Tube Dynamics, Lobe Dynamics, & Low Energy Tour of Jupiter's Moons

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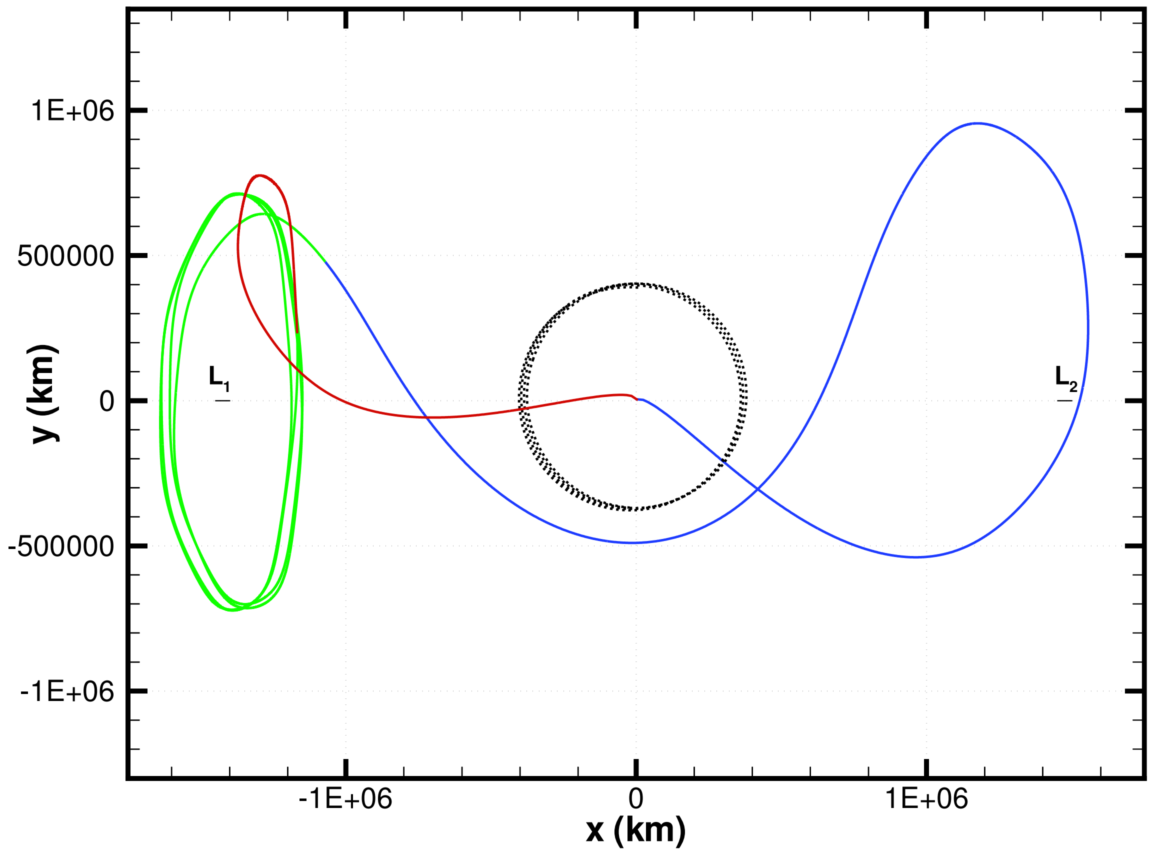
Outline of Presentation

- **Main Theme**
  - how to use dynamical systems theory of 3-body problem in space mission design.
- **Background** and **Motivation**:
  - NASA's Genesis Discovery Mission.
  - Jupiter Comets.
- **Circular Restricted 3-Body Problem**.
- **Major Results on Tube Dynamics**.
- **Lobe Dynamics** & **Navigating in Phase Space.**
  - A Low Energy Tour of Jupiter's Moons.
- **Conclusion** and **Ongoing Work**.
  - Two Full Body Problem and Astroid Pairs.
  - Set Oriented Method.

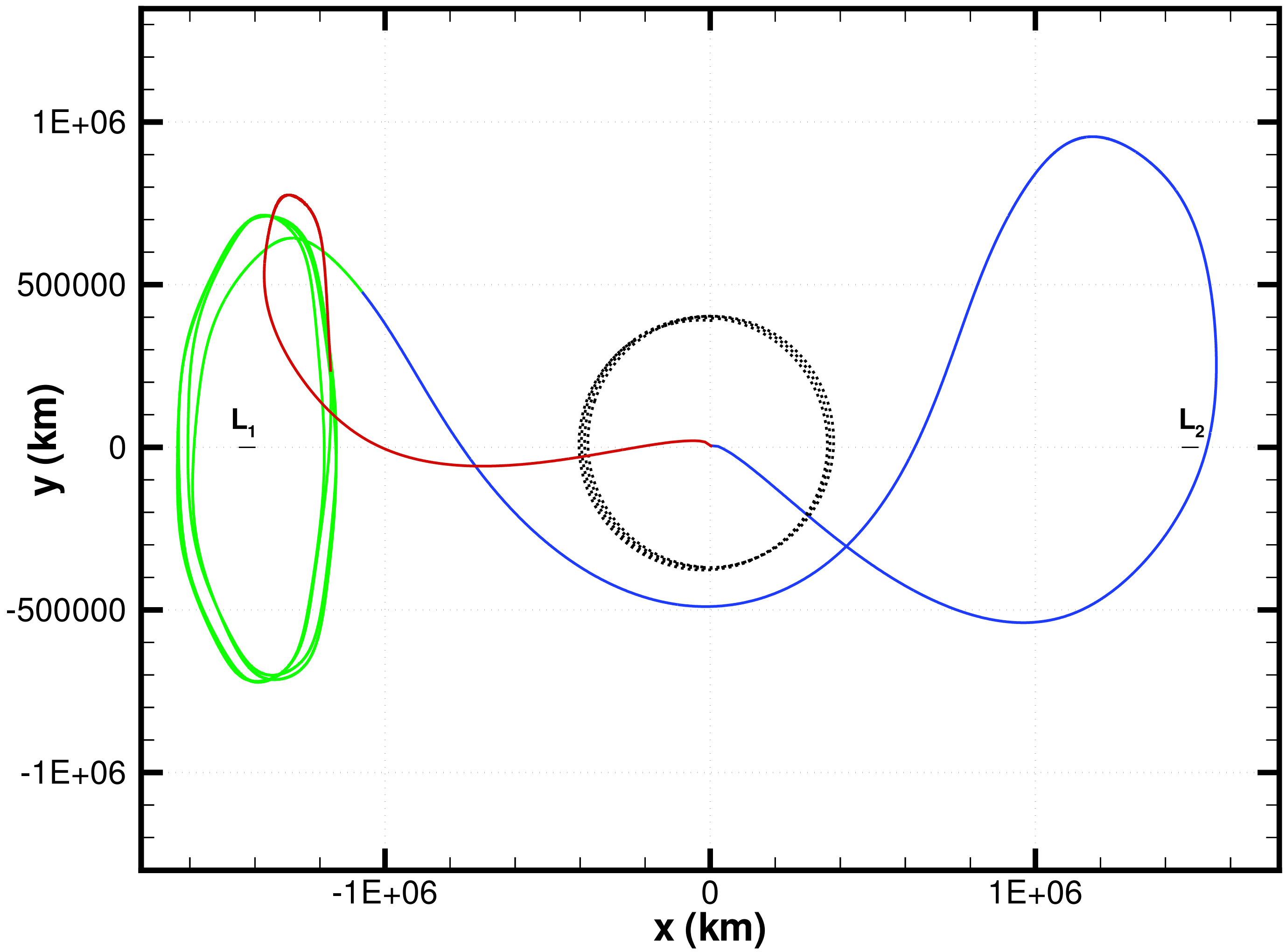
Genesis Discovery Mission

- **Genesis spacecraft**
  - is collecting solar wind sample from a L_1 **halo orbit**,
  - will **return** them to Earth next year.
- **Halo orbit**, **transfer**/ **return** trajectories in rotating frame.



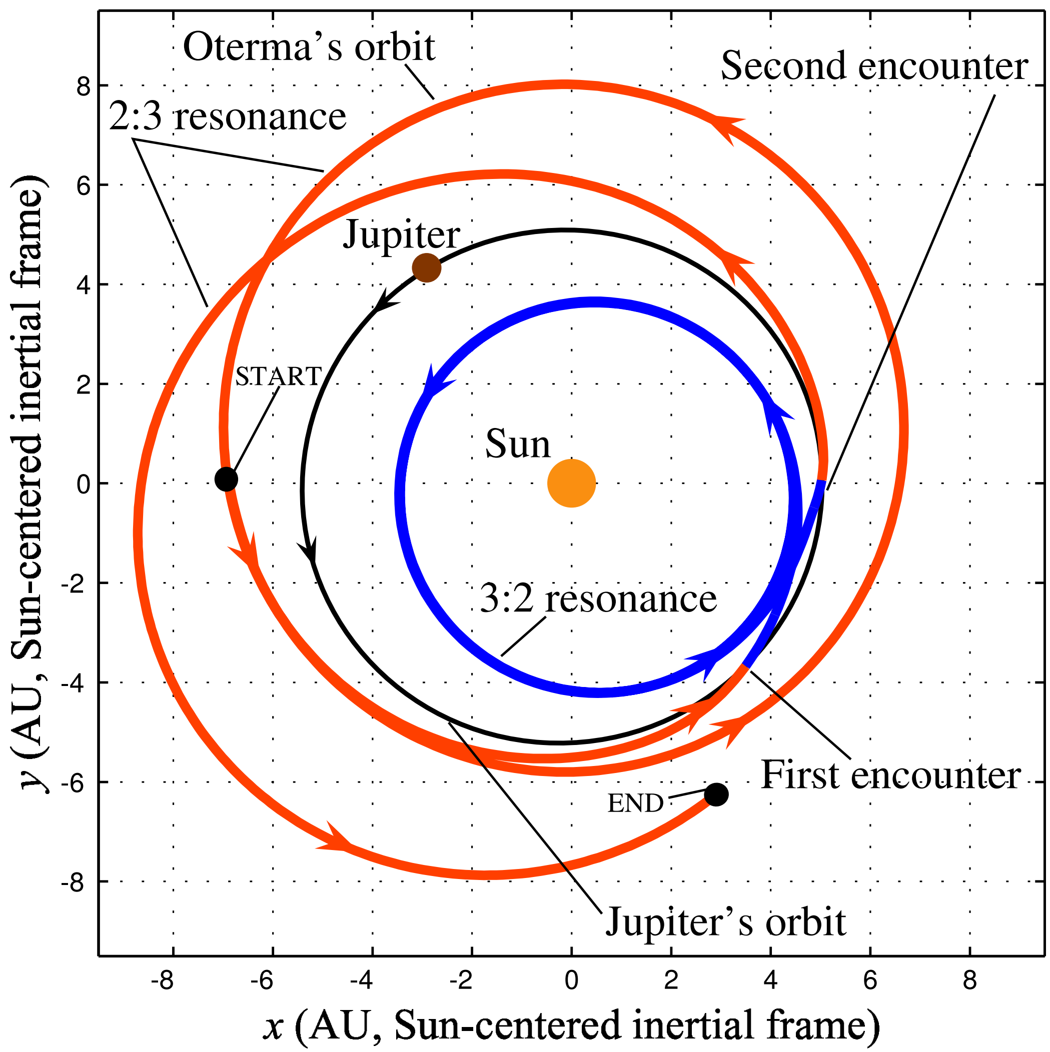
Genesis Discovery Mission

- Follows natural dynamics, little propulsion after launch.
- **Return-to-Earth portion** utilizes heteroclinic dynamics.



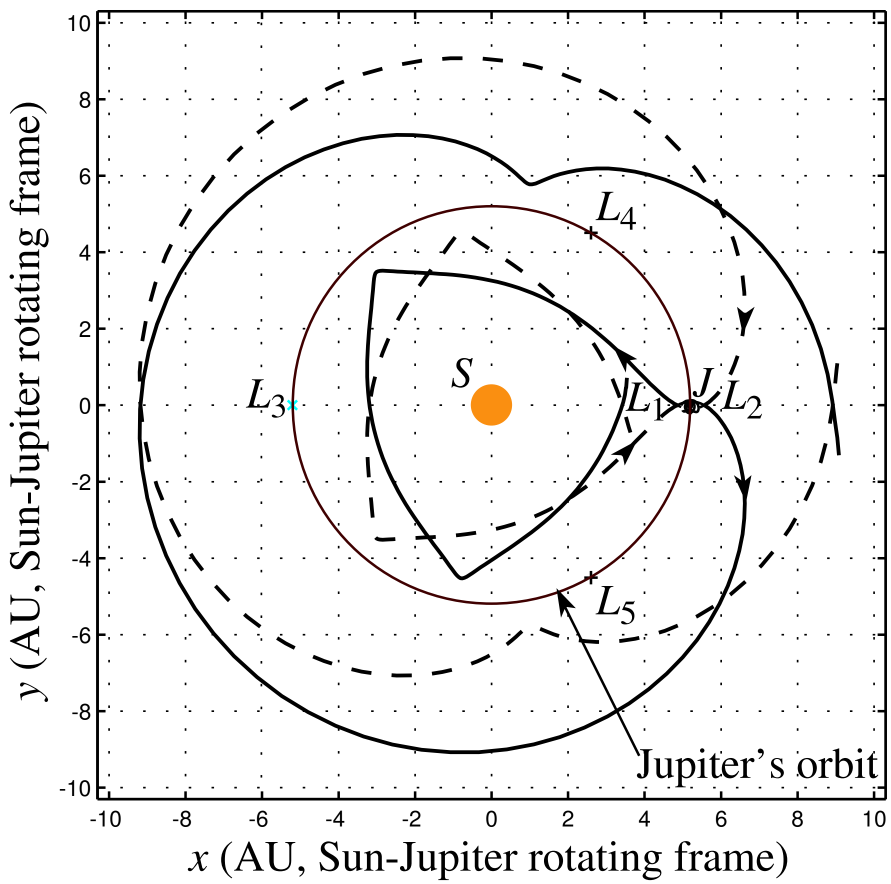
Jupiter Comets

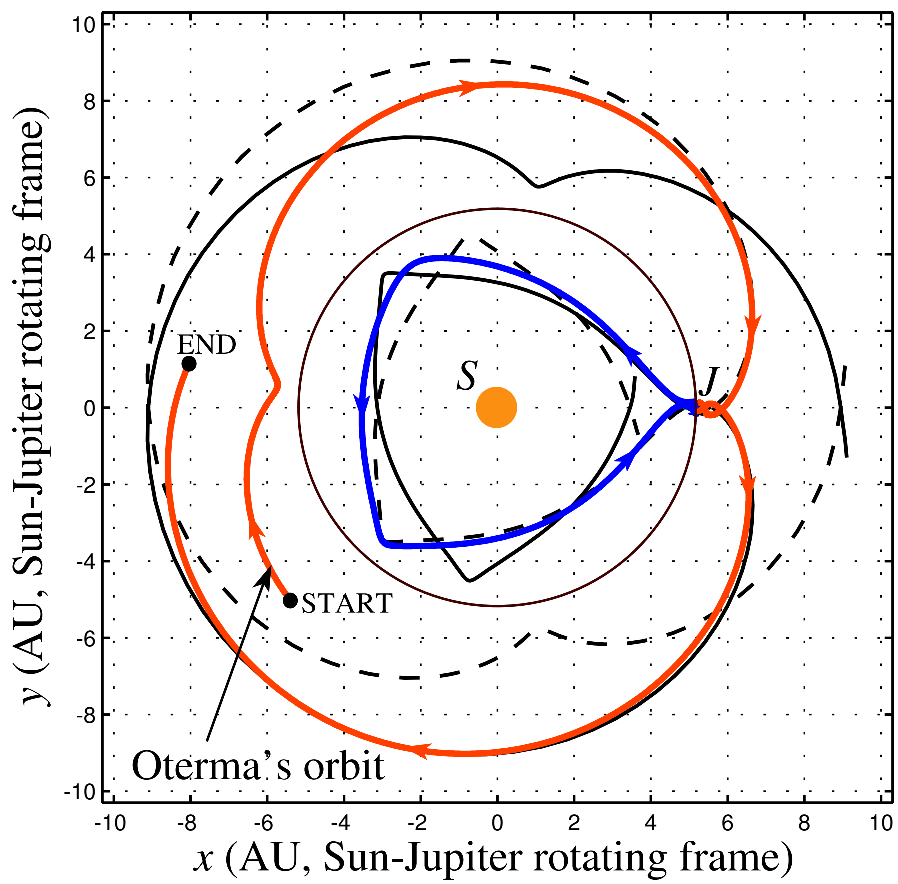
- **Rapid transition** from **outside** to **inside** Jupiter's orbit.
- **Captured temporarily** by Jupiter during transition.
- Exterior (**2:3 resonance**). Interior (**3:2 resonance**).



Jupiter Comets

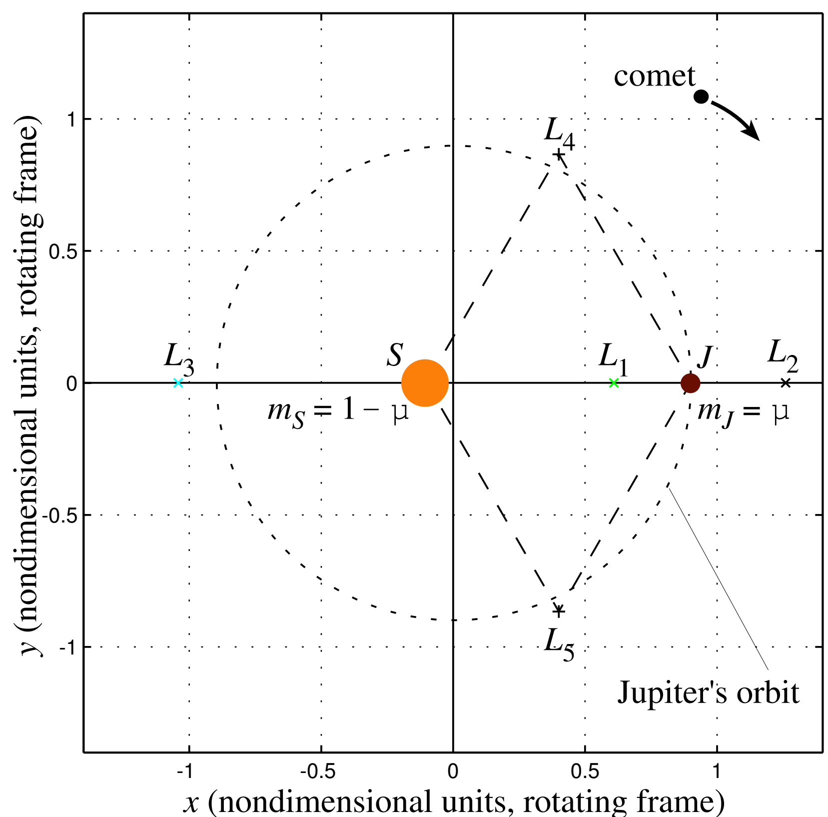
- Belbruno and B. Marsden [1997]
- Lo and Ross [1997]
  - Comet in **rotating frame** follows **invariant manifolds**.
- Jupiter comets make **resonance transition** near L_1 and L_2.





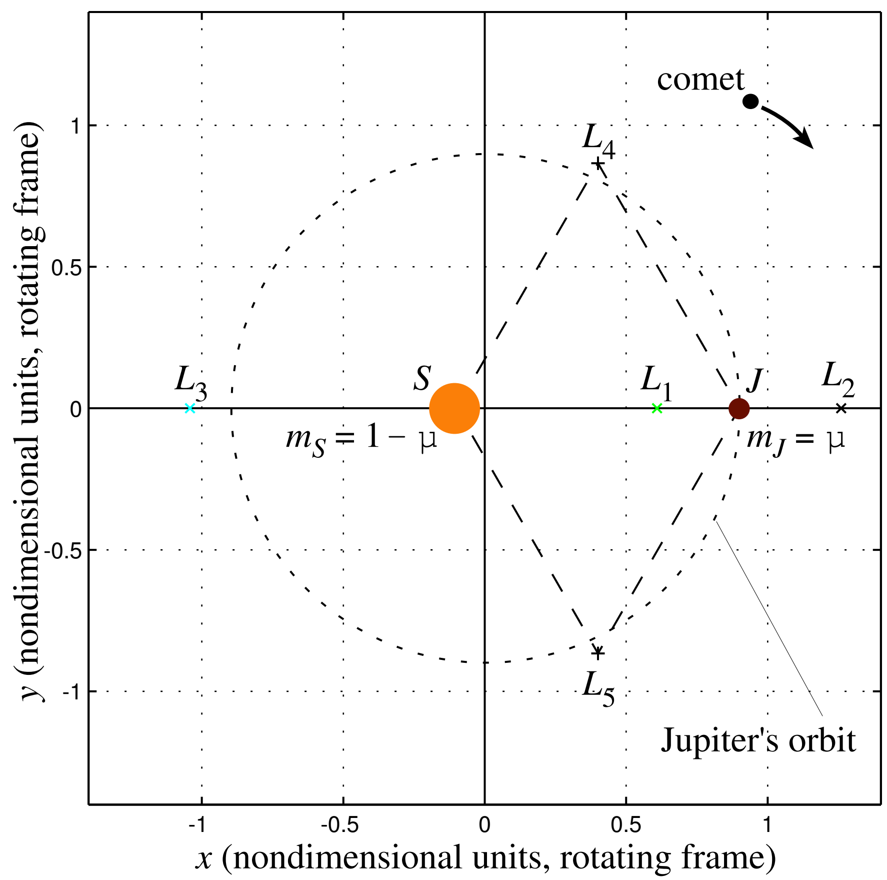
Circular Restricted 3-Body Problem

- **Planar CR3BP**. is a good starting model:
  - Comets mostly **heliocentric**, but their perturbation dominated by **Jupiter's gravitation**.
  - Jupiter's small **eccentricity** plays little role during transition.
  - Their motion nearly in Jupiter's **orbital plane**.
- Results generalized to **spatial CR3BP** (**planar** for illustration.)



Planar Circular Restricted 3-Body Problem

- 2 main bodies: **Sun** and **Jupiter**.
  - Total mass normalized to 1: $m_J = \mu, \quad m_S = 1 - \mu.$
  - Rotate about center of mass, angular velocity normalized to 1.
- Choose **rotating** coordinate system with origin at center of mass, **S** and **J** fixed at $(-\mu, 0)$ and $(1 - \mu, 0)$.



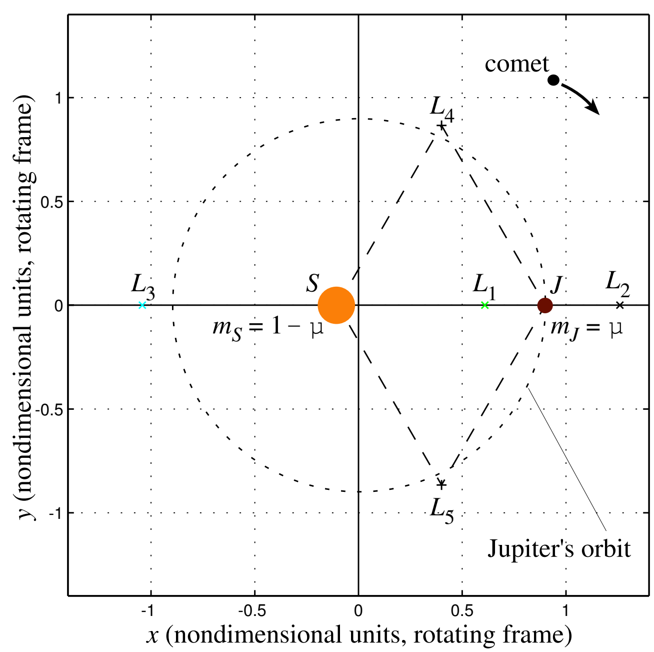
Equilibrium Points

- Comet's equations of motion are

$$\ddot{x} - 2\dot{y} = -\frac{\partial U}{\partial x}, \qquad \ddot{y} + 2\dot{x} = -\frac{\partial U}{\partial y},$$

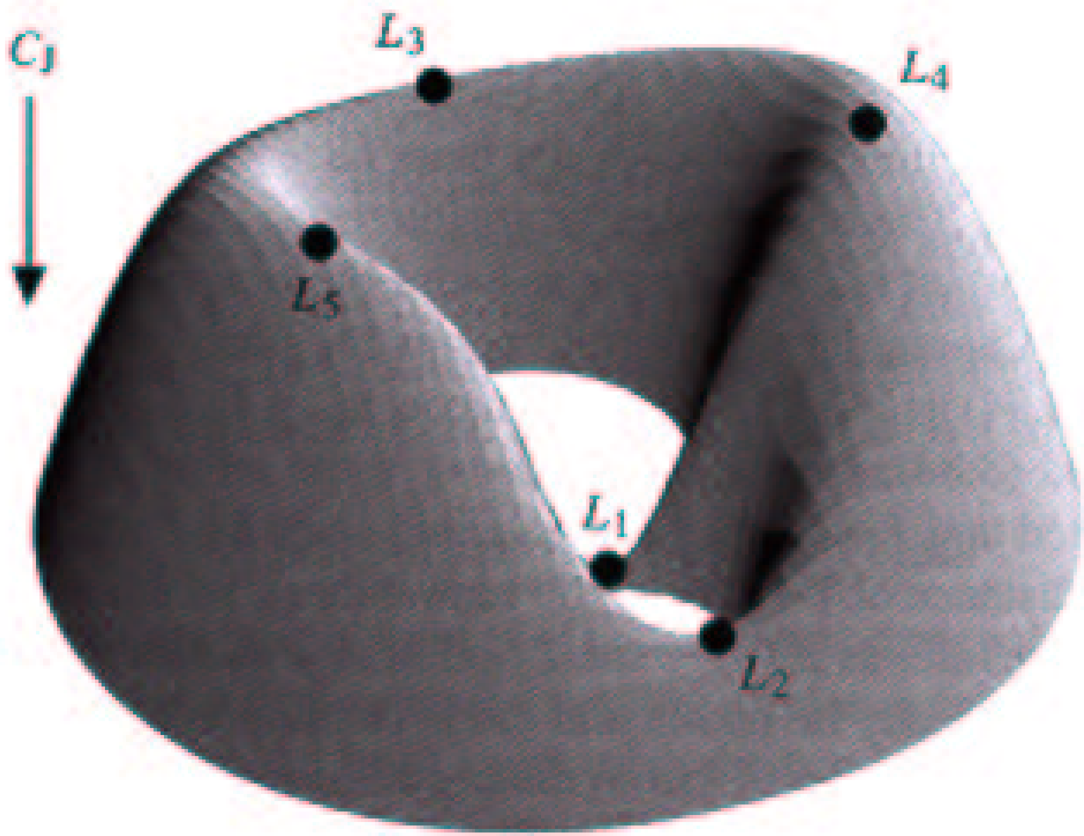
where $U(x, y) = -\frac{x^2+y^2}{2} - \frac{1-\mu}{r_s} - \frac{\mu}{r_j}$.

- Five equilibrium points:
  - 3 **unstable** equilibrium points on S-J line, L_1, L_2, L_3.
  - 2 equilateral equilibrium points, L_4, L_5.



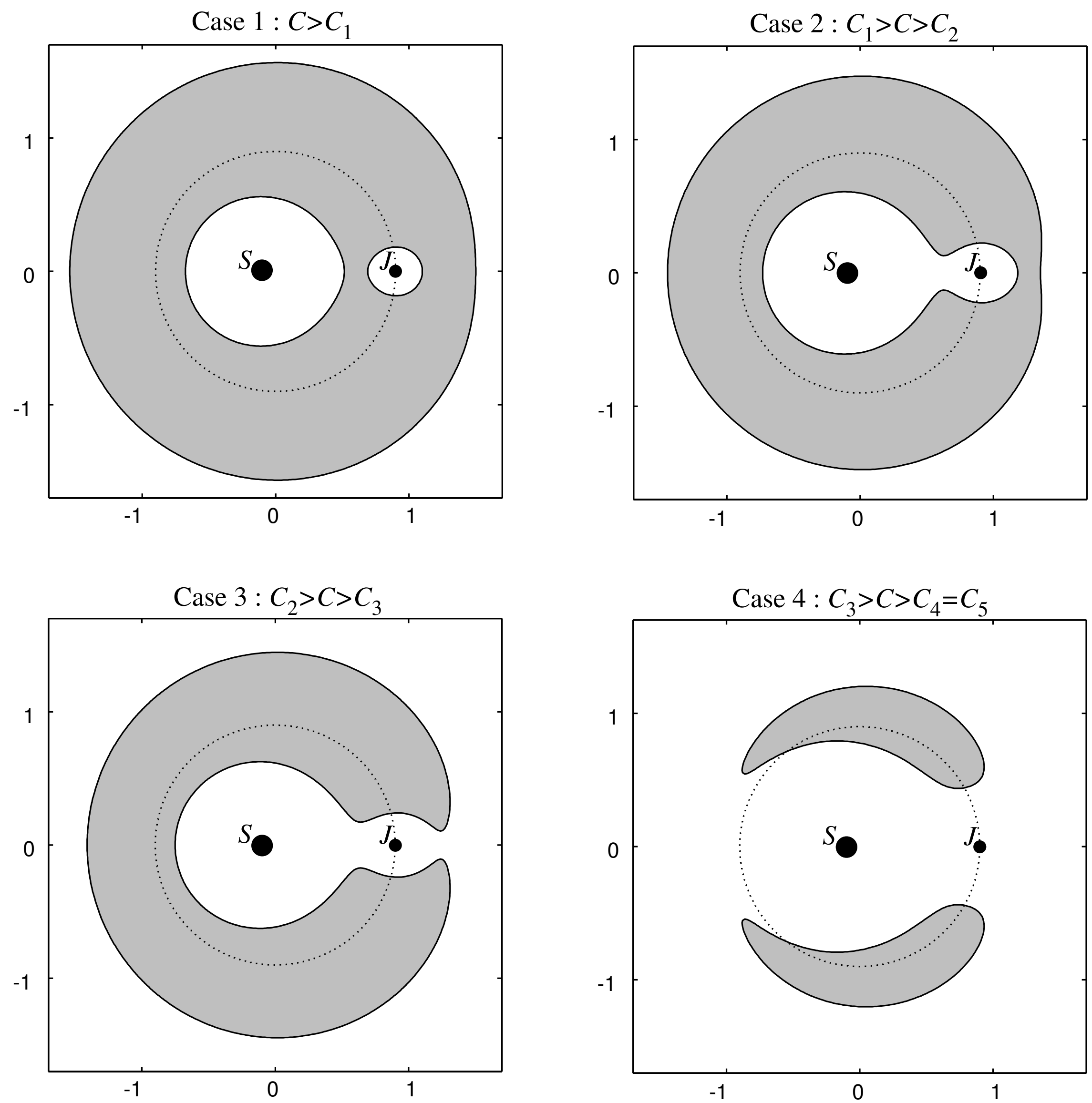
Hill's Realm

- **Energy integral**: $E(x, y, \dot{x}, \dot{y}) = (\dot{x}^2 + \dot{y}^2)/2 + U(x, y)$.
- E can be used to determine (**Hill's**) **realm** in position space where comet is energetically permitted to move.
- **Effective potential**: $U(x, y) = -\frac{x^2+y^2}{2} - \frac{1-\mu}{r_s} - \frac{\mu}{r_j}$.



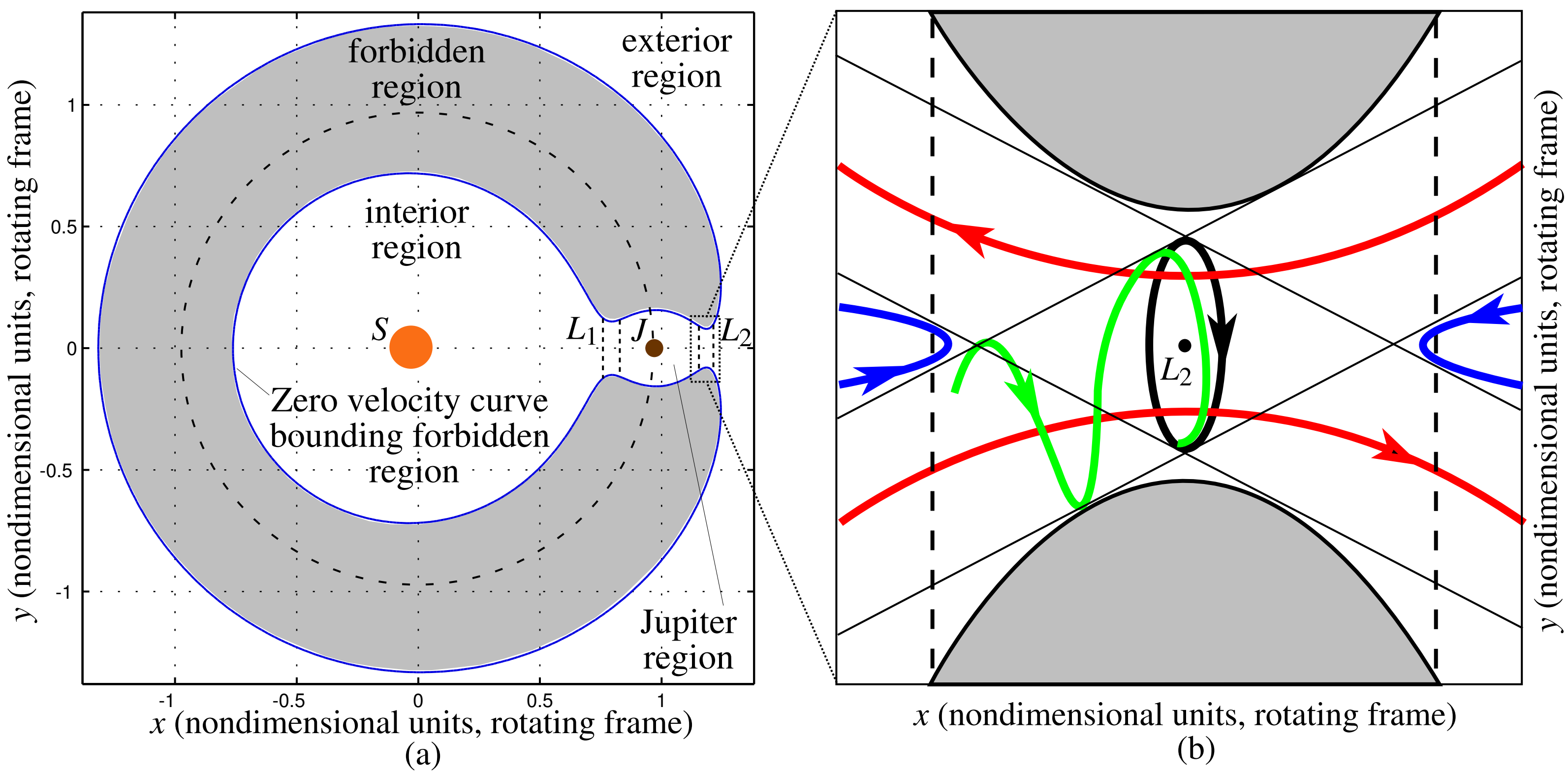
Hill's Realm

- To fix **energy value** E is to fix **height** of plot of $U(x, y)$. **Contour plots** give **5** cases of Hill's realm.



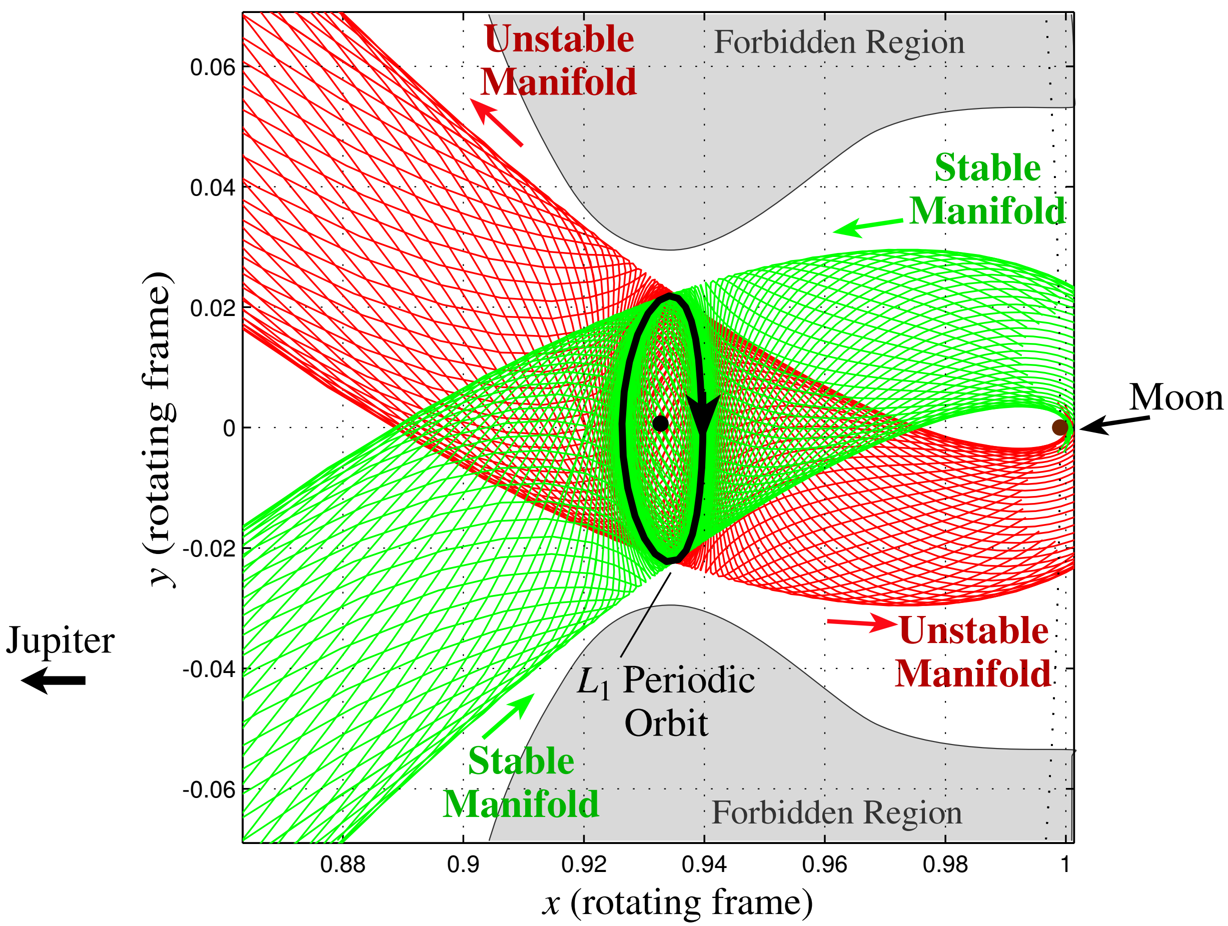
The Flow near L_1 and L_2

- For **energy value** just above that of L_2, **Hill's realm** contains a **"neck"** about L_1 & L_2.
- Comet can make **transition** through these equilibrium realms.
- Dynamics in equilibrium realm: **Saddle** X **Center**.
- 4 types of orbits: **periodic**, **asymptotic**, **transit** & **nontransit**.



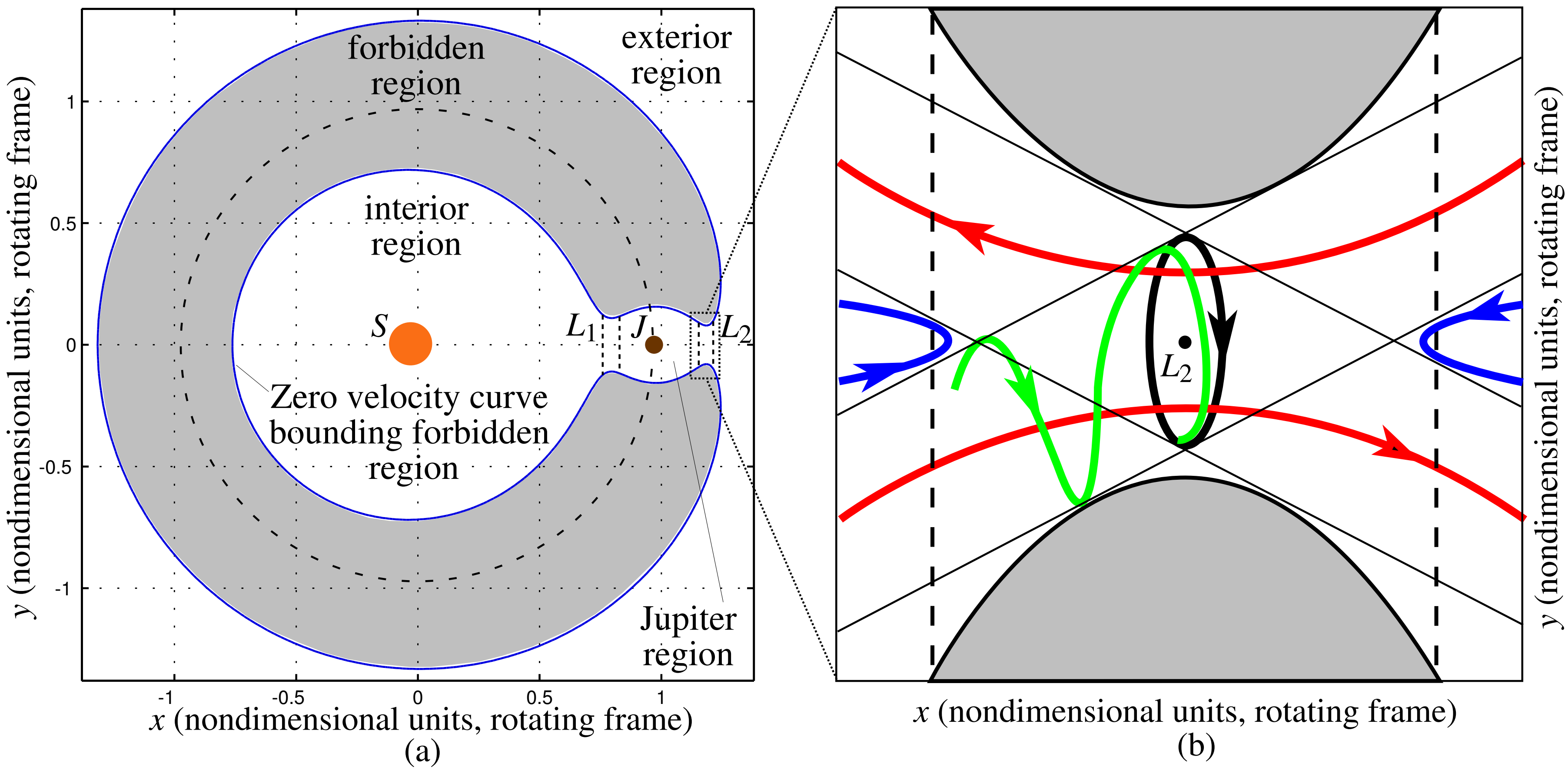
Planar: Invariant Manifold as Separatrix

- Asymptotic orbits form **2D invariant manifold tubes** in **3D energy surface**.
- They separate transit and non-transit orbits:
  - **Transit orbits** are those inside the tubes.
  - **Non-transit orbits** are those outside the tubes.



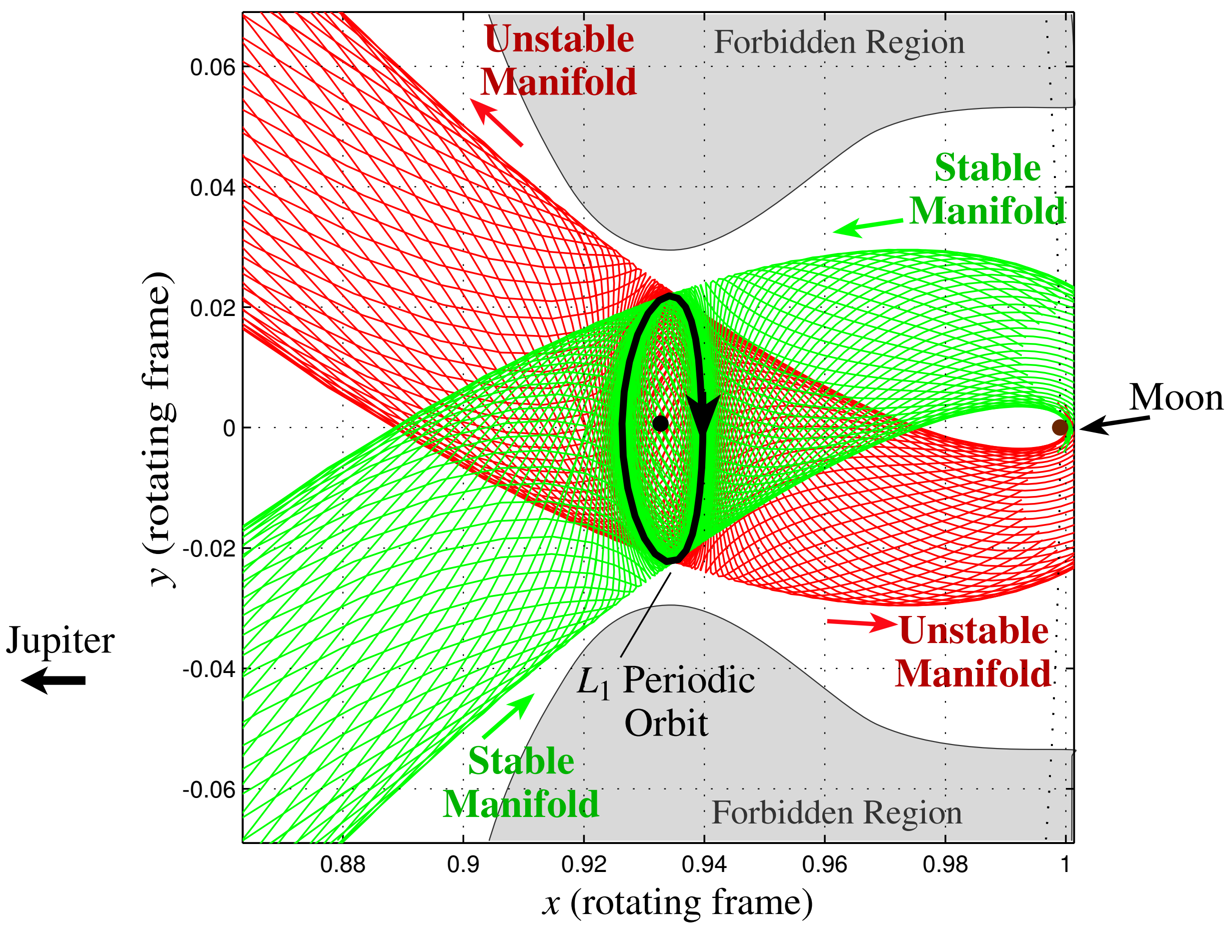
Spatial Restricted 3-Body Problem (CR3BP)

- Dynamics near equilibrium point: **Saddle** X **Center** X **Center**.
  - **NHIM** (periodic/quasi-periodic): 3-sphere S^3
  - **asymptotic orbits** to NHIM: $S^3 \times I$ (**"tubes"**)
  - **transit** and **nontransit** orbits.



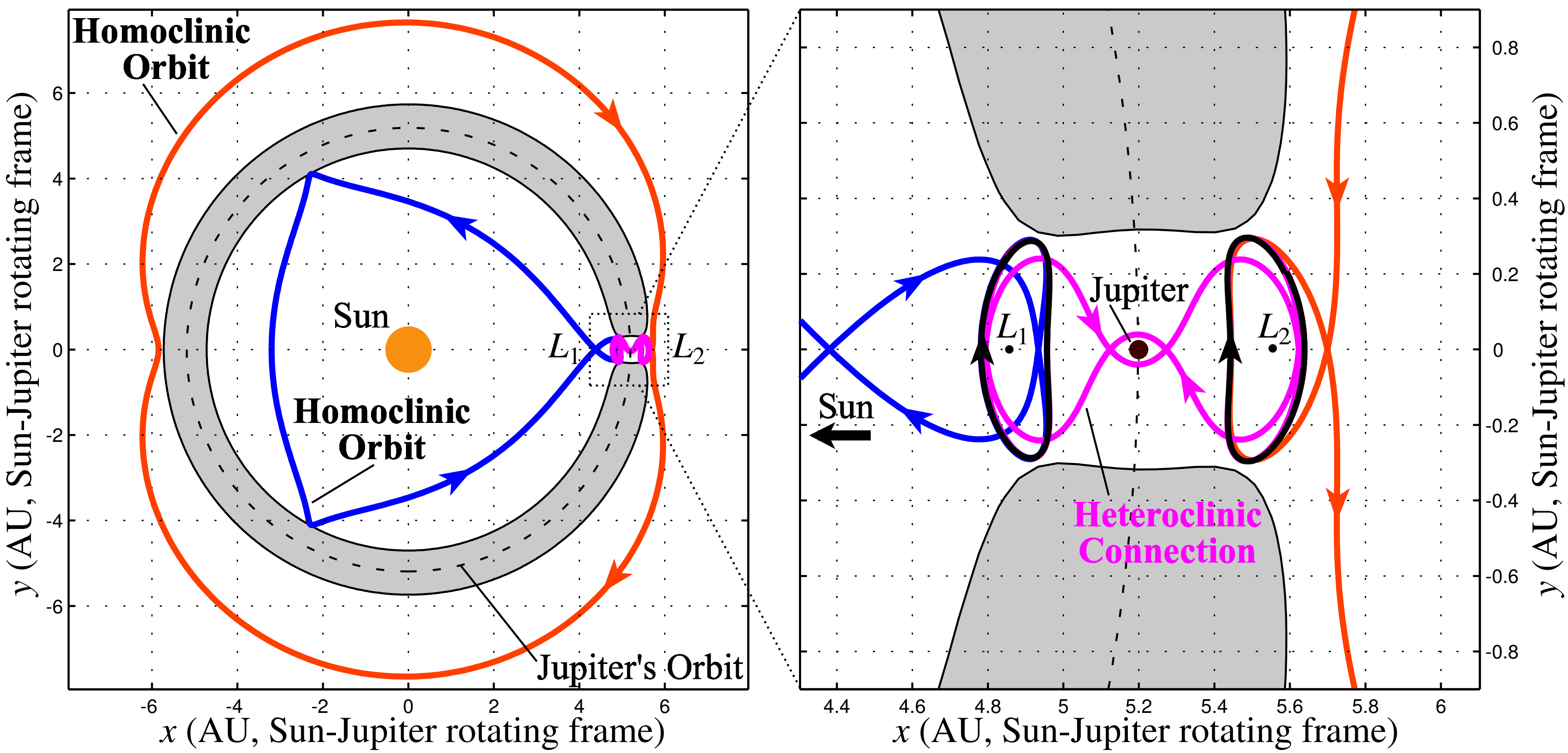
Spatial: Invariant Manifold as Separatrix

- Asymptotic orbits form **4D invariant manifold "tubes"** $(S^3 \times I)$ in **5D energy surface**.
- They separate transit and non-transit orbits:
  - **Transit orbits** are those inside the "tubes".
  - **Non-transit orbits** are those outside the "tubes".



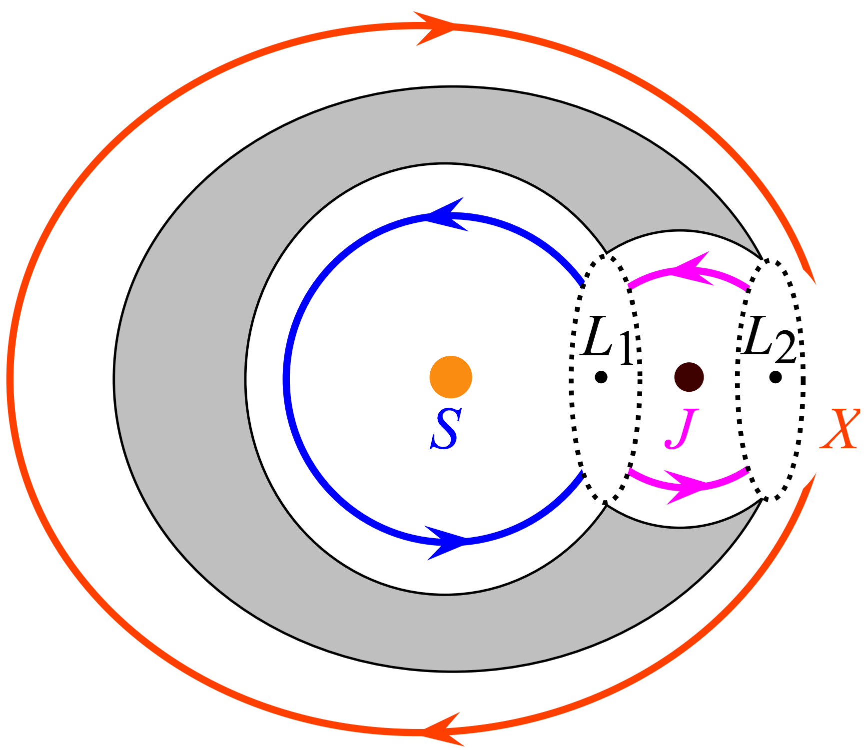
Major Result (A): Heteroclinic Connection

- Found **heteroclinic connection** between pair of periodic orbits.
- Found a large class of **orbits** near this (homo/heteroclinic) ***chain***.
- Comet can follow these ***channels*** in rapid transition.



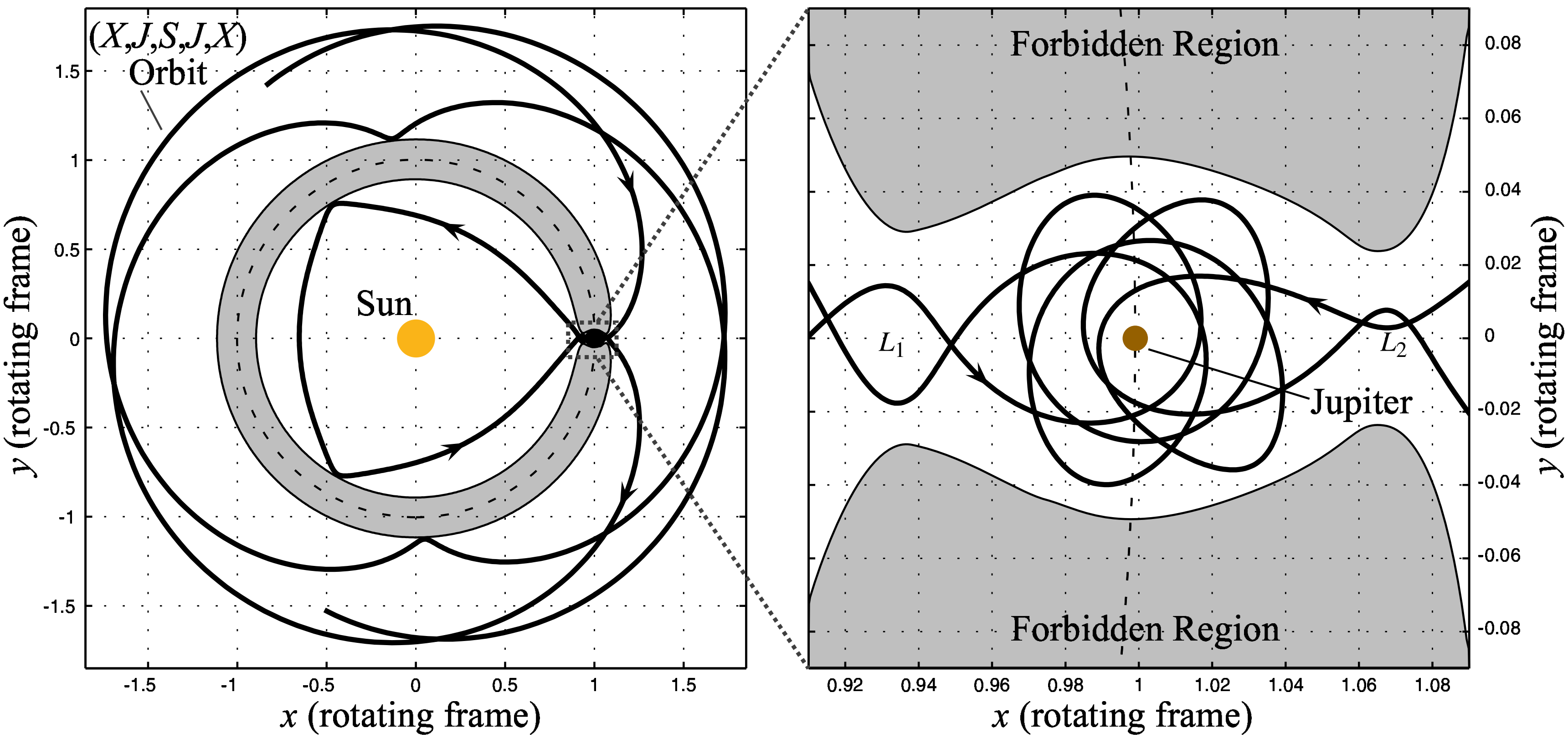
Major Result (B): Existence of Transitional Orbits

- **Symbolic sequence** used to label itinerary of each comet orbit.
- ***Main Theorem***: For any admissible **itinerary**, e.g., $(\ldots, \mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X}, \ldots)$, there exists an orbit whose **whereabouts** matches this **itinerary**.
- Can even specify **number of revolutions** the comet makes around Sun & Jupiter (plus L_1 & L_2).



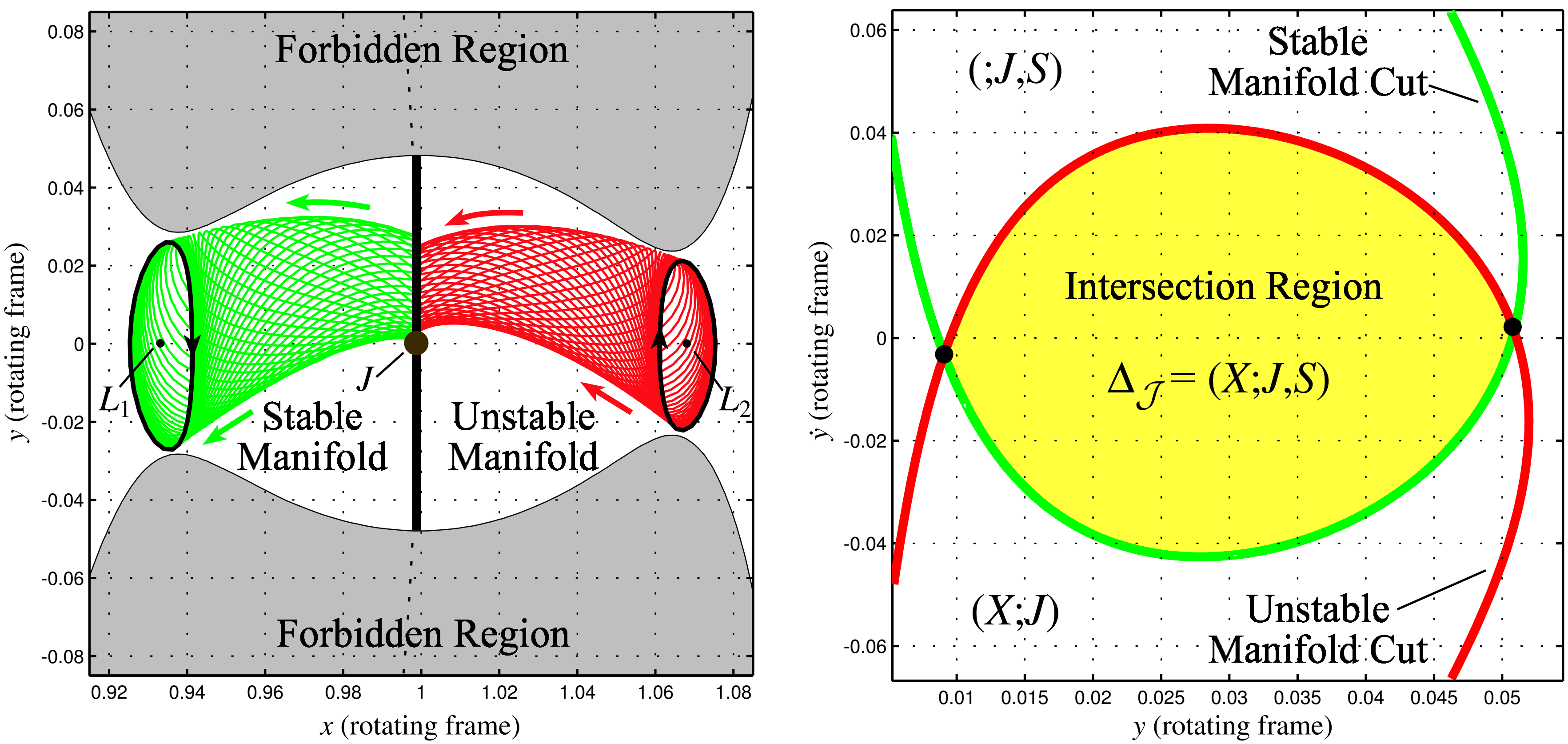
Major Result (C): Numerical Construction of Orbits

- Developed procedure to construct orbit with **prescribed itinerary**.
- Example: An orbit with itinerary $(\mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X})$.



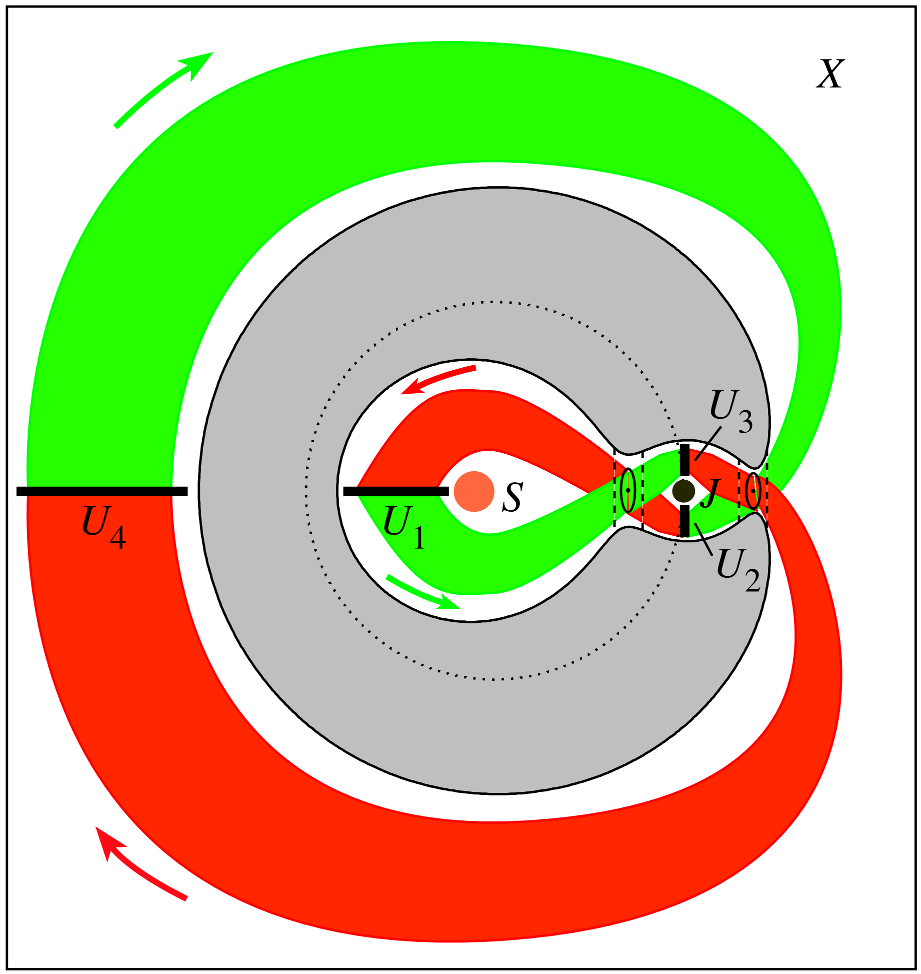
Details: Construction of $(\mathbf{J}, \mathbf{X}; \mathbf{J}, \mathbf{S}, \mathbf{J})$ Orbits

- Invariant manifold **tubes** separate transit from nontransit orbits.
- **Green curve** (Poincaré cut of L_1 **stable manifold**).
  Red curve (cut of L_2 **unstable manifold**).
- Any point inside the intersection region Δ_J is a $(\mathbf{X}; \mathbf{J}, \mathbf{S})$ orbit.



Details: Construction of (J, X; J, S, J) Orbits

- The desired orbit can be constructed by
  - Choosing appropriate **Poincaré sections** and
  - linking invariant **manifold tubes** in right order.

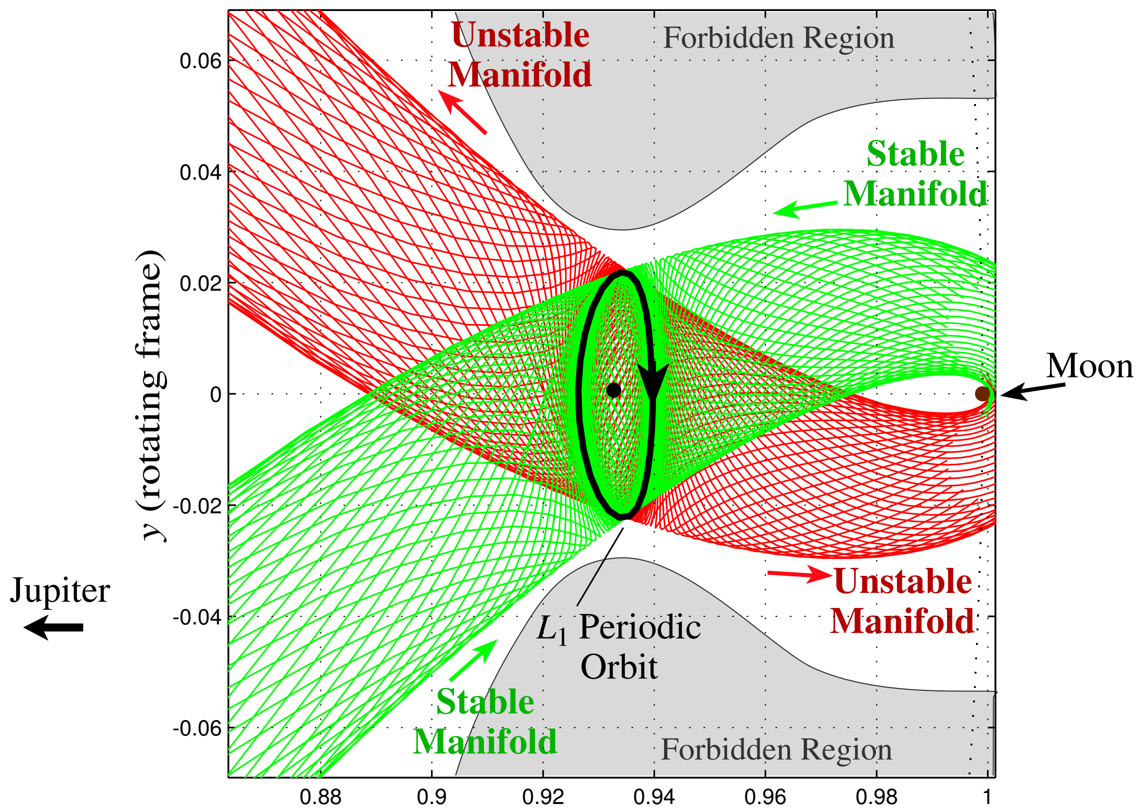


Computation of NHIM and Its Invariant Manifolds

- **Lie Transform** put CR3BP Hamiltonian into **normal form**

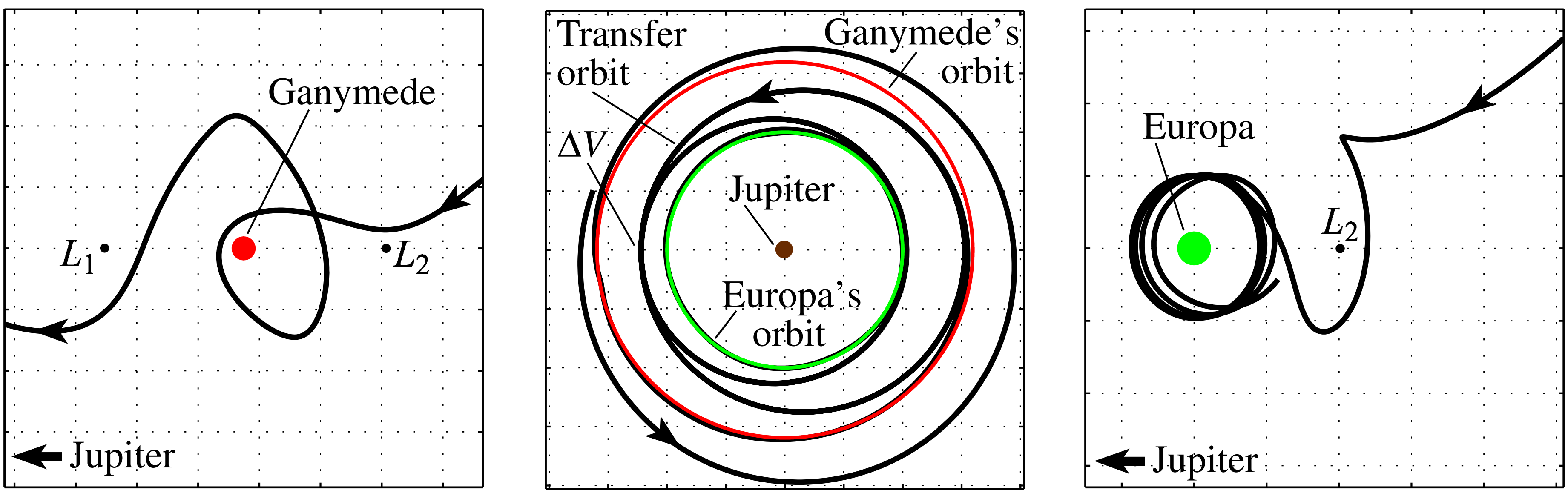
$$\bar{H}_N = \lambda q_1 p_1 + \frac{\nu}{2}(q_2^2 + p_2^2) + \frac{\omega}{2}(q_3^2 + p_3^2) + \sum_{n=3}^{N} H_n(q_1 p_1, q_2, p_2, q_3, p_3).$$

- Set $q_1 = p_1 = 0$, get **NHIM** (S^3).
- Set $q_1 = 0$ ($p_1 = 0$) and integrate, get stable (unstable) manifold.

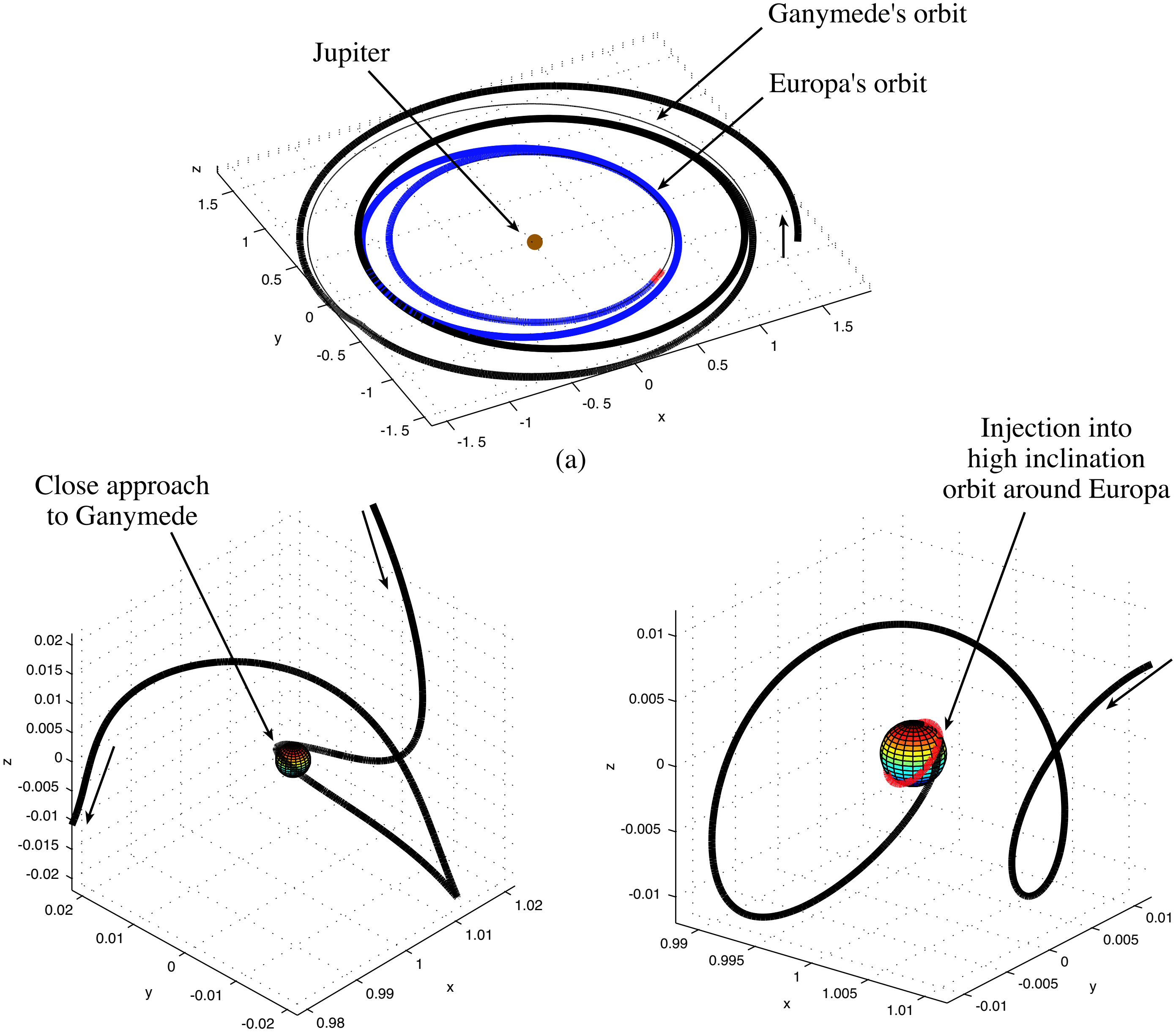


Petit Grand Tour of Jupiter's Moons (Planar Model)

- Used **invariant manifolds** to construct trajectories with interesting characteristics:
  - Petit Grand Tour of Jupiter's moons. 1 orbit around **Ganymede**. 4 orbits around **Europa**.
  - A ΔV nudges the SC from **Jupiter-Ganymede** system to **Jupiter-Europa** system.
- Instead of **flybys**, can orbit several moons for **any duration**.



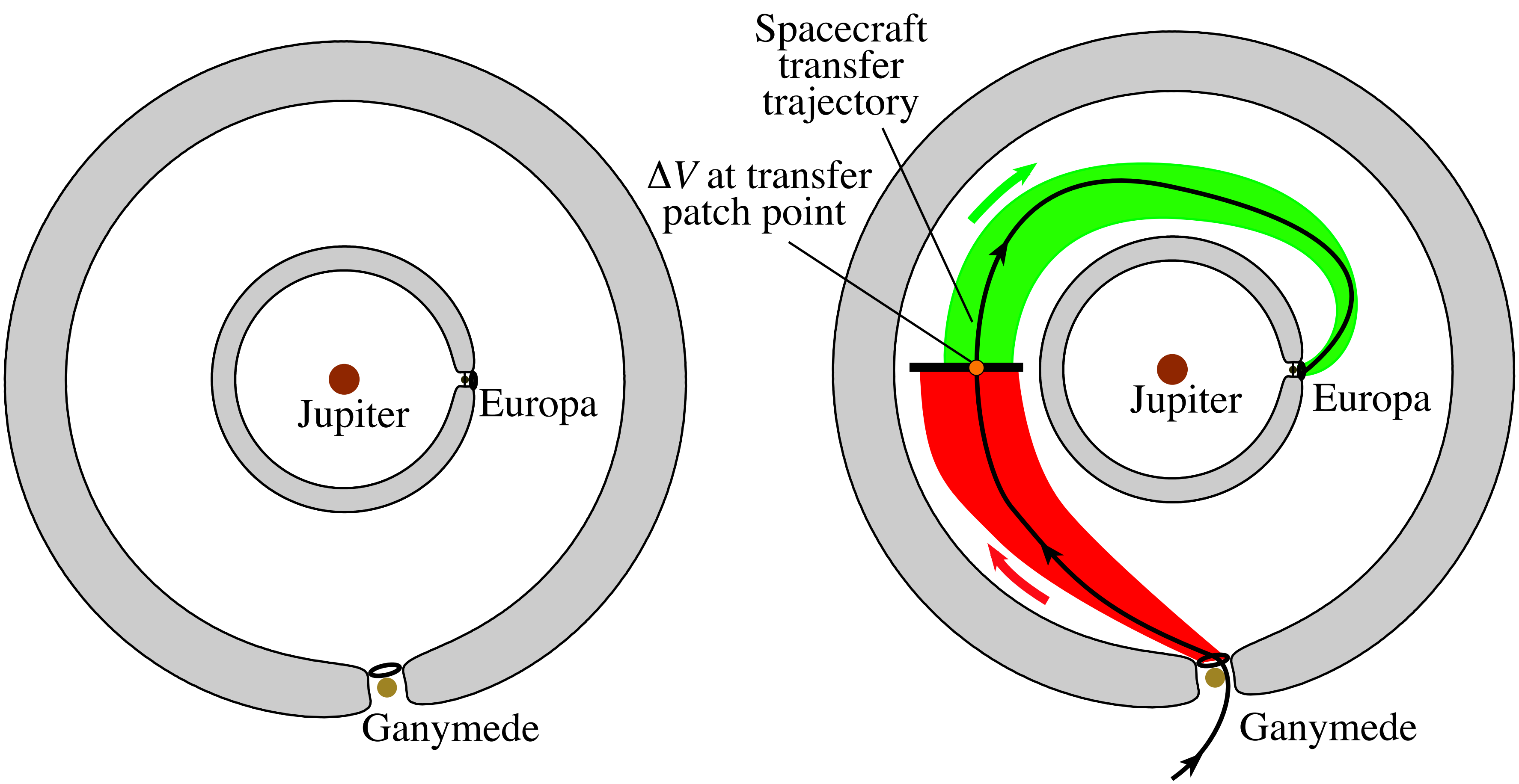
Extend from Planar Model to Spatial Model



(a) (b) (c)

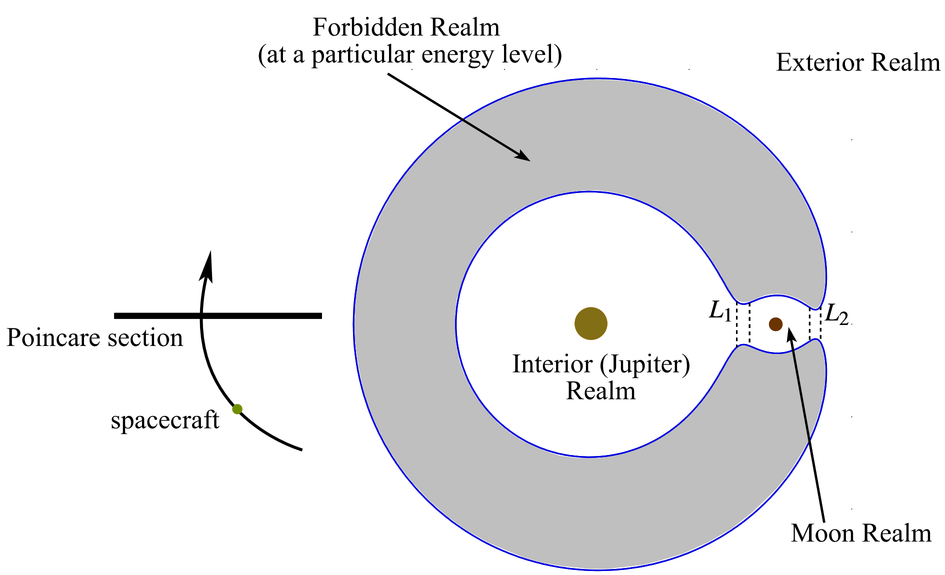
Petit Grand Tour of Jupiter's Moons

- Jupiter-Ganymede-Europa-SC 4-body system approximated as 2 **coupled 3-body systems**
- **Invariant manifold tubes** of two 3-body systems are linked in right order to construct orbit with desired itinerary.
- Initial solution refined in **4-body model**.

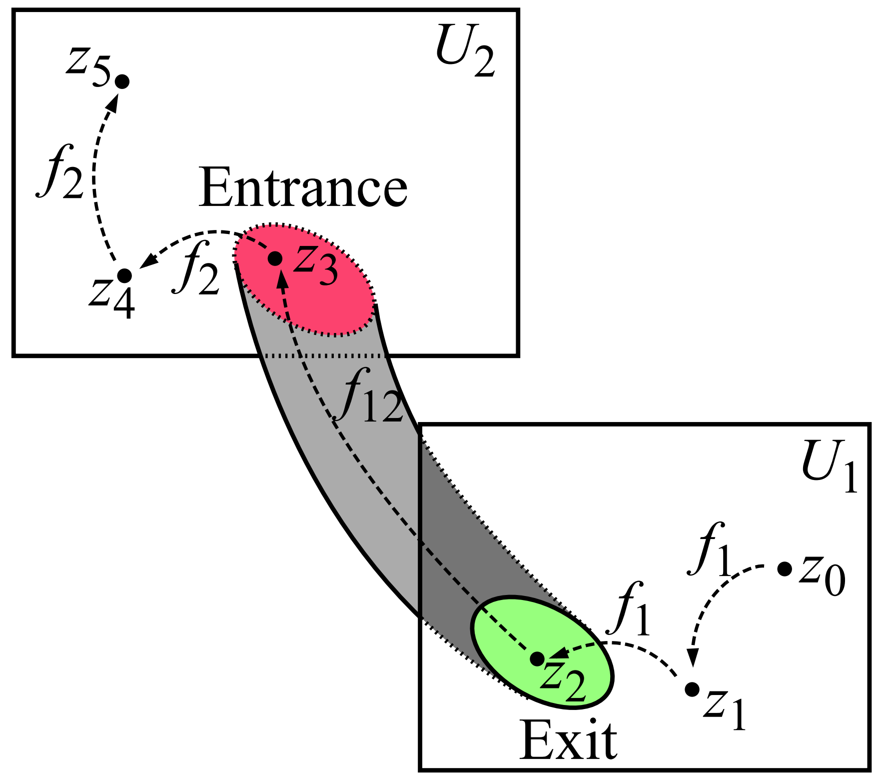


Look for Natural Pathways to Bridge the Gap

- **Tubes** of two 3-body systems **may not intersect** for awhile. May need large ΔV to "jump" from one tube to another.
- Look for **natural pathways** to bridge the gap by "hopping" through **phase space**
  - between z_0 where tube of one system **enters** and z_2 where tube of another system **exits** (into Europa realm).



(a)

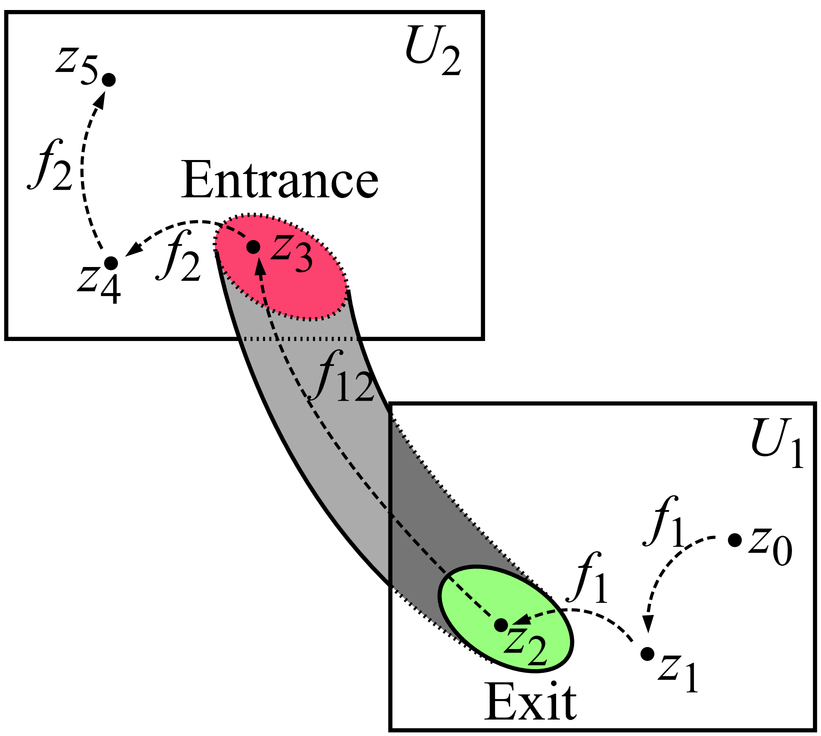


(b)

Transport in Phase Space via Tube & Lobe Dynamics

- By using
  - **tubes** of rapid transition that connect different **realms**
  - **lobe dynamics** to hop through phase space within a **realm**,

  New tour only needs $\Delta V = 20\text{m/s}$ (50 times less).



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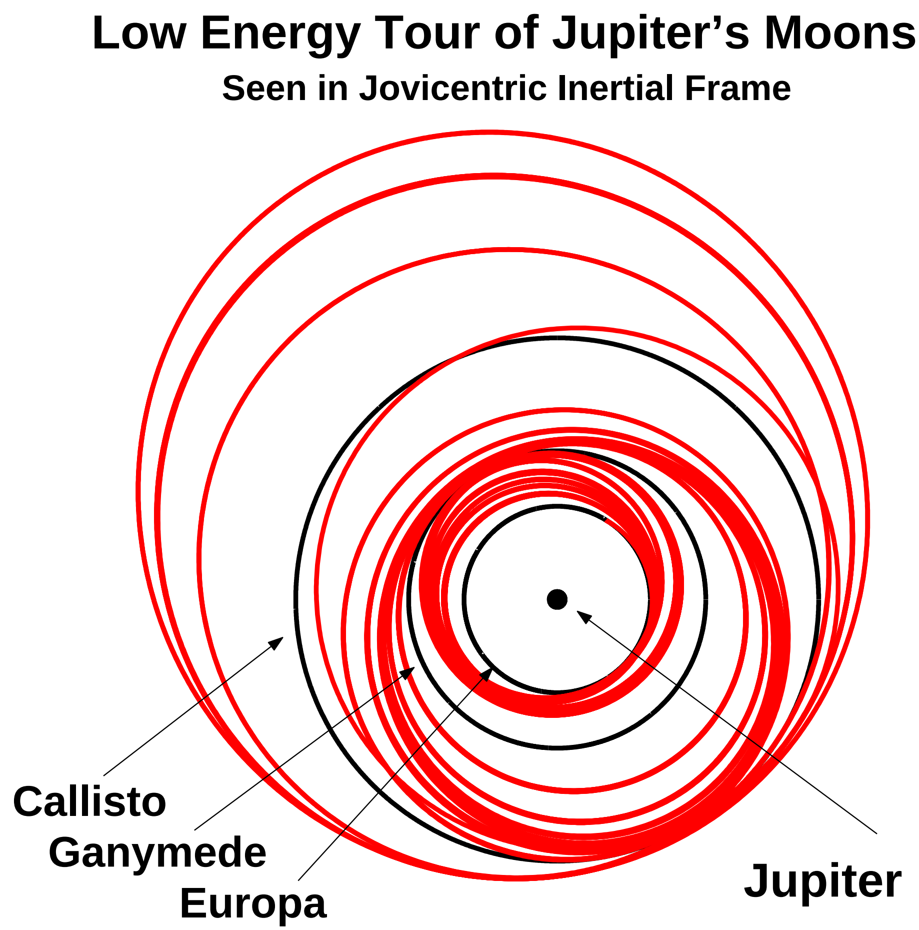
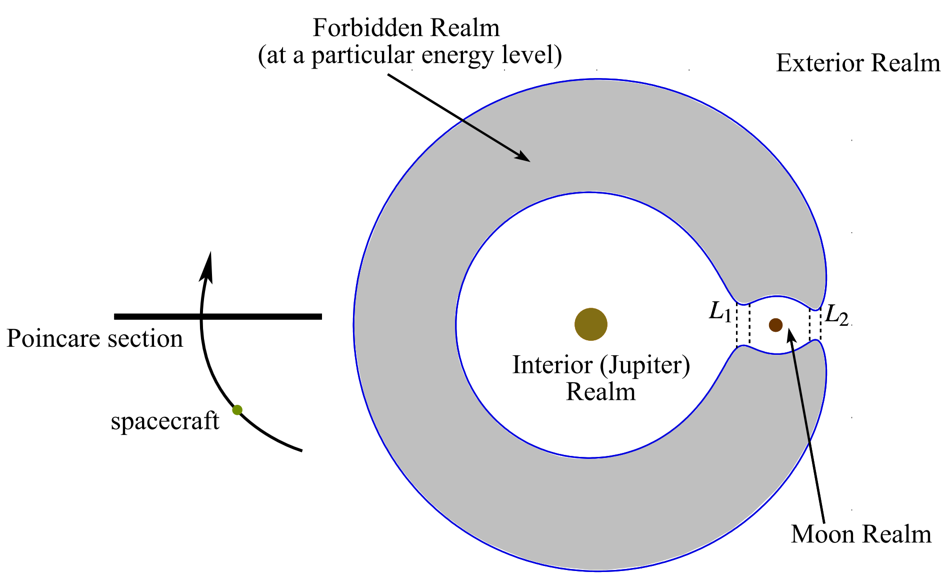


(b)

Lobe Dynamics: Mixed Phase Space

- Poincaré section reveals **mixed phase space**:
  - **resonance regions** and
  - "**chaotic sea**".



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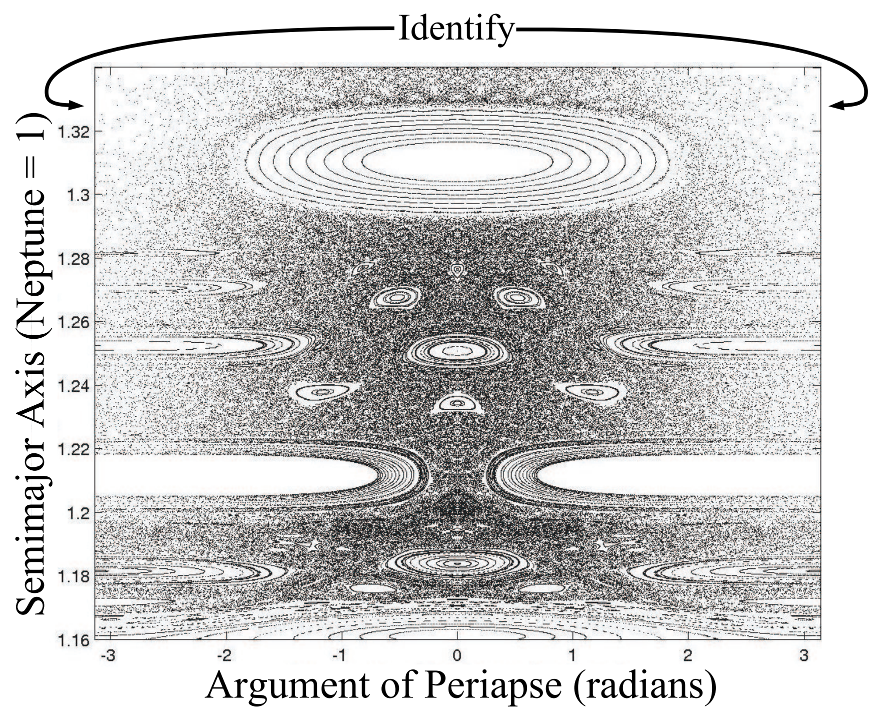
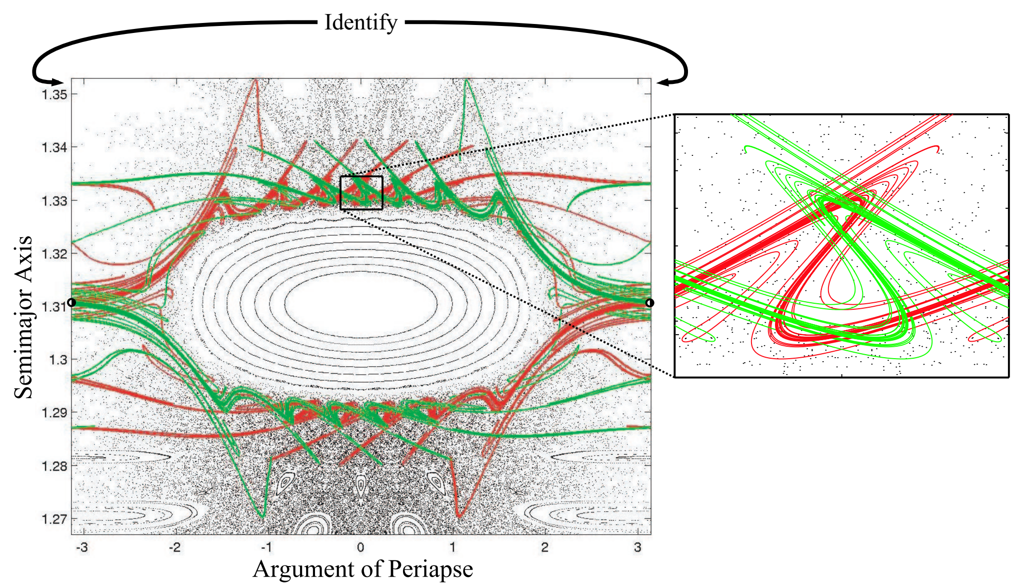
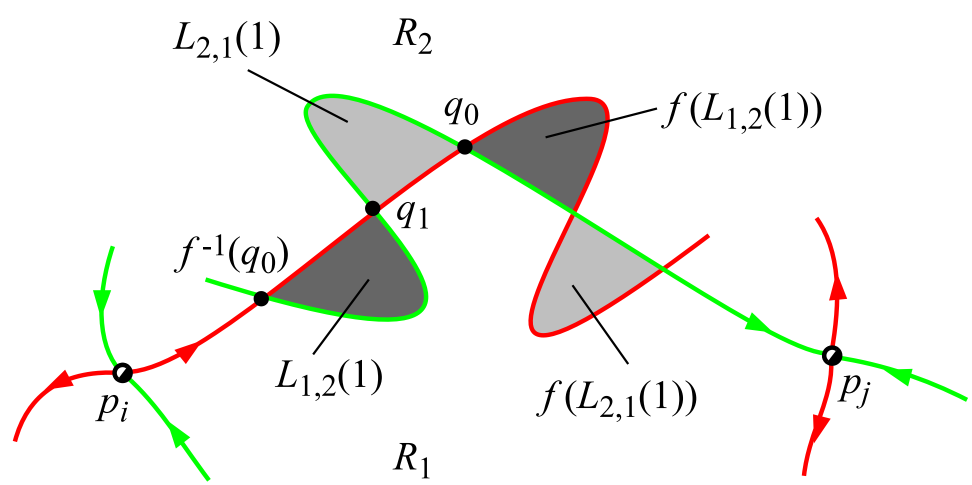


(b)

Transport between Regions via Lobe Dynamics

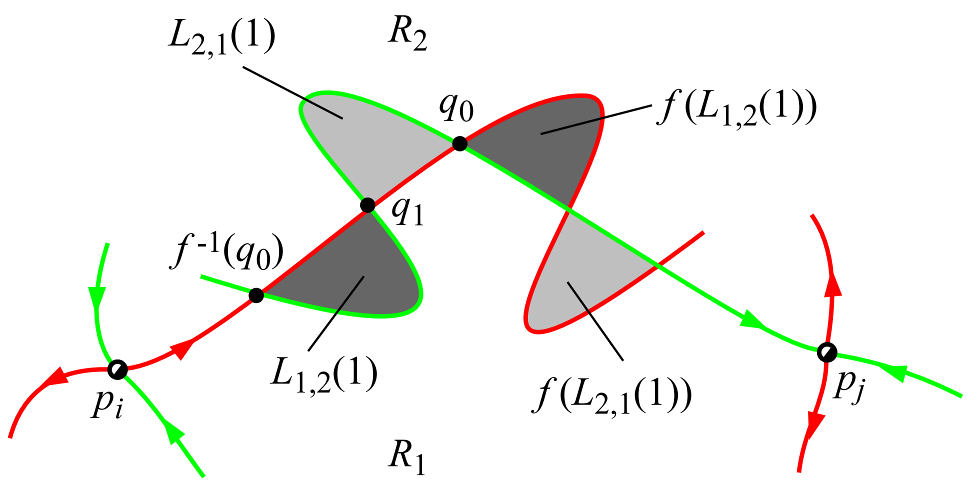
- **Invariant manifolds** divide phase space into resonance regions.
- Transport between regions can be studied via **lobe dynamics**.





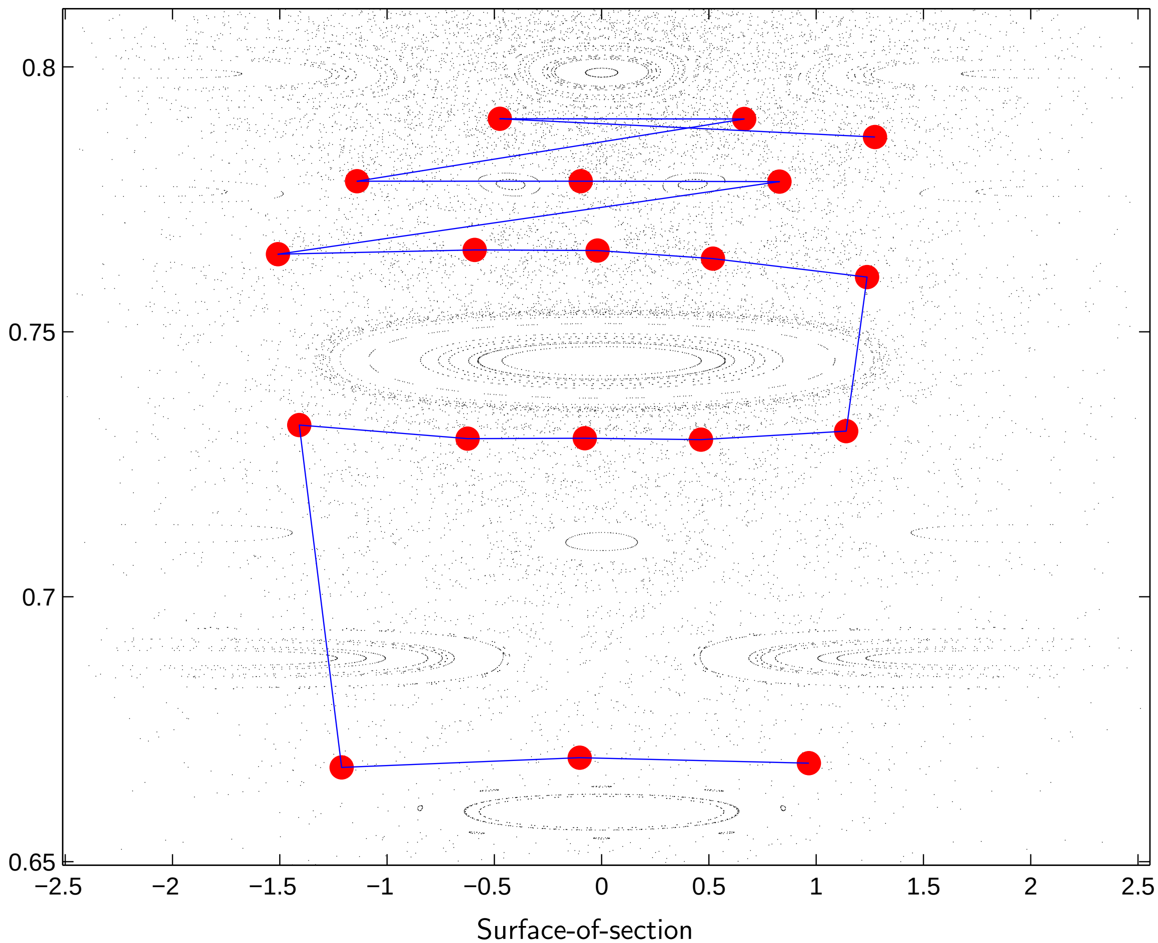
Transport between Regions via Lobe Dynamics

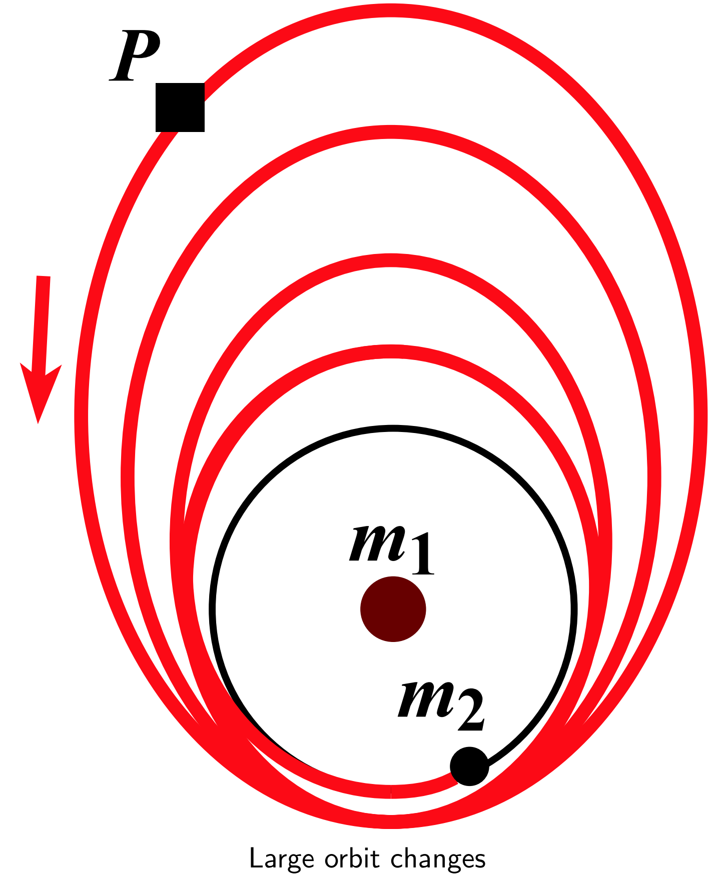
- Segments of **unstable** and **stable** manifolds form **partial barriers** between regions R_1 and R_2.
- $L_{1,2}(1), L_{2,1}(1)$ are **lobes**; they form a **turnstile**.
  - In one iteration, only points from R_1 to R_2 are in $L_{1,2}$
  - only points from R_2 to R_1 are in $L_{2,1}(1)$.
- By studying pre-images of $L_{1,2}(1)$, one can find efficient way from R_1 to R_2.



Hopping through Resonaces in Low Energy Tour

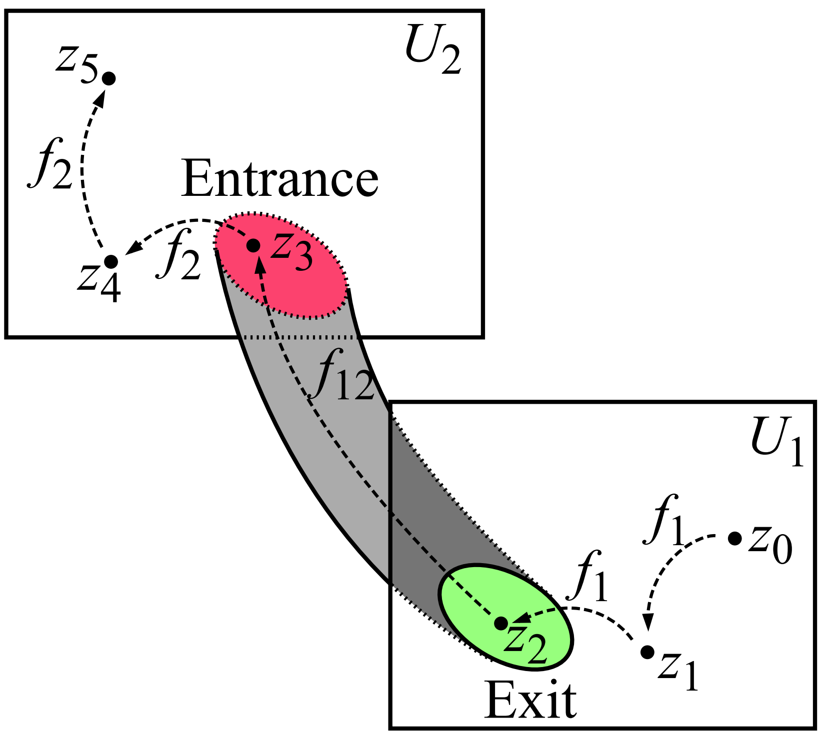
- Guided by lobe dynamics, **hopping** through resonances (essential for low energy tour) can be performed.





Tube/Lobe Dynamics: Transport in Solar System

- To use **tube/lobe** dynamics of **spatial** 3-body problem to **systematically** design low-fuel trajectory.
- Part of our program to study transport in solar system using **tube** and **lobe dynamics**.



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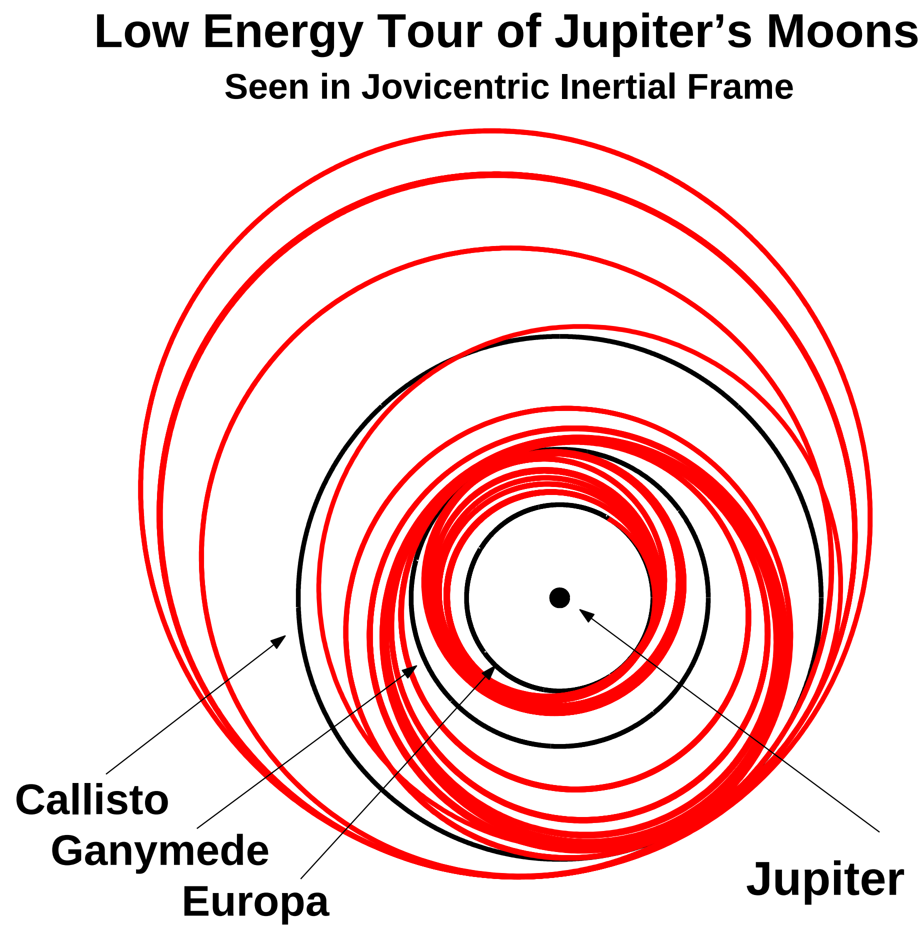
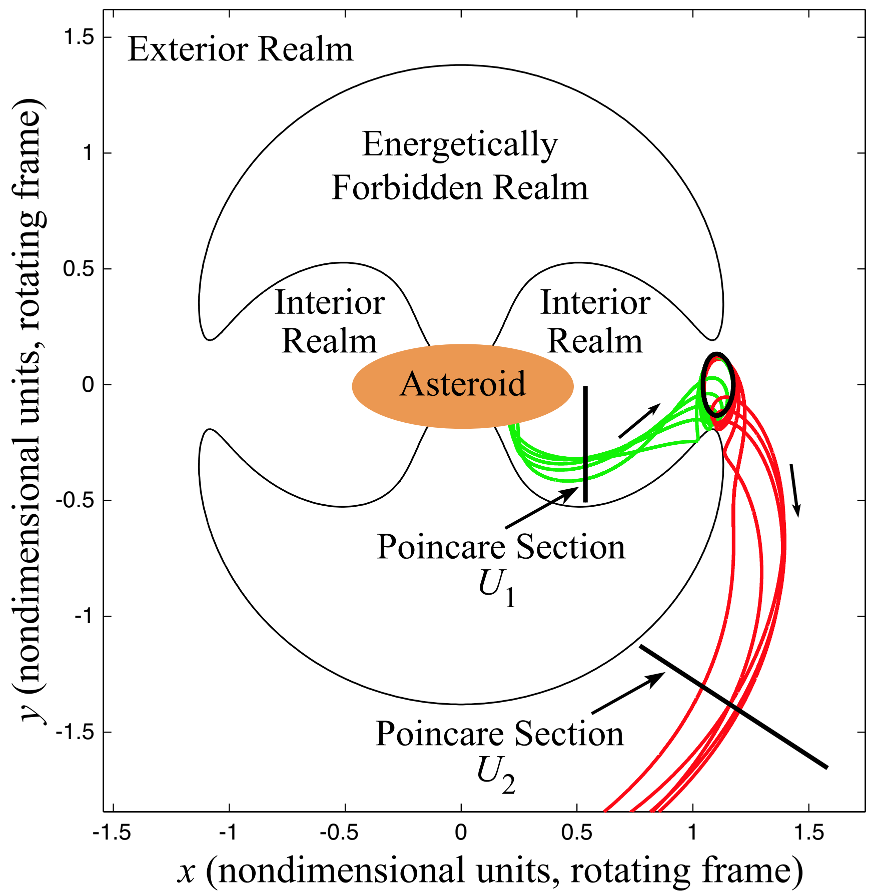


(b)

Tube/Lobe Dynamics: 2FBP/Asteroid Pairs

- To study dynamical interacton between 2 rigid bodies where their **rotational** and **translational** motions are coupled.
  - formation of binary asteroids (shown below are Ida and Dactyl)
  - evolution of asteroid rotational states

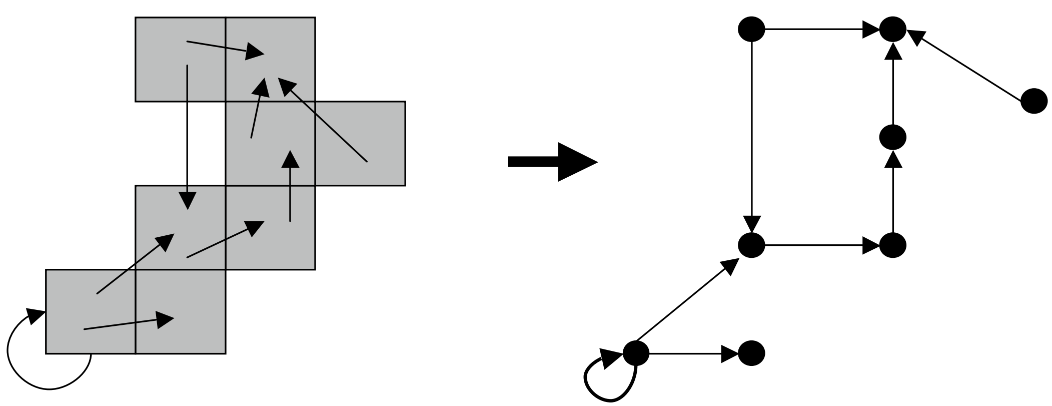
(a)



(b)

Techniques of Almost Invariant Sets (AIS)

- Combine techniques of **almost invariant sets** (tree structured box elimination/graph partitioning algorithms) with **lobe dynamics** techniques.
- Compute almost invariant sets, resonance regions, bottlenecks and **transport rates** (Sun-Jupiter system).
- Besides astronomy, applicable to multibody problems: **molecular modeling**, **chemical reaction rates**, etc.



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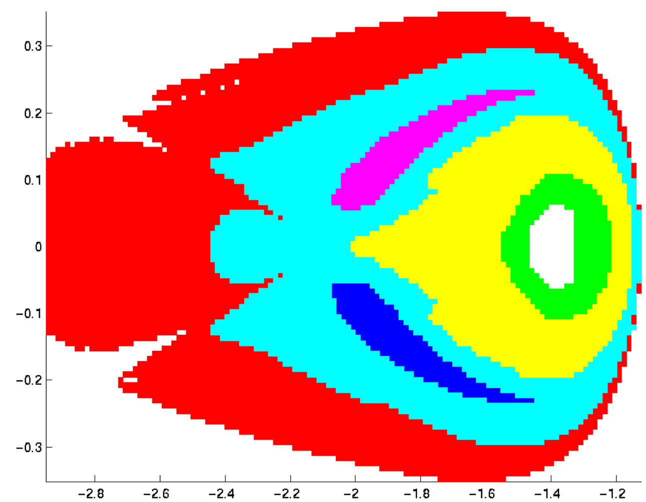

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