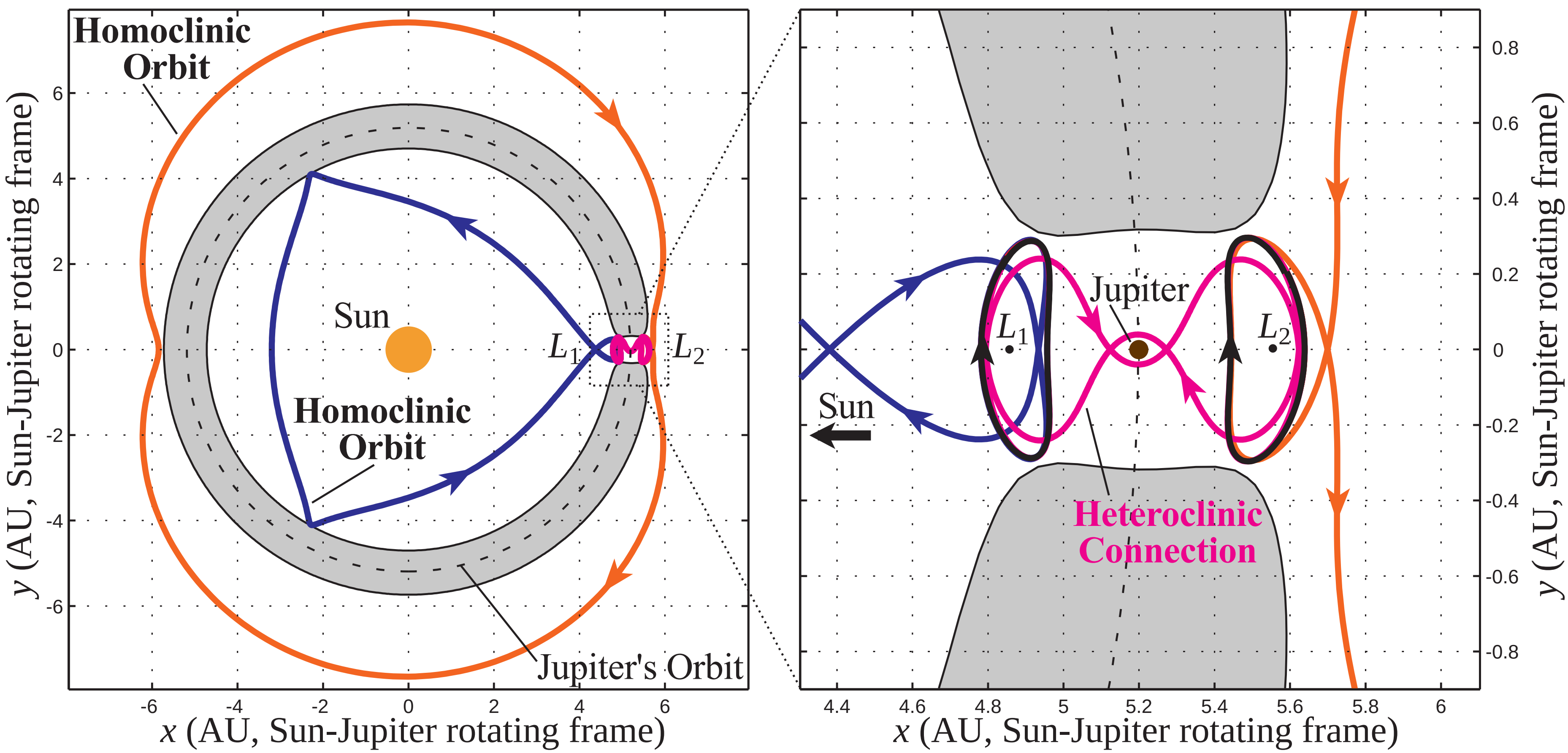
The Flow near L_1 and L_2

- For **energy value** just above that of L_2, **Hill's region** contains a **"neck"** about L_1 & L_2.
- Comet can make **transition** through these equilibrium regions.
- 4 types of orbits: **periodic**, **asymptotic**, **transit** & **nontransit**.



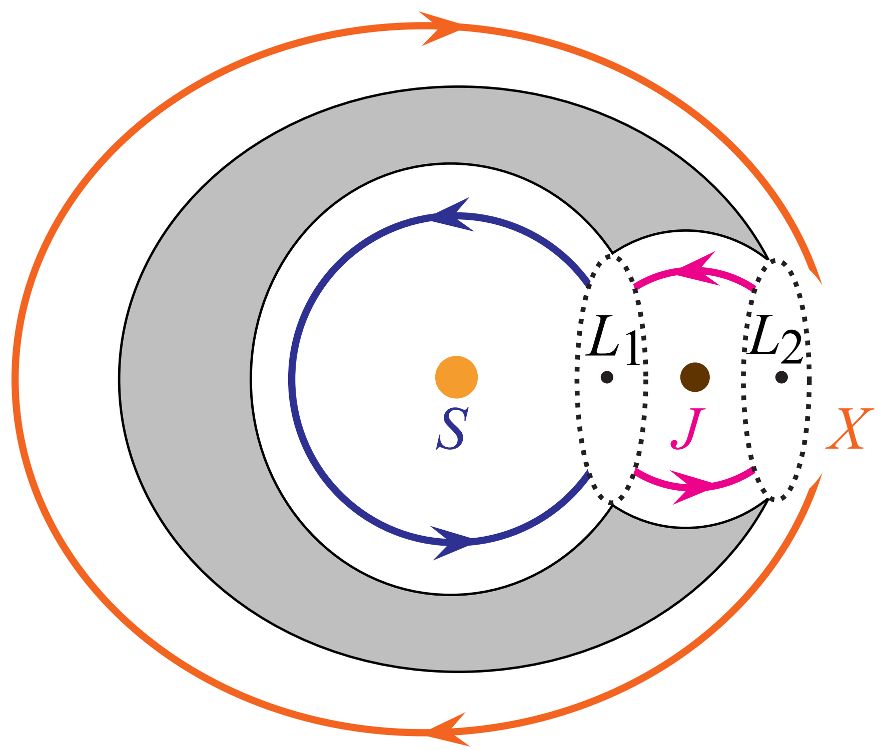
Major Result (A): Heteroclinic Connection

- Found **heteroclinic connection** between pair of periodic orbits.
- Found a large class of **orbits** near this (homo/heteroclinic) ***chain***.
- Comet can follow these ***channels*** in rapid transition.



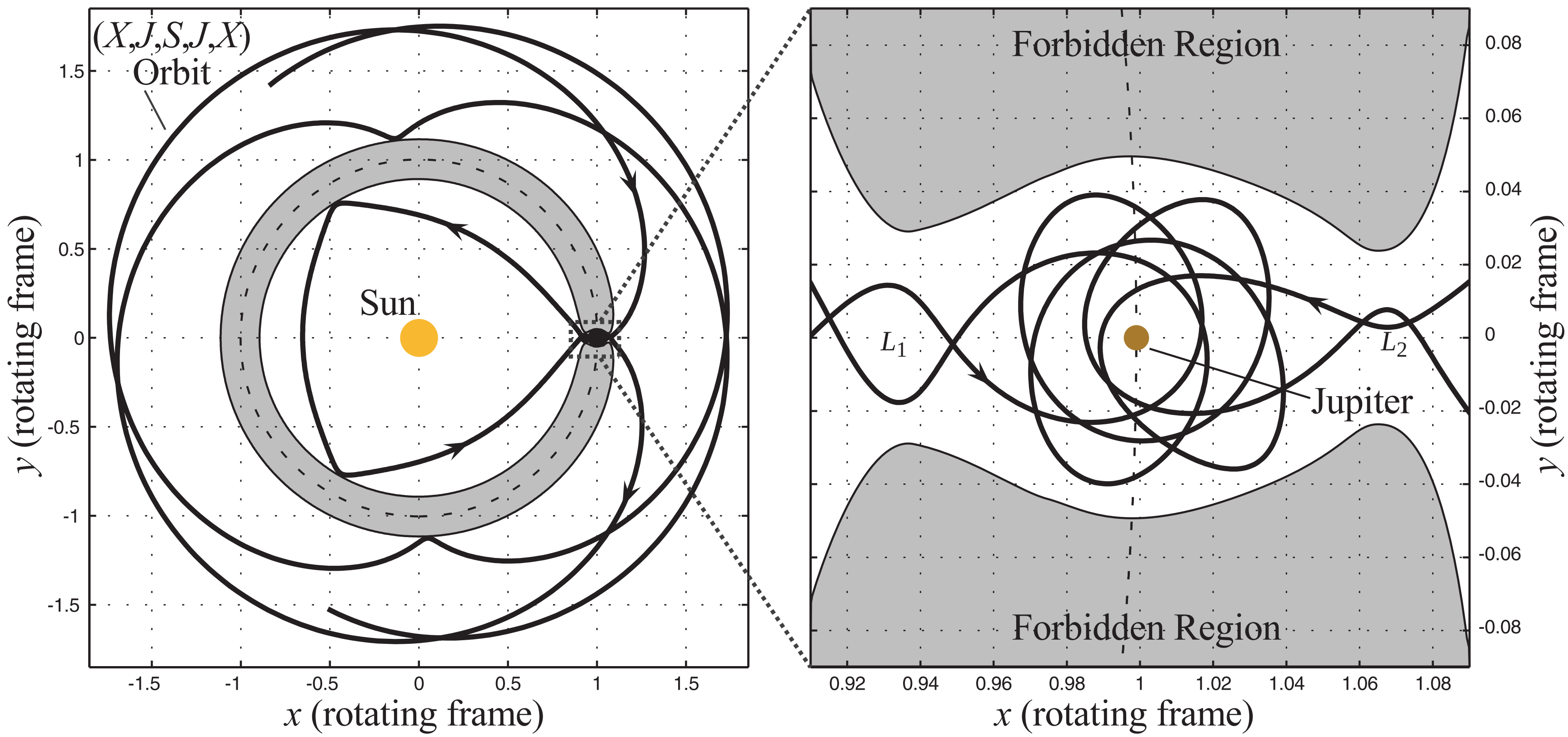
Major Result (B): Existence of Transitional Orbits

- **Symbolic sequence** used to label itinerary of each comet orbit.
- ***Main Theorem***: For any admissible **itinerary**, e.g., $(\ldots, \mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X}, \ldots)$, there exists an orbit whose **whereabouts** matches this **itinerary**.
- Can even specify **number of revolutions** the comet makes around Sun & Jupiter (plus L_1 & L_2).



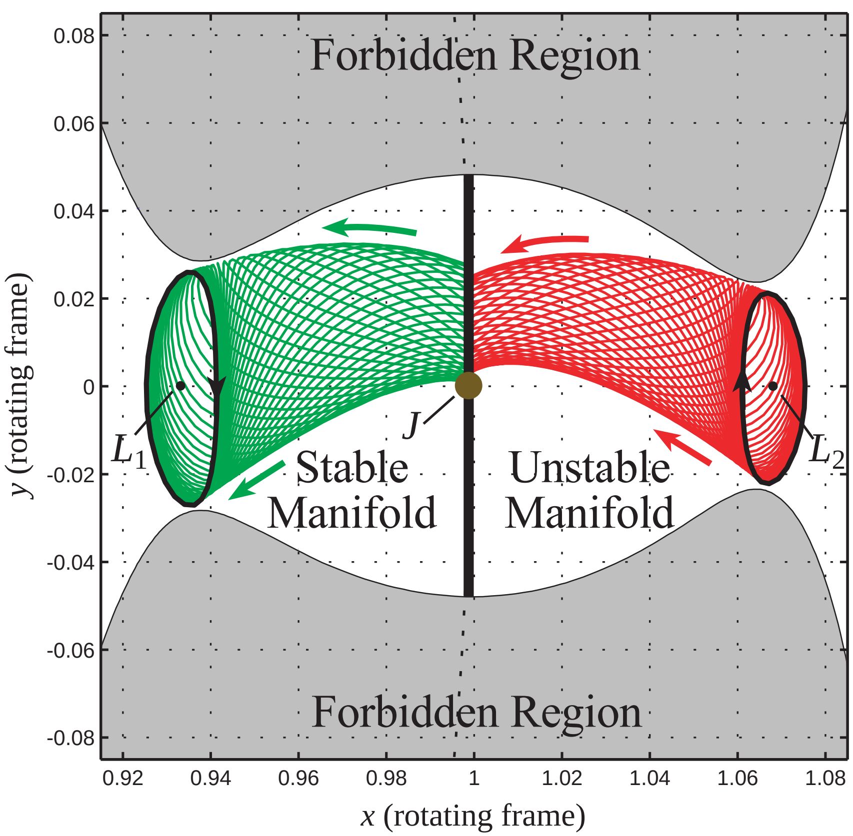
Major Result (C): Numerical Construction of Orbits

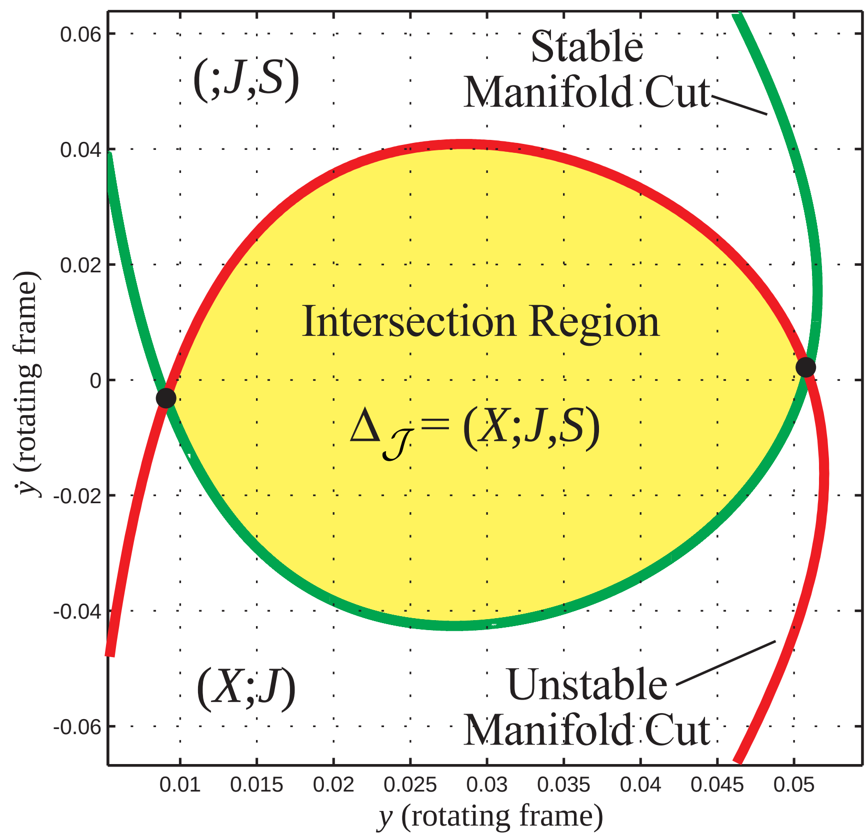
- Developed procedure to construct orbit with **prescribed itinerary**.
- Example: An orbit with itinerary $(\mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X})$.



■ Details: Construction of $(\mathbf{J}, \mathbf{X}; \mathbf{J}, \mathbf{S}, \mathbf{J})$ Orbits

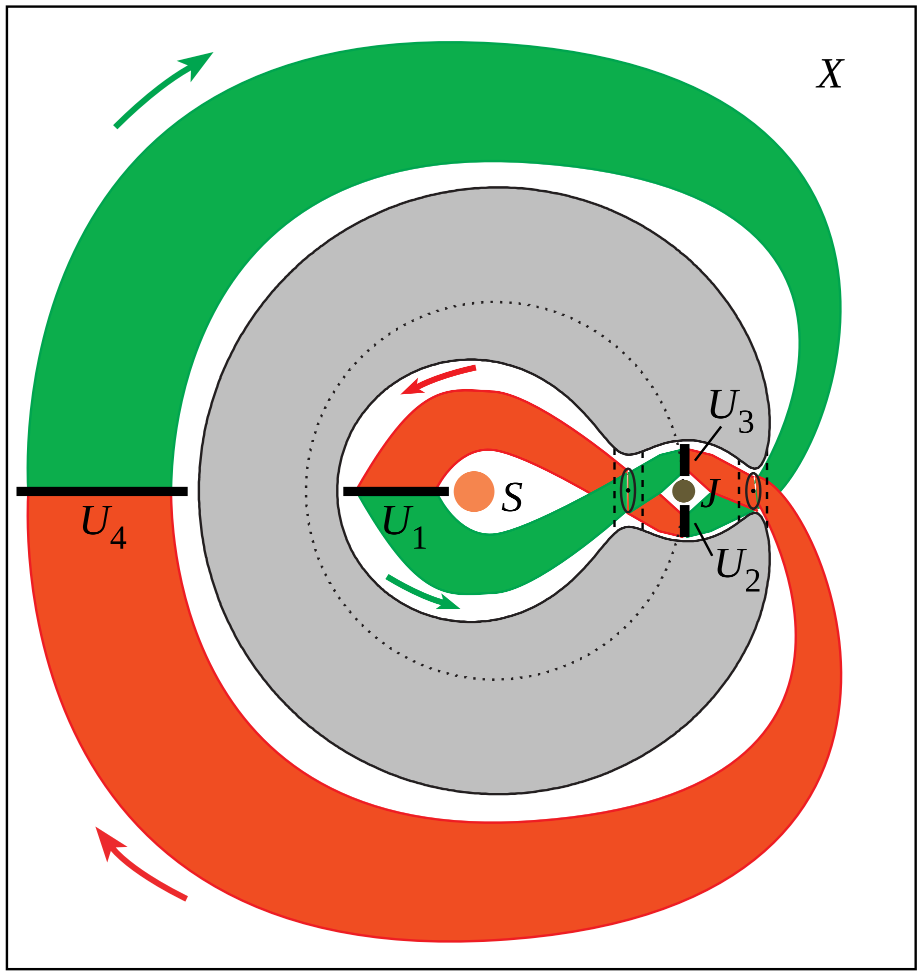
- Invariant manifold **tubes** separate transit from nontransit orbits.
- **Green curve** (Poincaré cut of L_1 **stable manifold**).
  Red curve (cut of L_2 **unstable manifold**).
- Any point inside the intersection region Δ_J is a $(\mathbf{X}; \mathbf{J}, \mathbf{S})$ orbit.





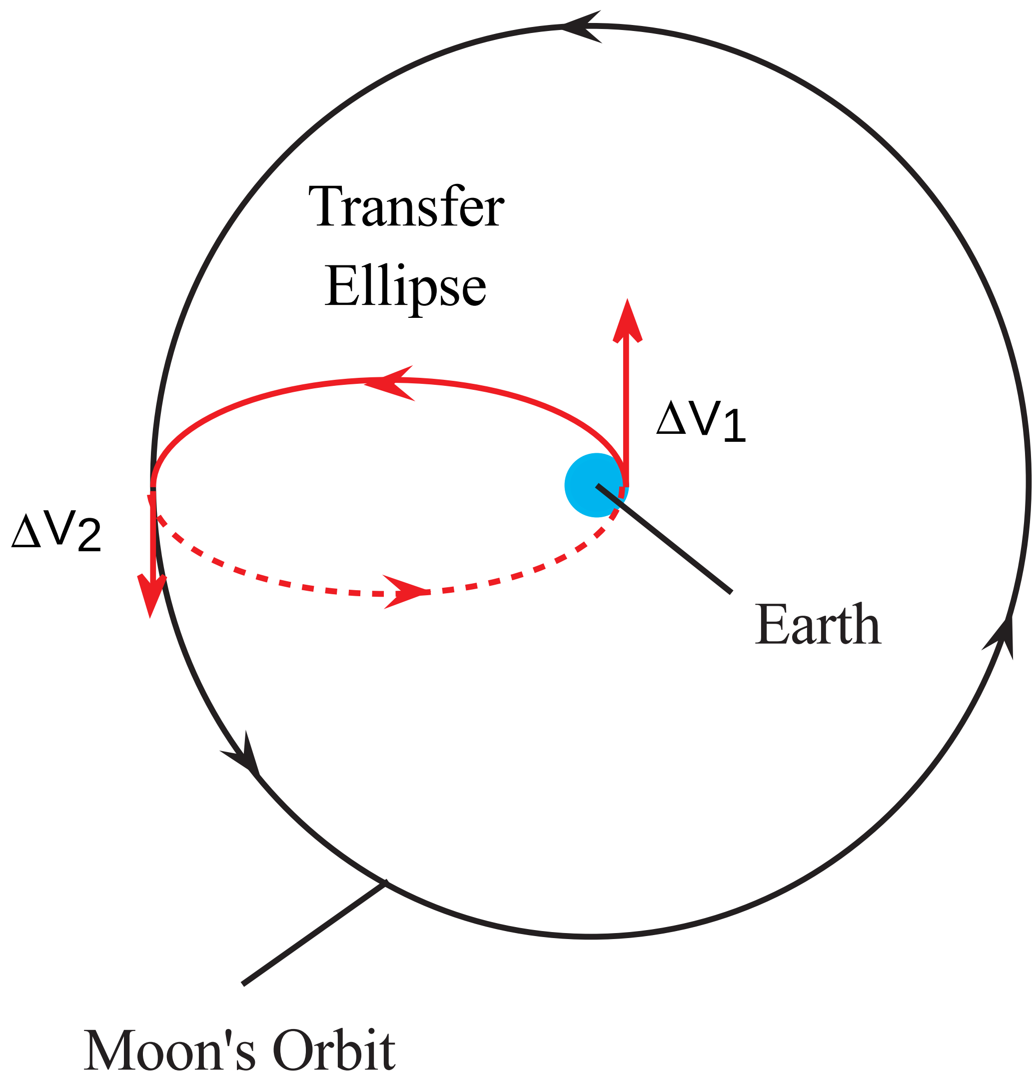
Details: Construction of $(J, X; J, S, J)$ Orbits

- The desired orbit can be constructed by
  - Choosing appropriate **Poincaré sections** and
  - linking invariant **manifold tubes** in right order.



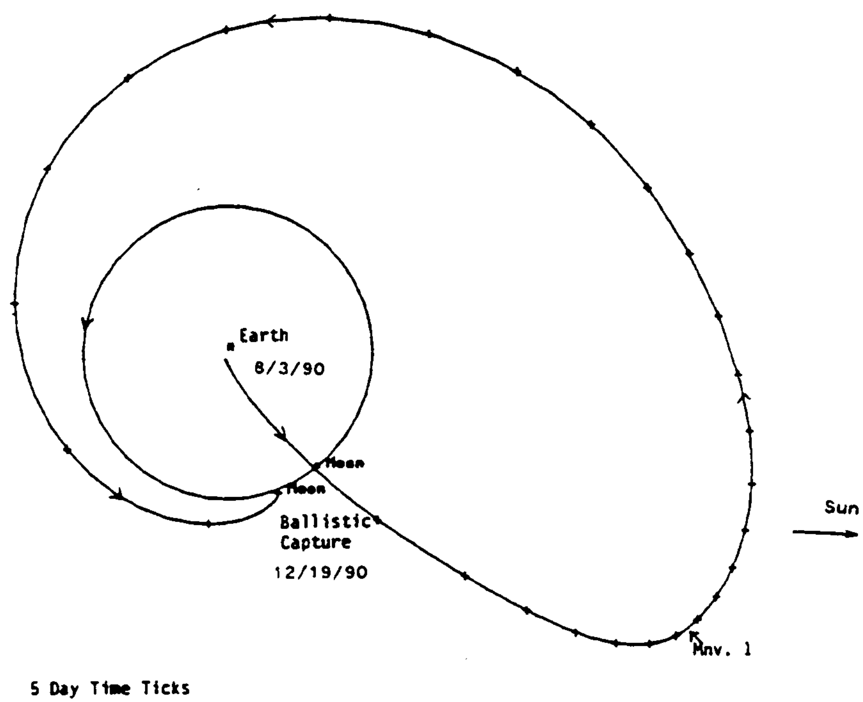
Low Energy Transfer to the Moon

- Traditional transfer from Earth to Moon is by **Hohmann transfer**. See Apollo mission.
- 2 body Keplerian **ellipse** from Earth to Moon. Need 2 $\Delta V s$.



Low Energy Transfer to the Moon

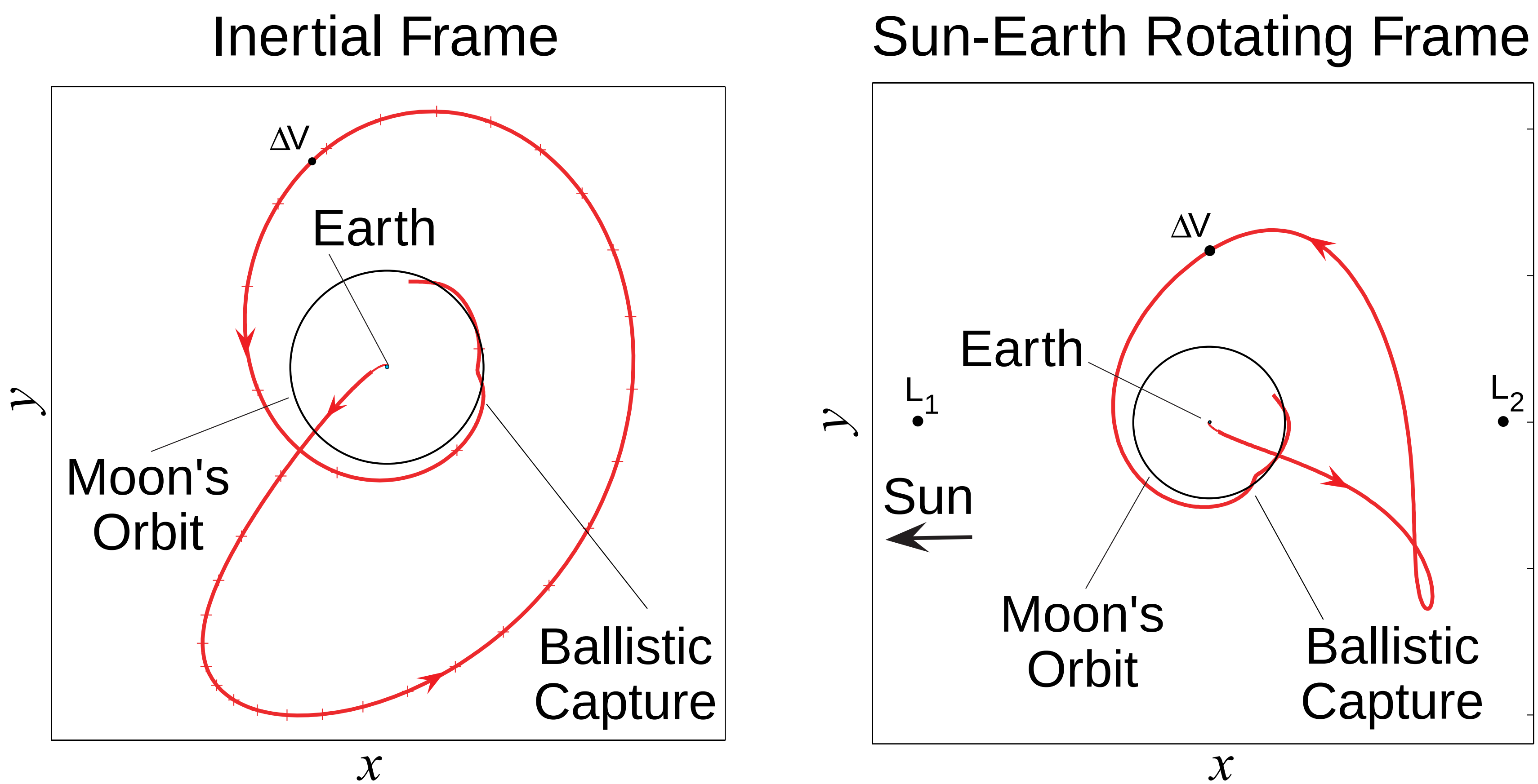
- In 1991, Muses-A did not have enough propellant to reach Moon by **Hohmann transfer**.
- Belbruno/Miller designed a **Sun**-assisted **Earth-to-Moon transfer** with **ballistic capture** at Moon.
- Similar techniques used by Japanese team to save mission.



Numerical simulation of a ballistic capture transfer trajectory for the Japanese spacecraft Hiten: ecliptic plane projection, sun's direction indicated at Earth injection. (from Belbruno and Miller [1993])

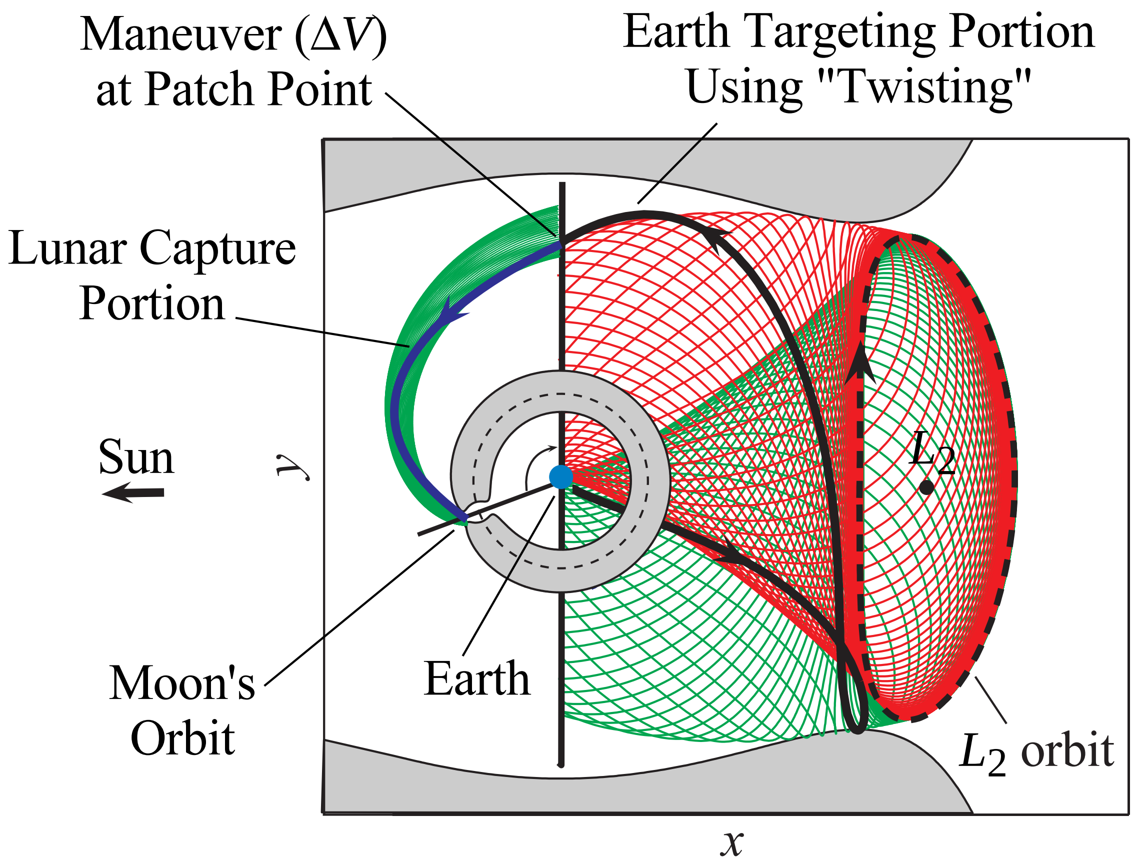
Two Coupled 3-Body Systems

- We provide a theoretical basis and a numerical procedure for constructing such ballistic capture transfer.
- By considering **Sun-Earth-Moon-SC 4-body system** as **2 coupled 3-body systems**.
- Better seen in Sun-Earth **rotating frame**.



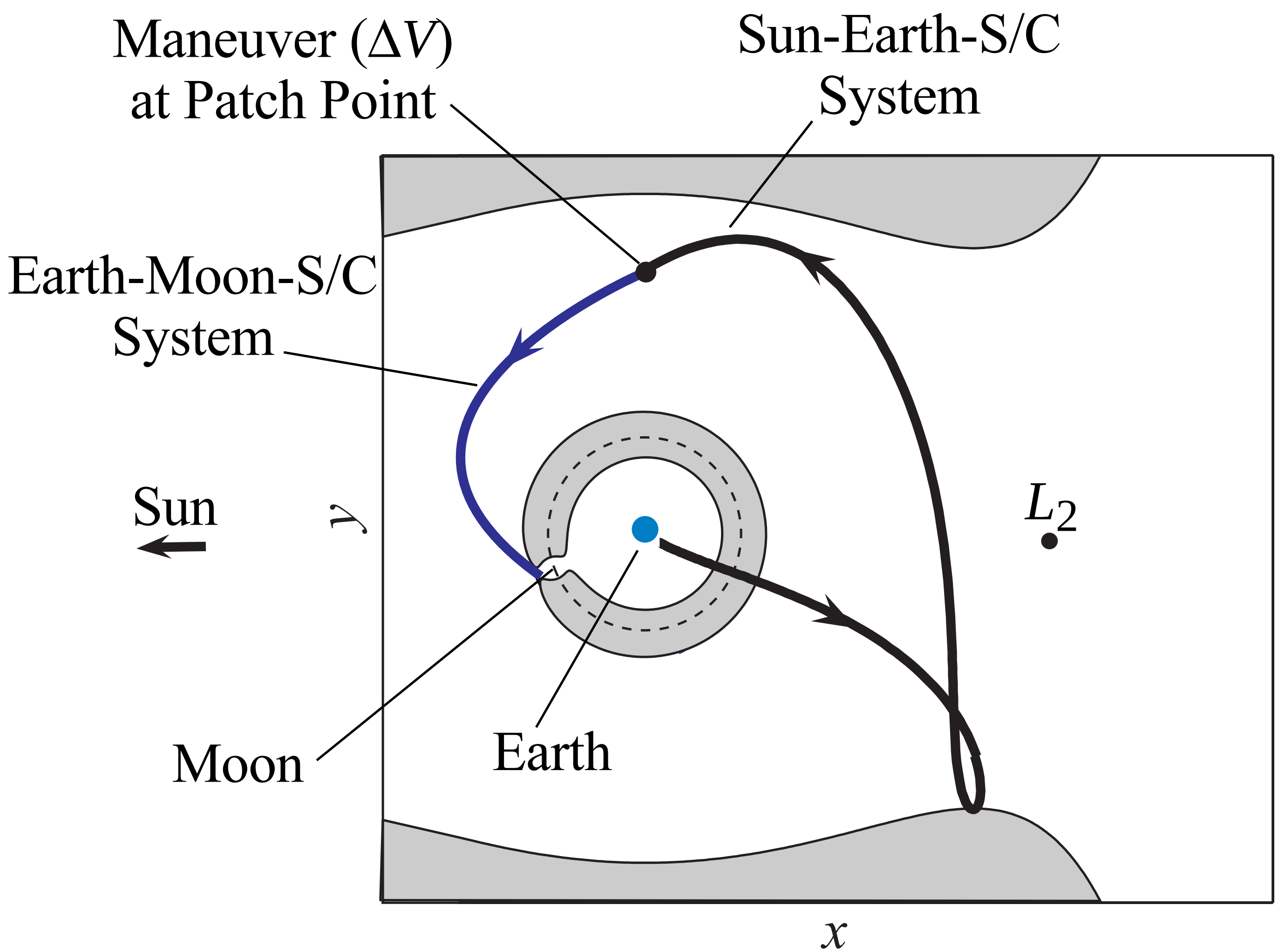
Two Coupled 3-Body Systems

- Find **position/velocity** for spacecraft
  - integrating **forward**, SC guided by **Earth-Moon manifold** and get ballistically captured at Moon;
  - integrating **backward**, SC hugs **Sun-Earth manifolds** with a twist and return to Earth.



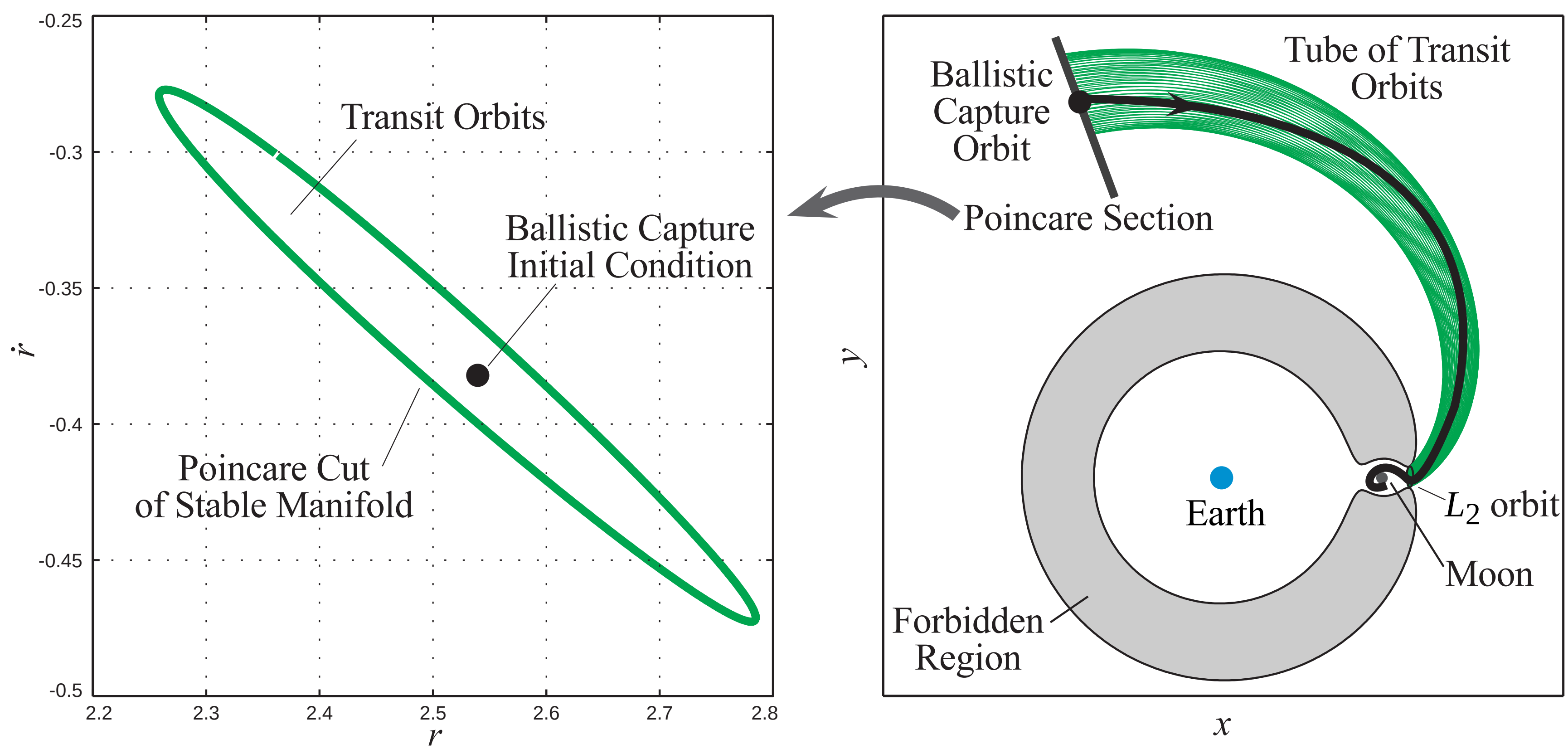
Two Coupled 3-Body Systems

- In **Sun-Earth rotating frame**, we have
  - **Sun-Earth libration point portion**.
  - **Lunar ballistic capture portion**.



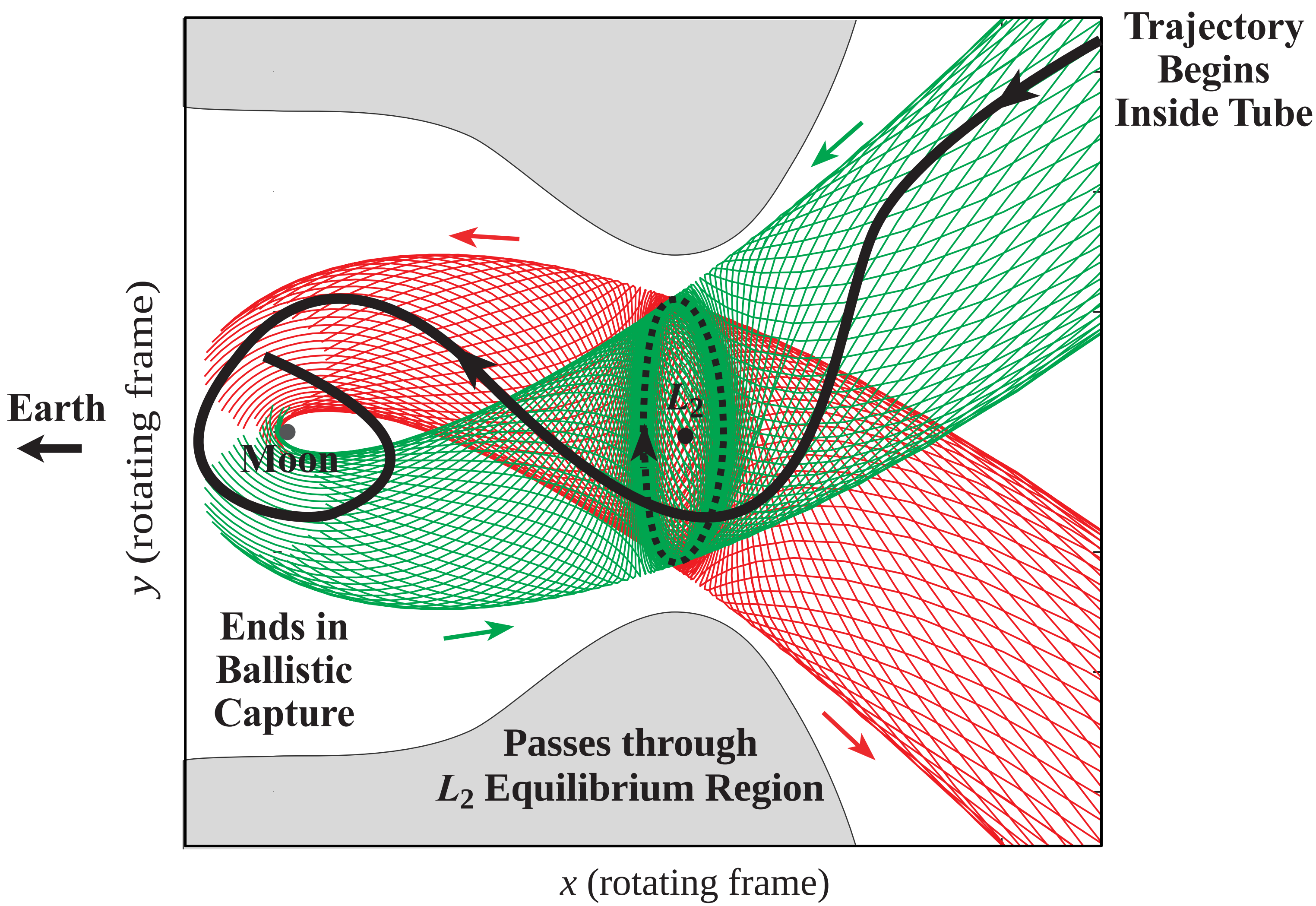
Lunar Ballistic Capture Portion

- **Stable manifold tube** provides **temporary ballistic capture** mechanism by the Moon.
- Picking a point inside **stable manifold cut** and integrating forward, spacecraft gets **ballistic capture** by Moon.



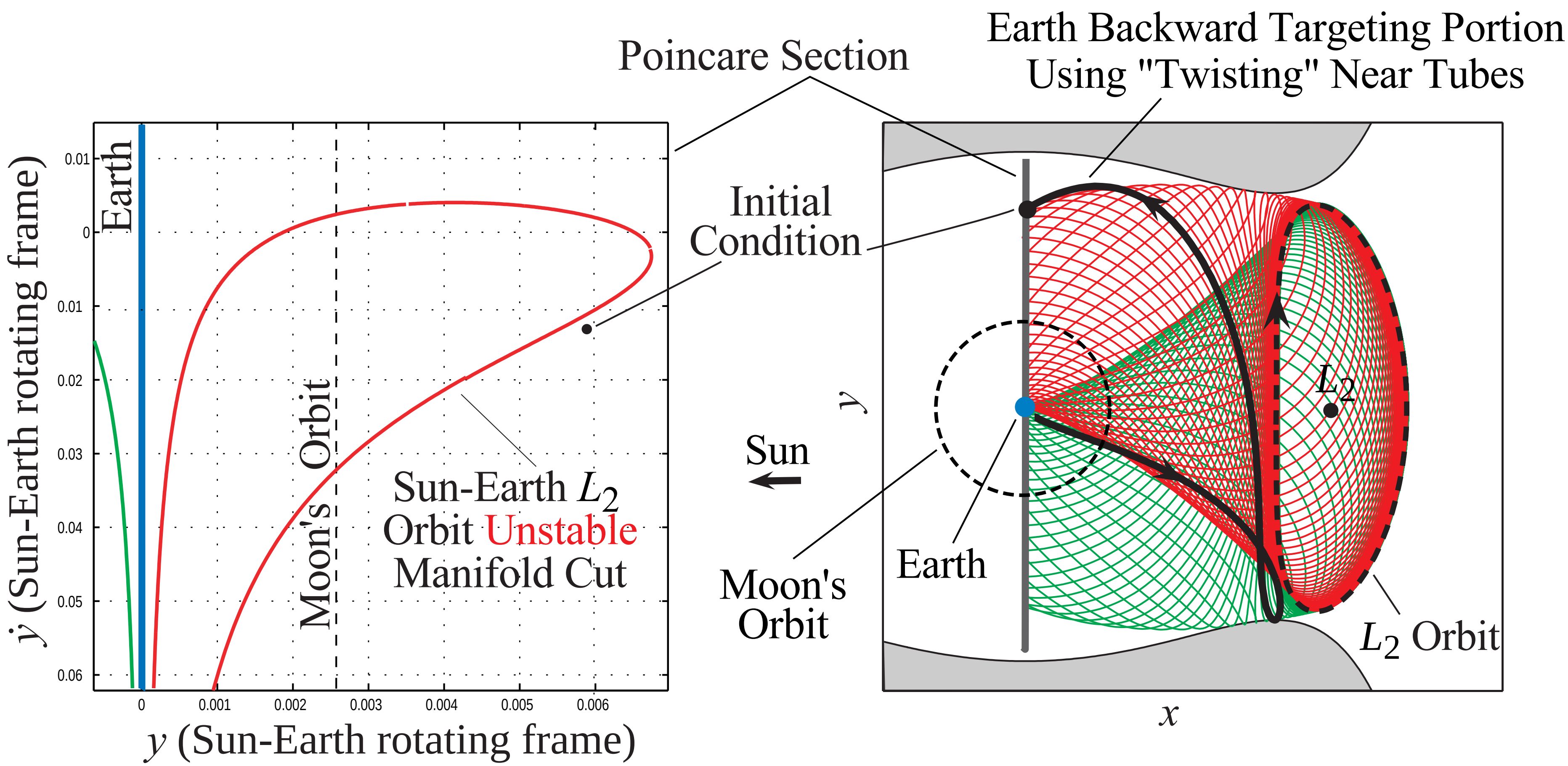
Lunar Ballistic Capture Portion

- By saving (on-board) fuel for lunar **ballistic capture** portion, this design uses **less** fuel than Earth-to-Moon **Hohmann transfer**.



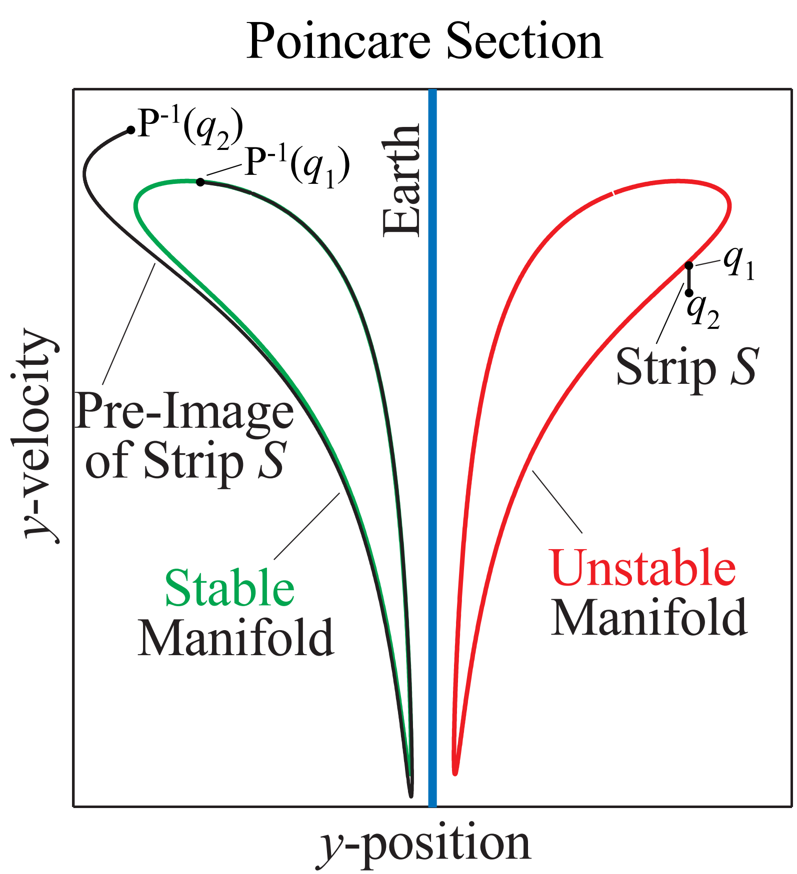
Sun-Earth Libration Point Portion

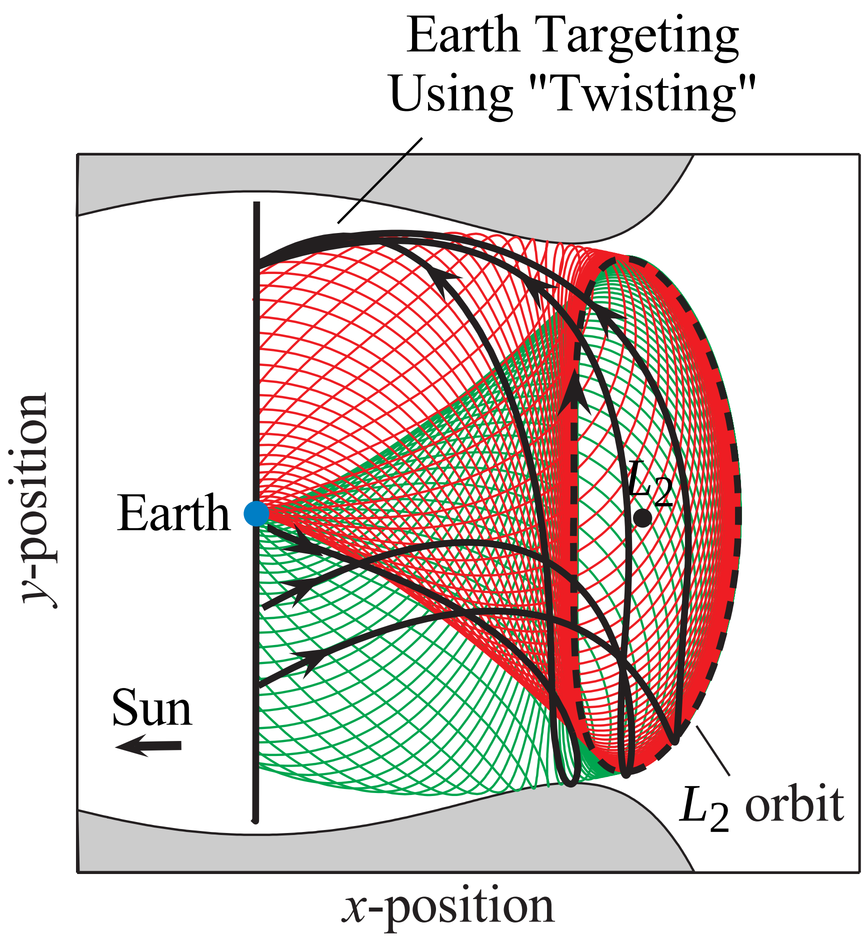
- Pick **initial condition** outside **Poincaré cut**, backward integrate to produce a trajectory:
  - hugs **unstable** manifold back to L_2 region with a **twist**,
  - hugs **stable** manifold back towards Earth.



Sun-Earth Libration Point Portion

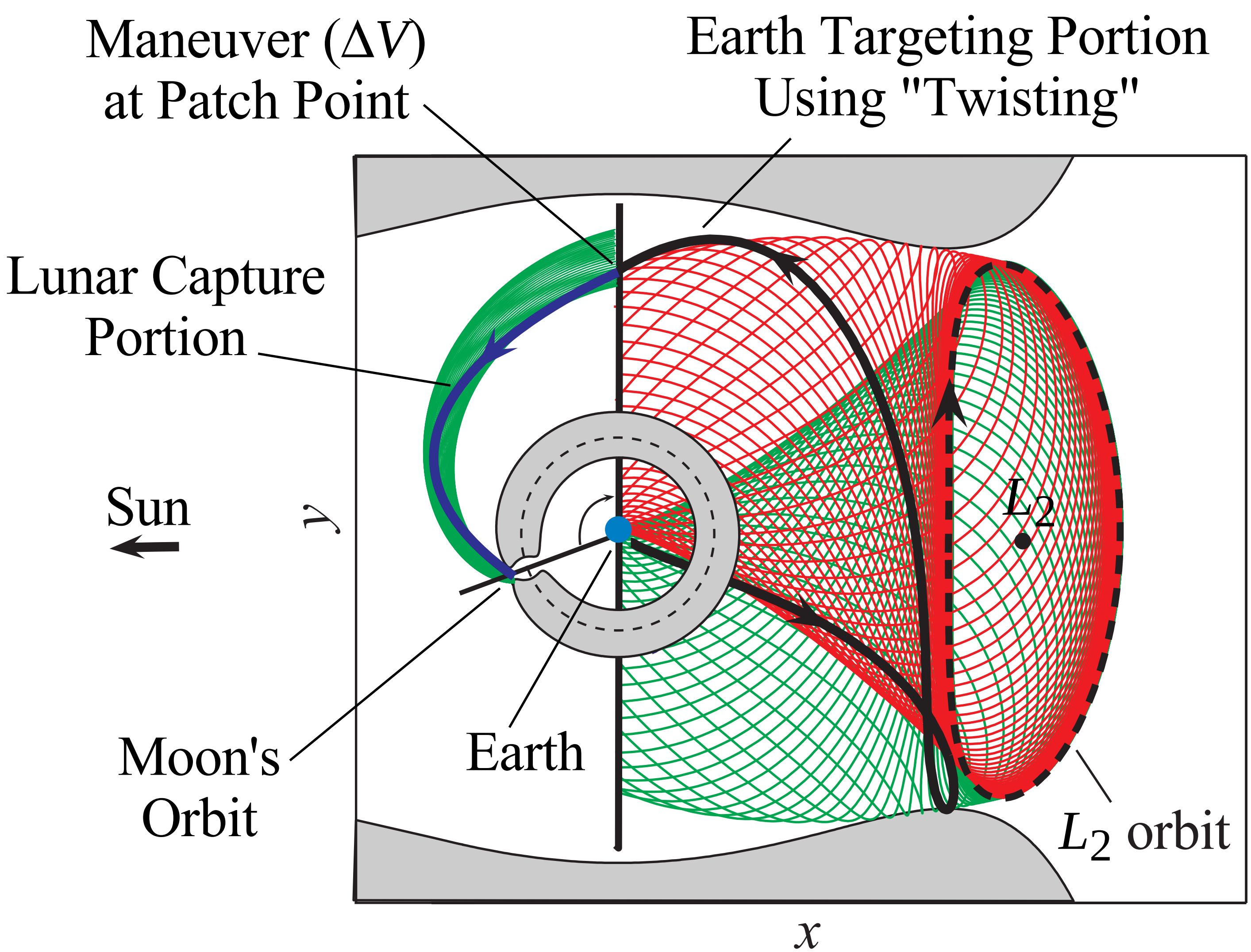
- Amount of **twist** depends sensitively on **distance** from manifold, can change dramatically with small ΔV.
- With small ΔV, can target back to (200 km) Earth **parking orbit**.





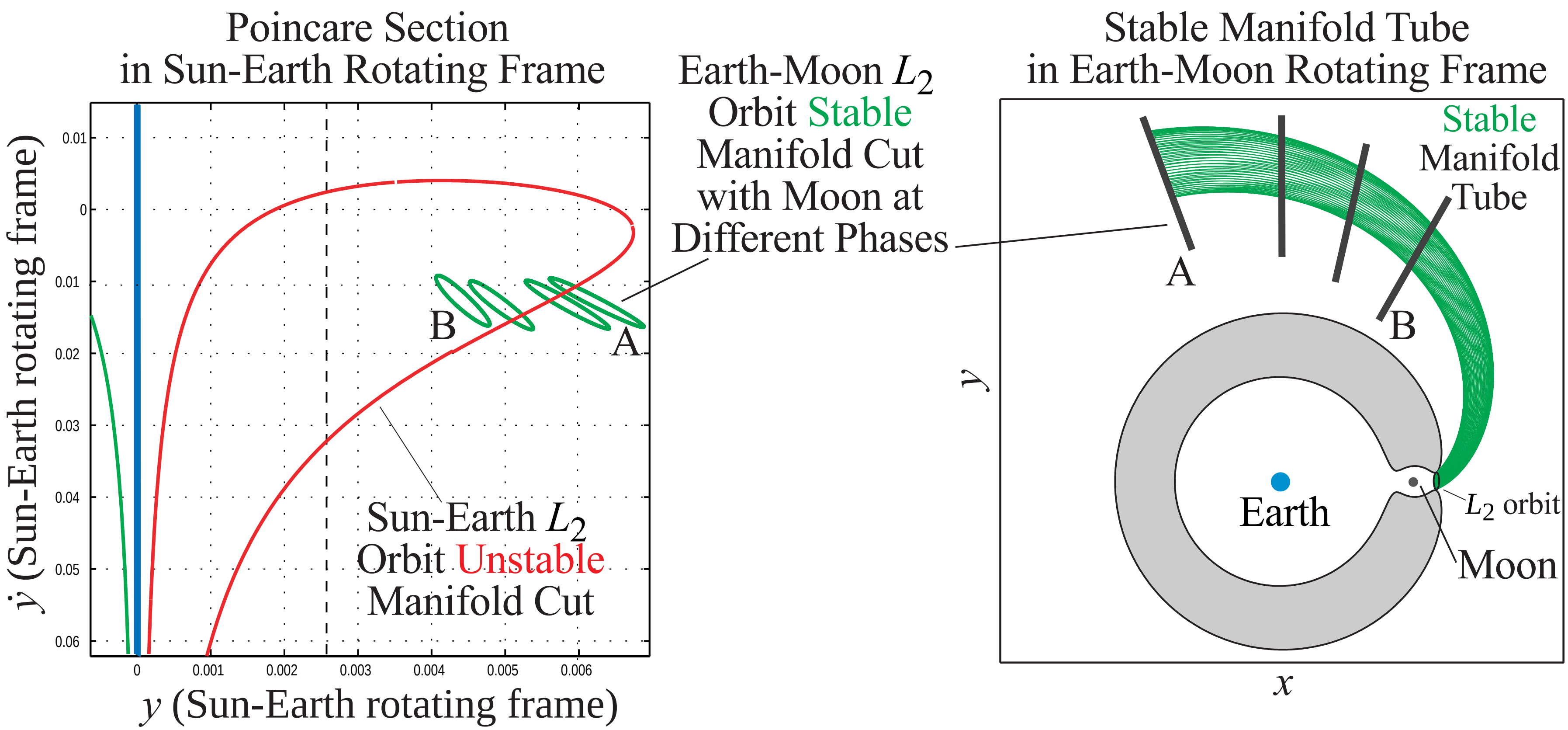
Connecting Two Portions

- Recall: Sun-Earth-Moon-SC **4-body system** as **2 coupled 3-body systems**.



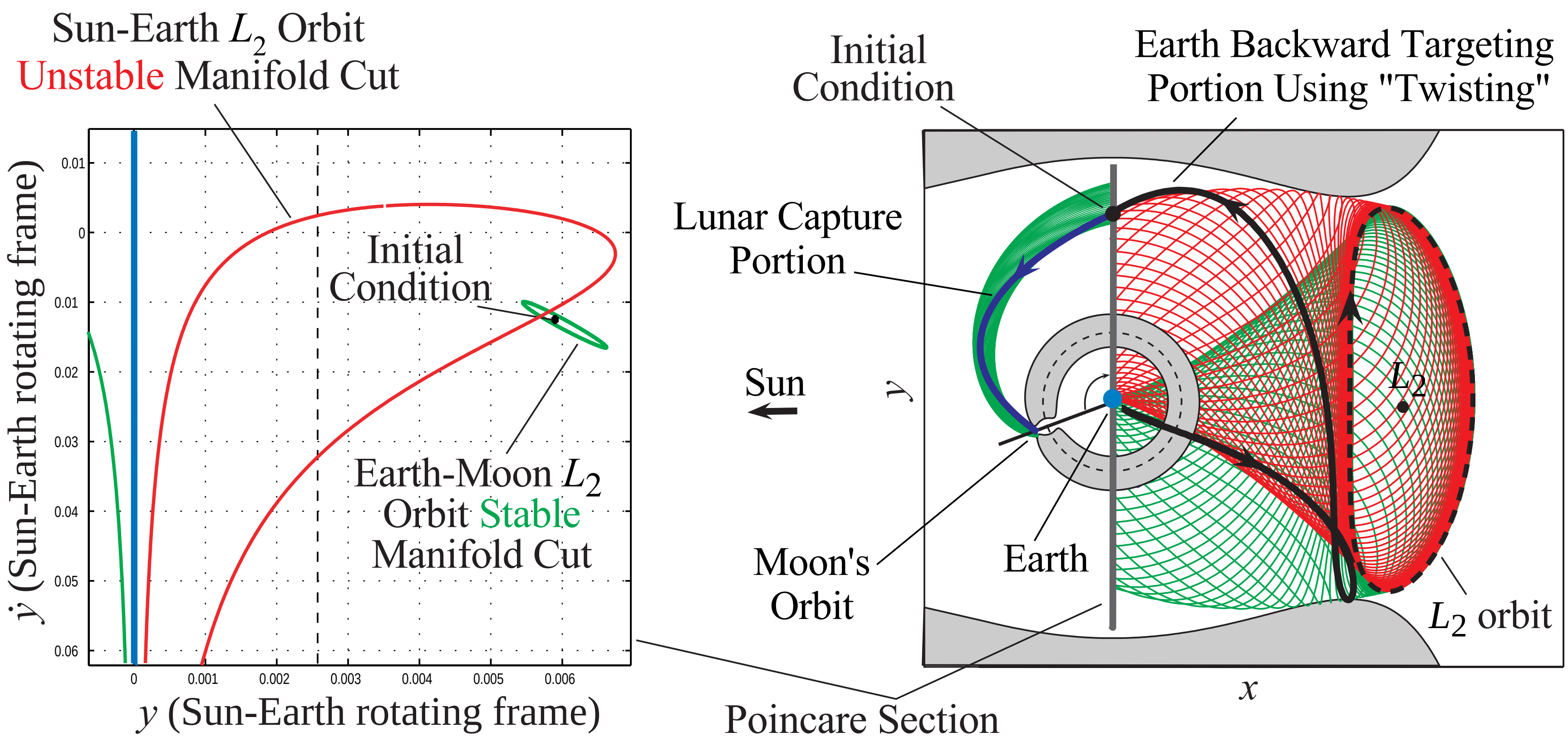
Connecting Two Portions

- Vary phase of Moon until **Earth-Moon L_2 manifold cut** intersects **Sun-Earth L_2 manifold cut**.



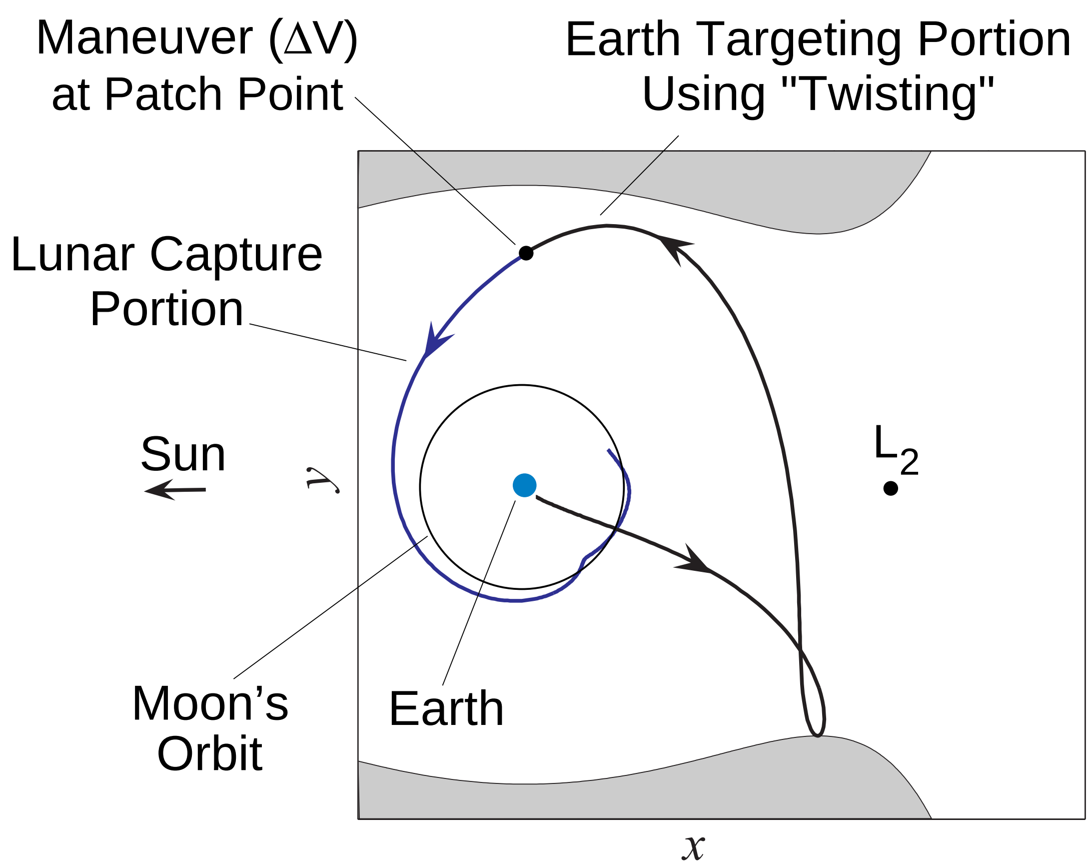
Connecting Two Portions

- Pick **initial condition** in region
  - in interior of **green curve**
  - but in exterior of **red curves**.



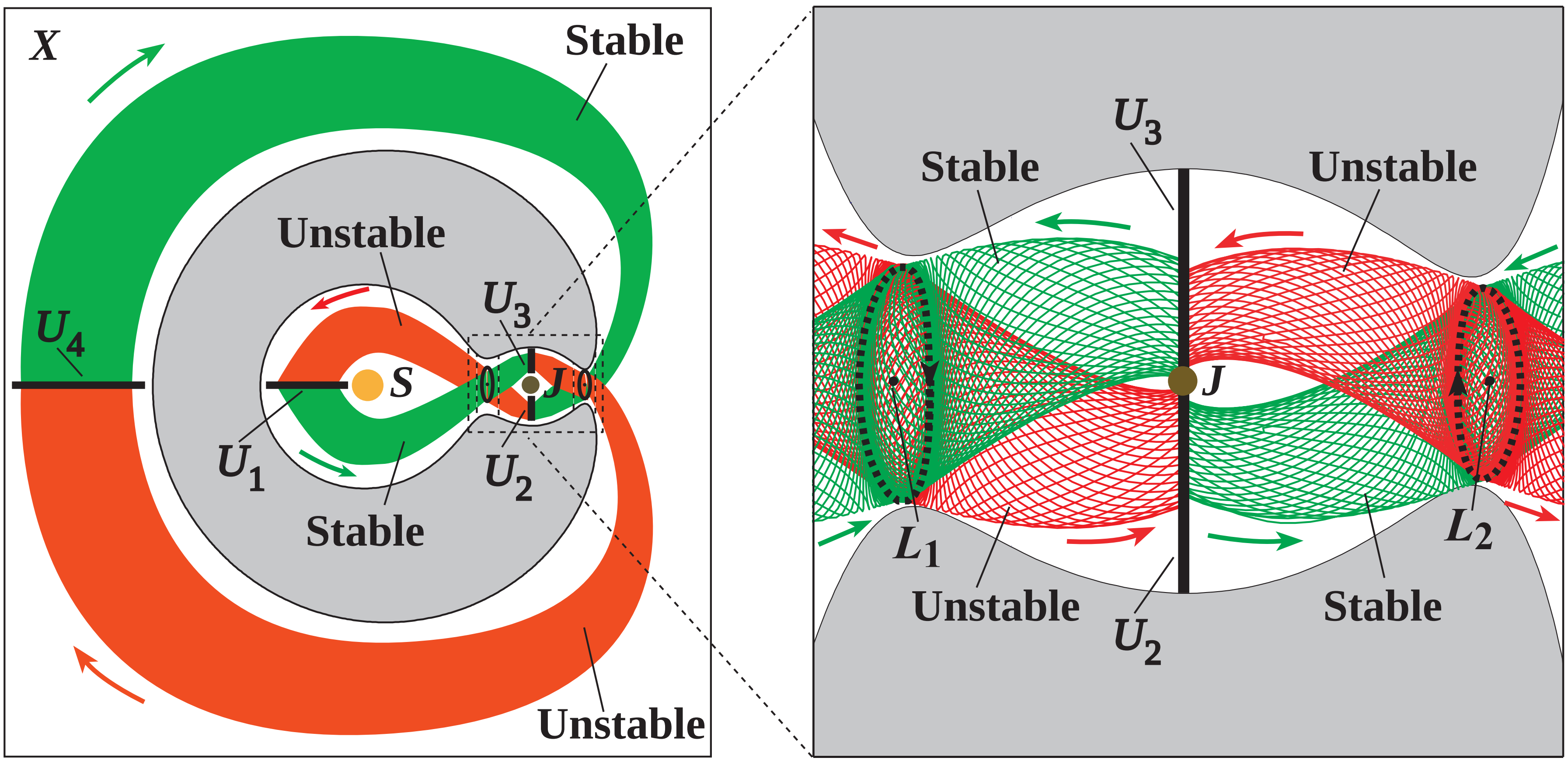
Connecting Two Portions

- With slight modification (a 34 m/s ΔV at patch point), this produces a solution in bicircular 4-body problem.
- Since capture at Moon is **natural** (zero ΔV), amount of on-board ΔV needed is lowered (by about 20%).



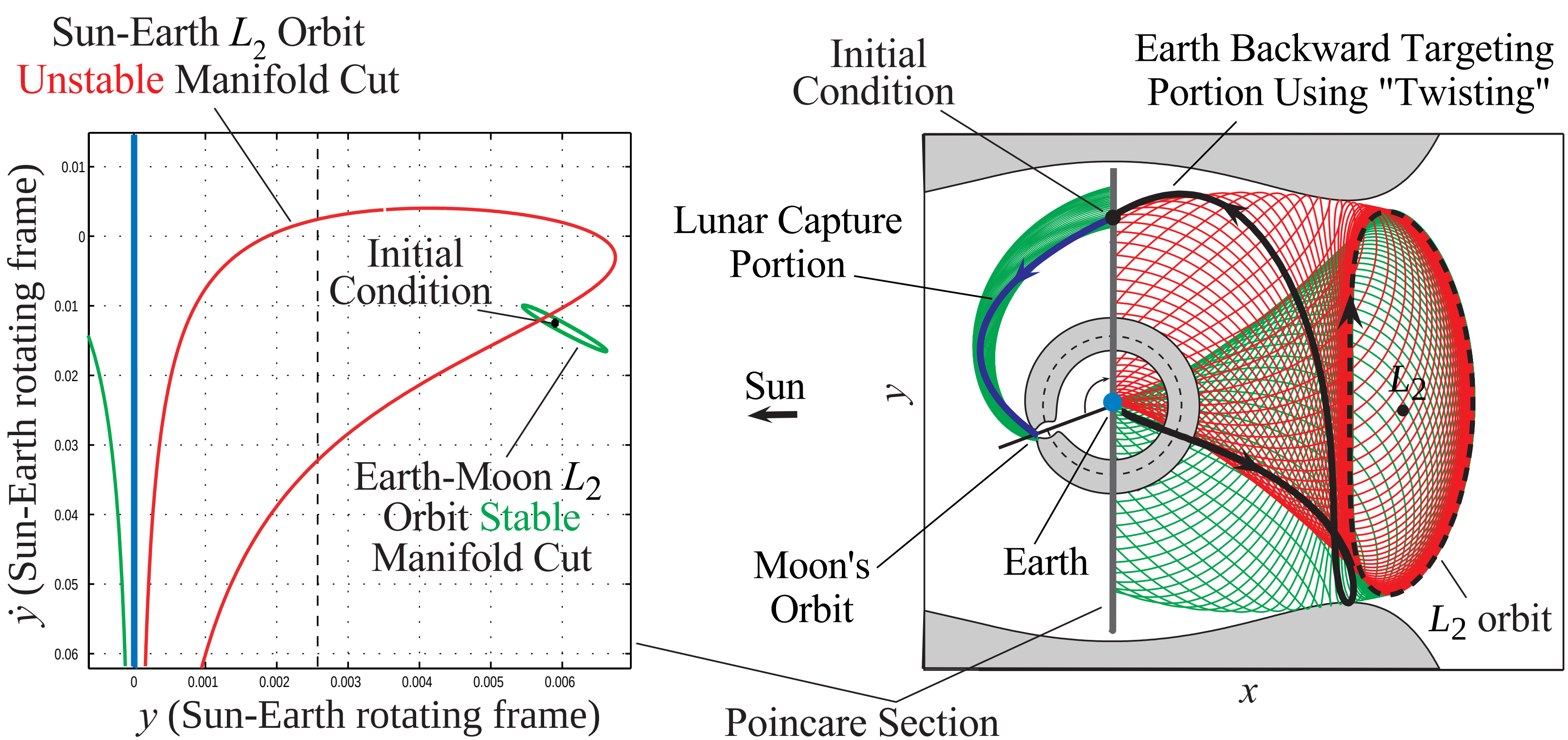
Conclusion

- Review: theory of libration point dynamics of 3-body system.
  - Invariant manifold structure determines material transport (comets) in 3-body system,
  - It can be used in space mission design.



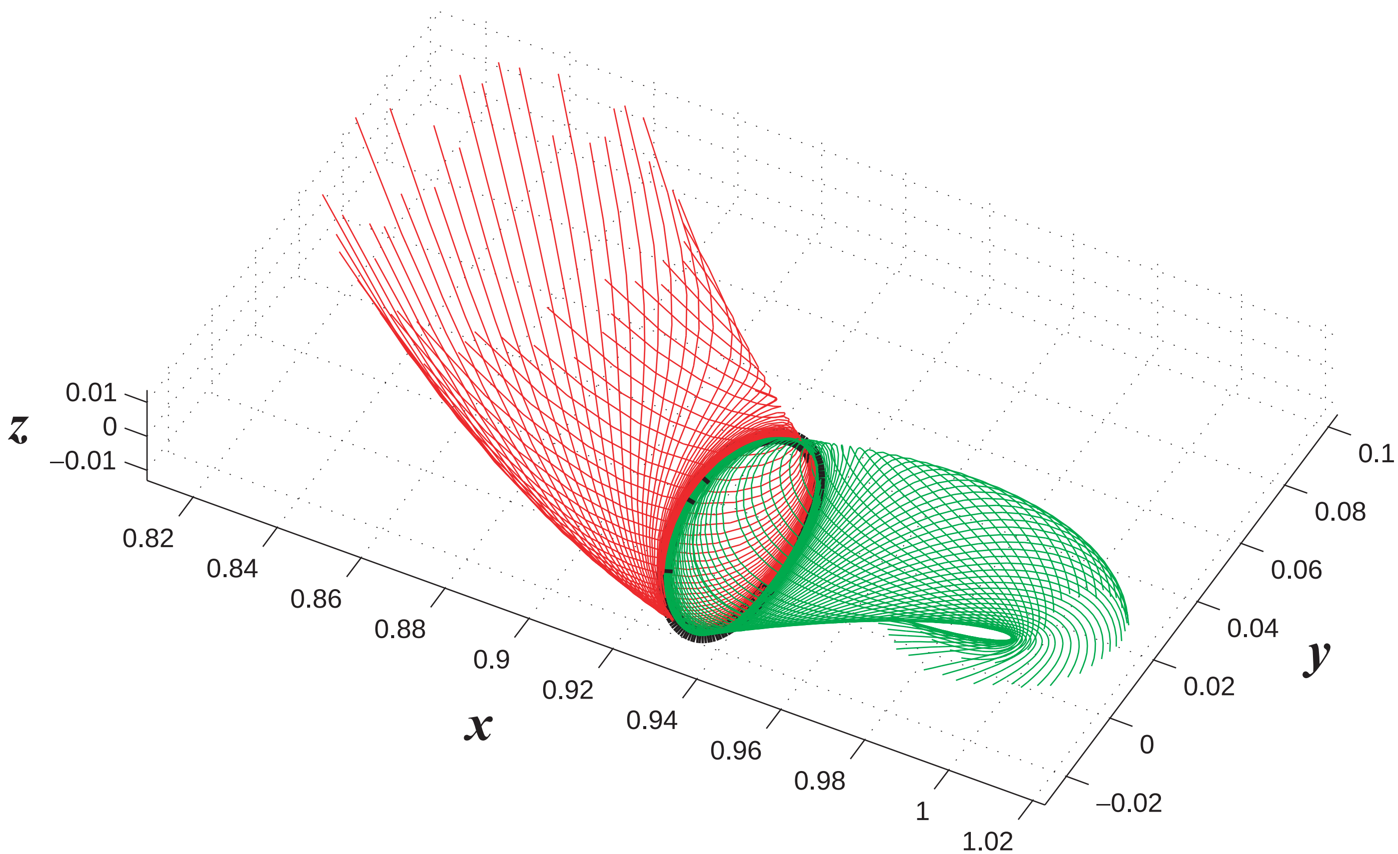
Conclusion

- Reveal the dynamics for "Low Energy Transfer to Moon."
  - Tubular regions, regions exterior to manifolds, and manifolds themselves all may be used.
  - Can pick and choose a variety of trajectories to suit almost any purpose at hand.



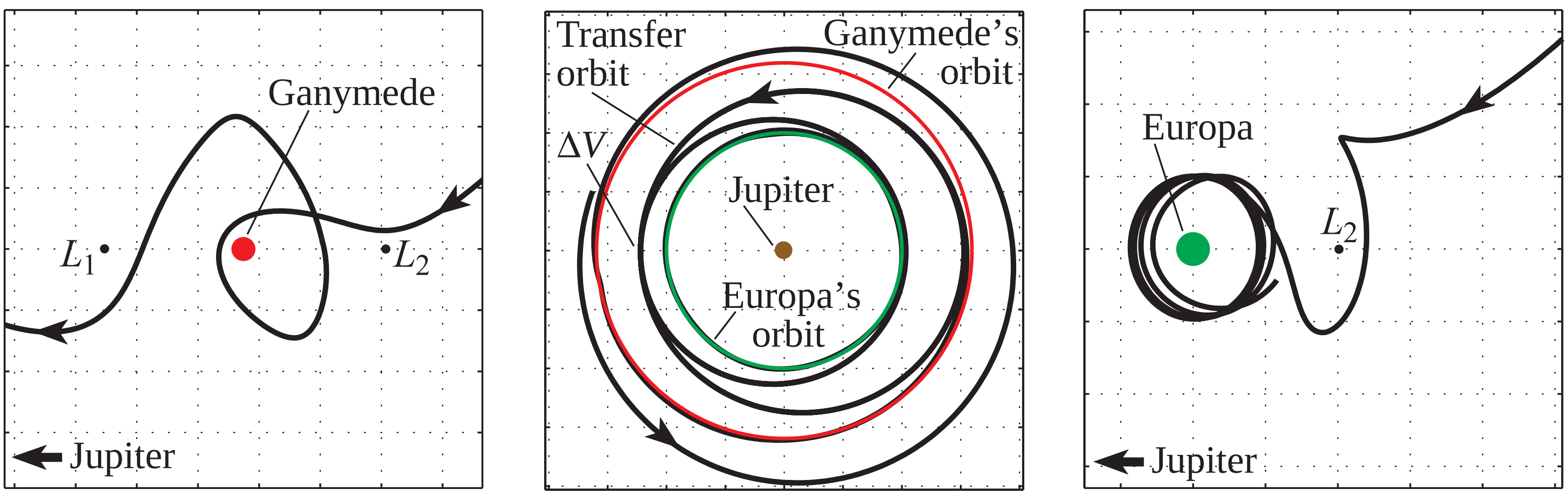
Ongoing Work: Extension to 3 Dimensions

- Find **chains/dynamical channels** for 3D periodic orbits, use them for **low fuel** deployment of spacecraft.
- Understand phase space geometry near L_1 & L_2; use it to design/control **constellations** of spacecraft.



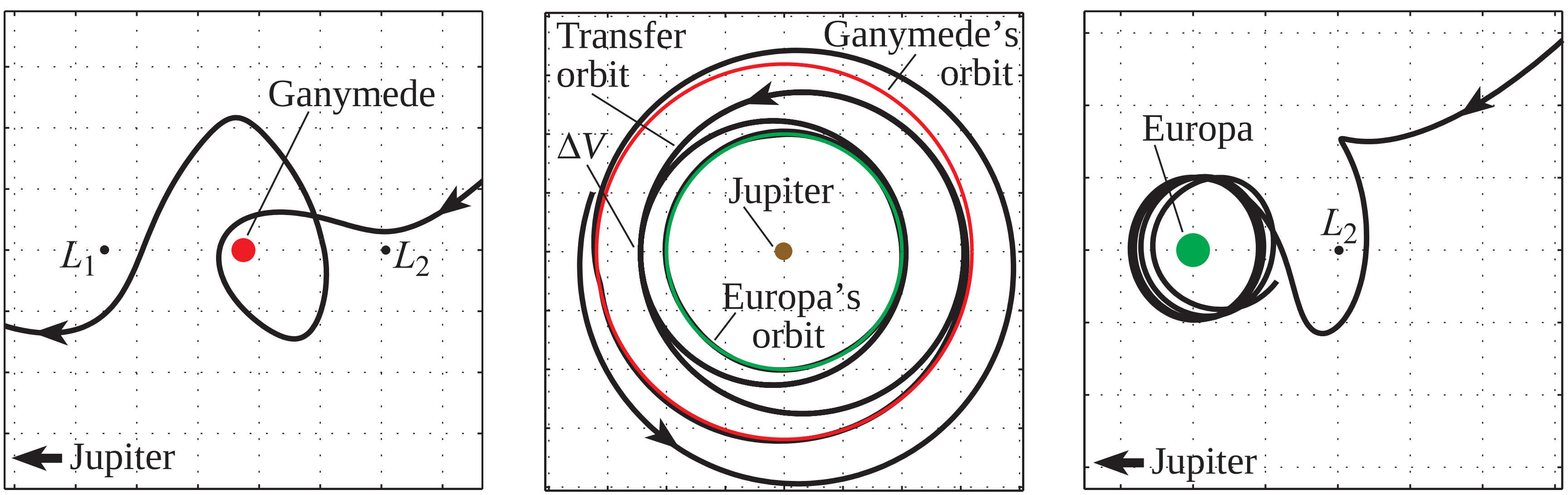
Ongoing Work: Coupling Two 3-Body Systems

- To understand
  dynamics governing **transport** between adjacent planets.
- Preliminary result on a **“Petit Grand Tour”** of Jupiter’s moons.
  - 1 orbit around **Ganymede**.
  - 4 orbits around **Europa**, etc.
  - Less than half of Hohmann transfer.



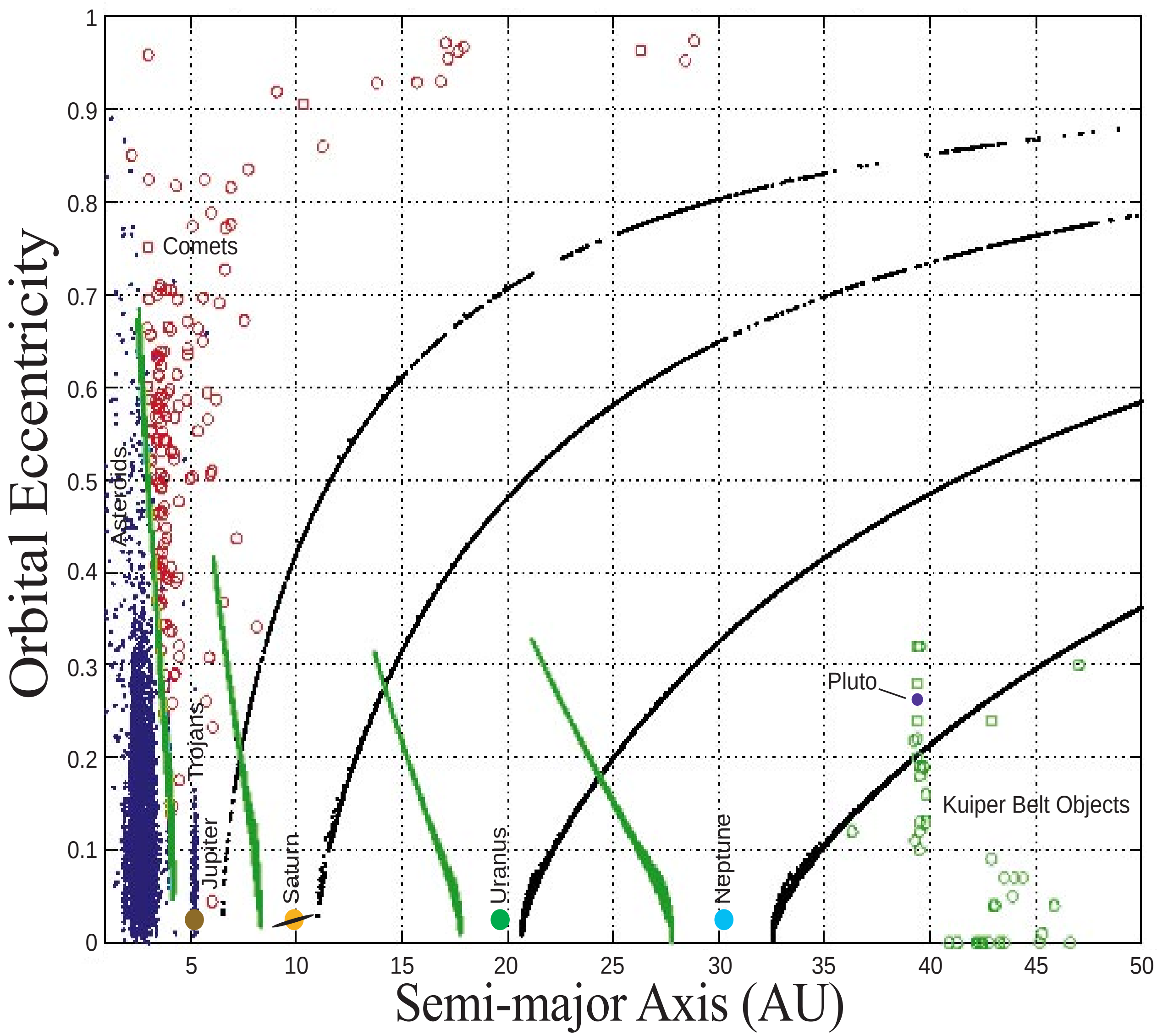
Ongoing Work: Coupling Two 3-Body Systems

- Using **differential correction**,
  can utilize this trajectory as initial guess
  - to find **3-dimensional** "Petit Grand Tour" trajectory,
  - with full solar system model.
- **Optimize trajectory**
  - by applying optimal control (e.g., COOPT),
  - with continuous (low) thrust.



Ongoing Work: 4 or More Body Problems

- **Interplanetary transport** and distribution of material.



References and Other Informations

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  - **http://www.cds.caltech.edu/~koon/**
  - **http://ojps.aip.org/chaos/**

- **Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross**
  Low Energy Transfer to the Moon.

- Email: **koon@cds.caltech.edu**