Design of Low Energy Space Missions using Dynamical Systems Theory

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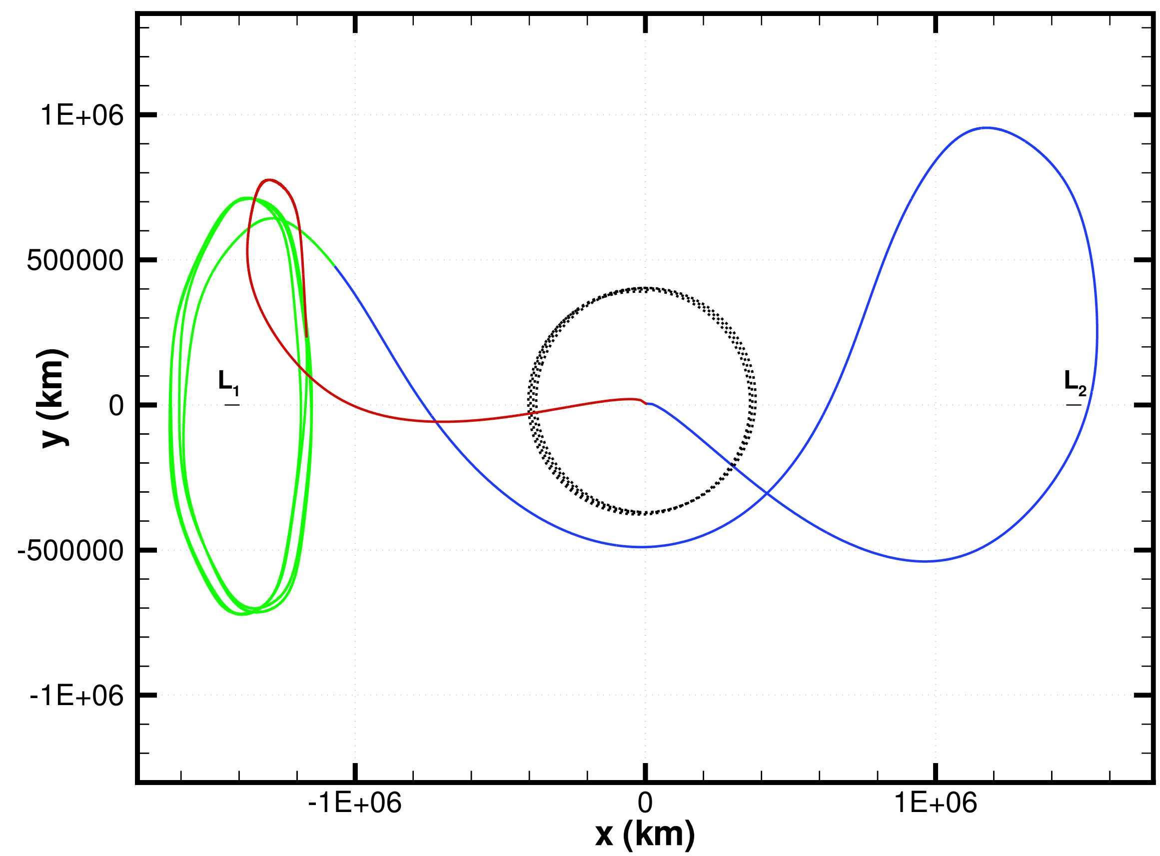
Outline

- **Main Theme**
  - how to use dynamical systems theory of 3-body problem in low energy trajectory design.
- **Background** and **Motivation**:
  - NASA's Genesis Discovery Mission.
  - A Low Energy Tour of Jupiter's Moons.
- **Restricted 3-Body Problem**.
- **Main Results**.
- **Ongoing Work**.
  - Low Thrust Trajectories in a Multi-Body Environment.
  - Parking a Satellite near an Asteroid Pair.

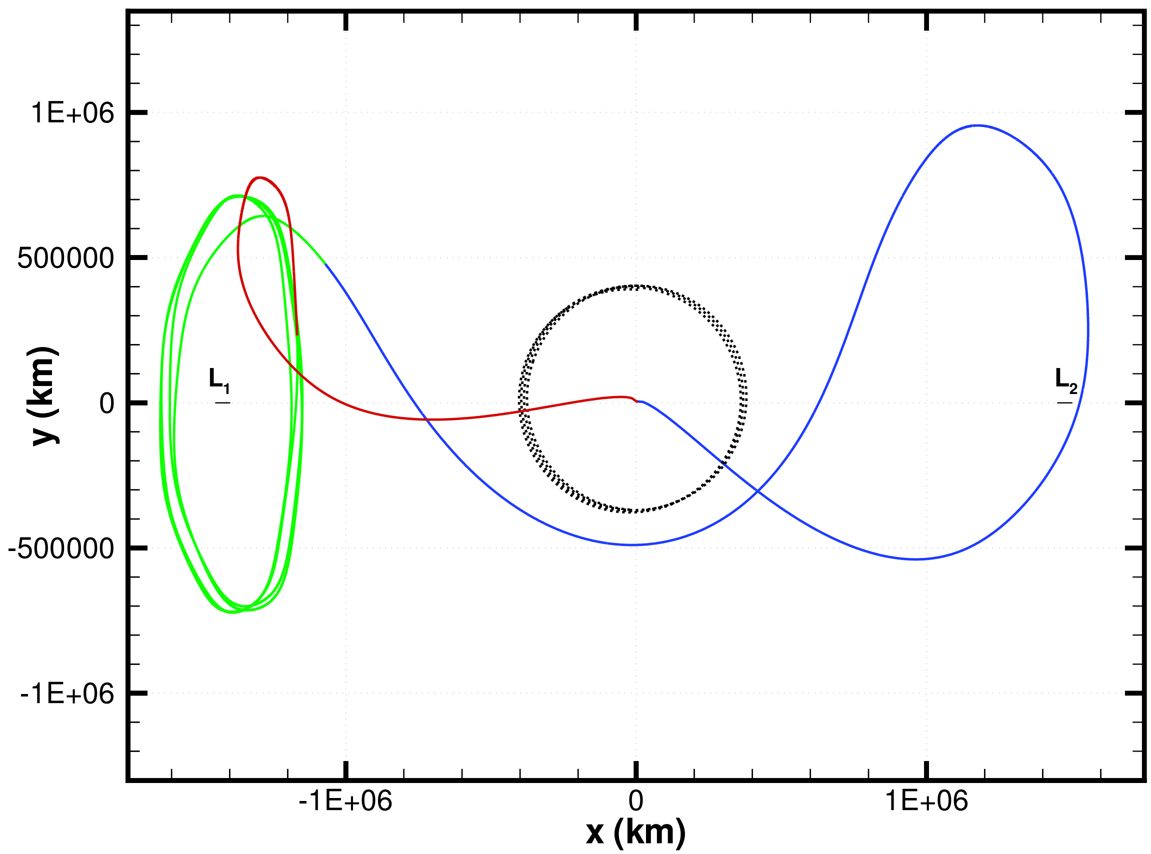
Motivation: Genesis Discovery Mission

- **Genesis spacecraft**
  - collected solar wind sample from a L_1 **halo orbit**,
  - **returned** them to Earth.
- **Halo orbit**, **transfer**/ **return** trajectories in rotating frame.



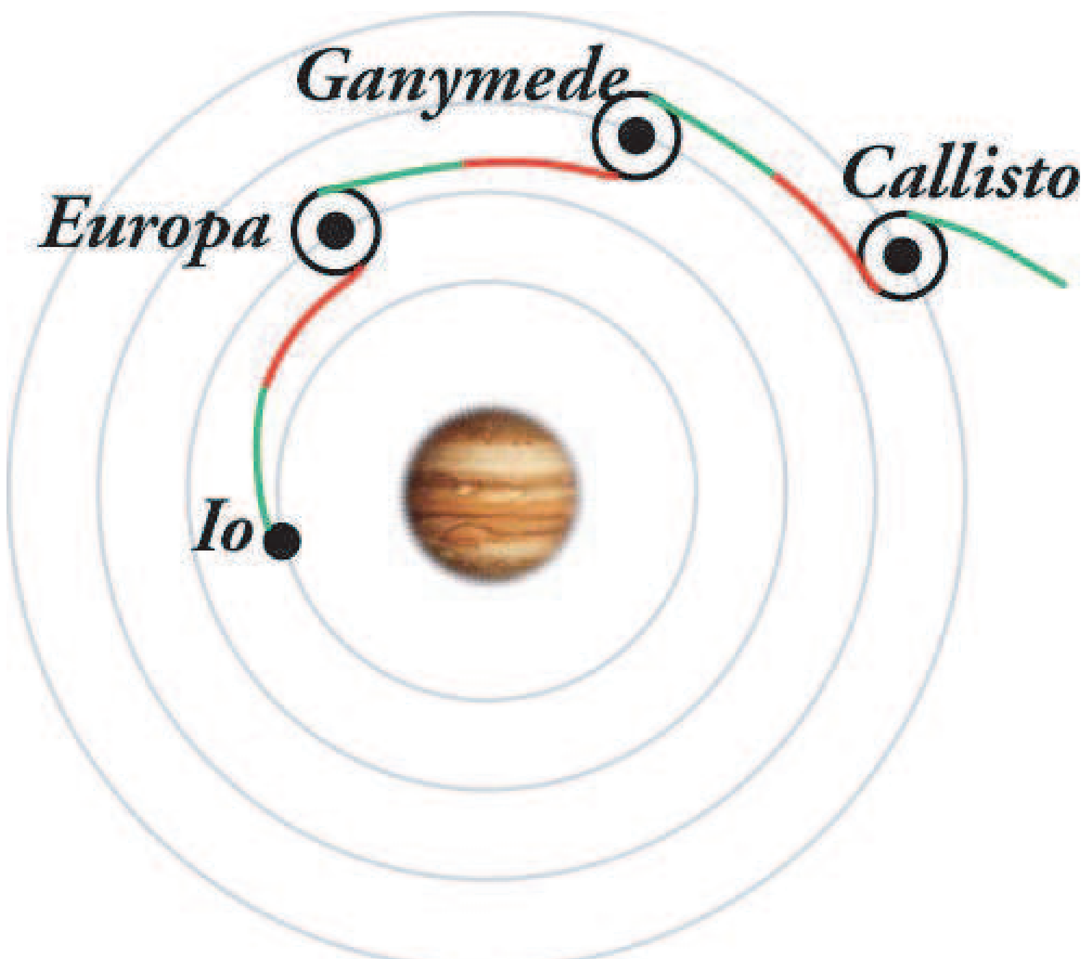
Motivation: Genesis Discovery Mission

- Designed using **dynamical systems theory** (Barden, Howell, and Lo).
- Followed **natural dynamics**, little propulsion after launch.
- **Return-to-Earth portion** utilized heteroclinic dynamics.



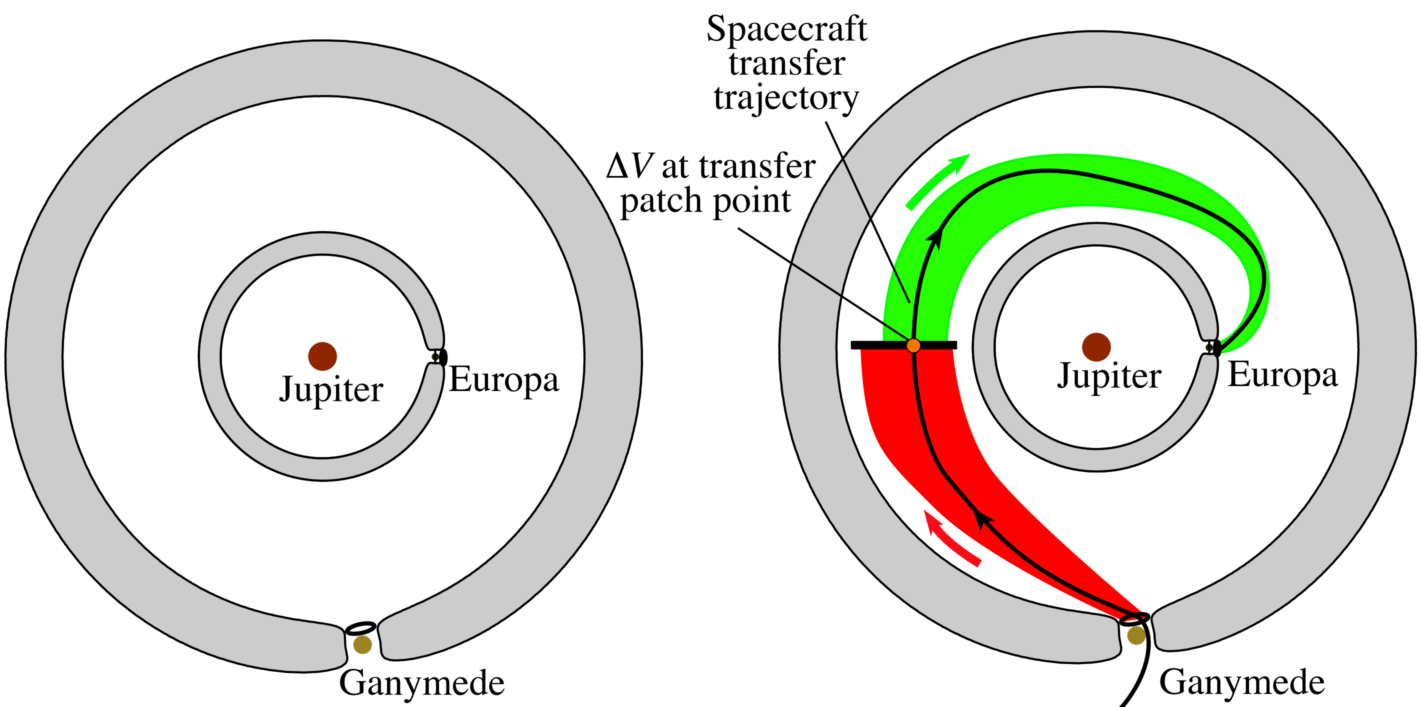
Motivation: Petit Grand Tour of Jupiter's Moons

- Construct a **low energy trajectory** to visit **several moons** in one mission.
- Instead of flybys, can **orbit each moon for any duration**.
- NASA is considering a Jupiter Icy Moon Orbiter (**JIMO**).



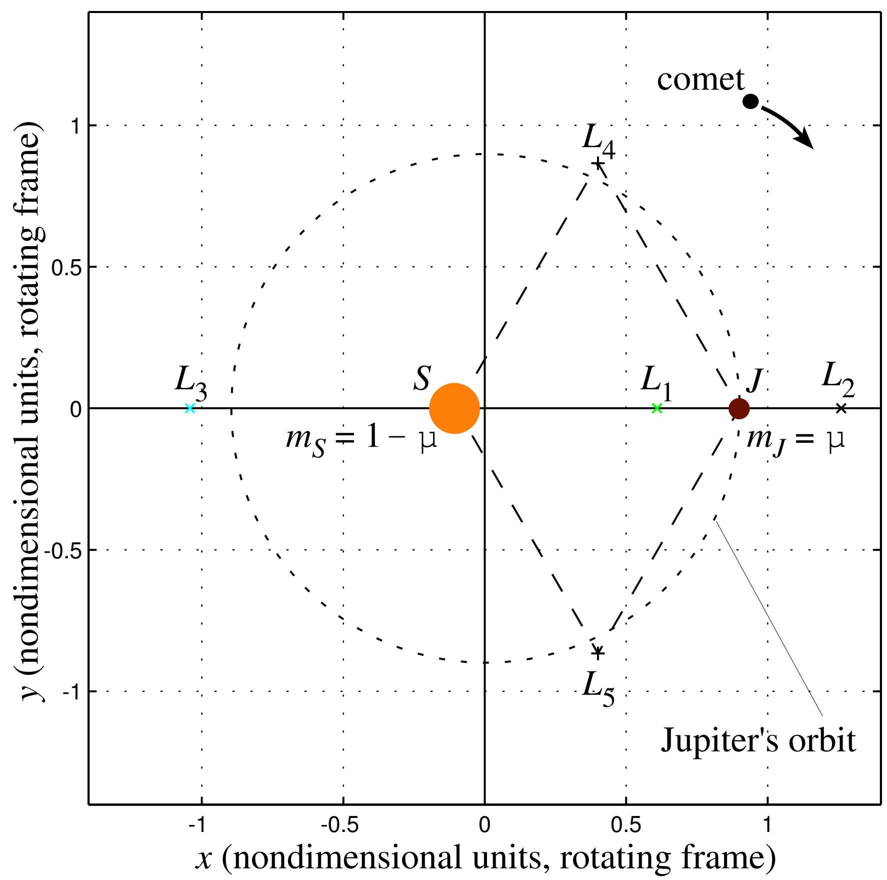
Design Strategy: Patched 3-Body Solutions

- Jupiter-Ganymede-Europa-SC 4-body system approximated as 2 **coupled 3-body systems**
- **3-body solutions** of each 3-body systems are linked in right order to construct orbit with desired itinerary.
- Try to **minimize** ΔV at each transfer patch point.
- Initial solution refined in **4-body model**.
- 3-body solutions offer a large class of low energy trajectories.



Planar Circular Restricted 3-Body Problem

- 2 main bodies
  - Total mass normalized to 1: $m_J = \mu, \quad m_S = 1 - \mu.$
  - Rotate about center of mass, angular velocity normalized to 1.
- Choose **rotating** coordinate system with origin at center of mass, 2 main bodies fixed at $(-\mu, 0)$ and $(1 - \mu, 0)$.



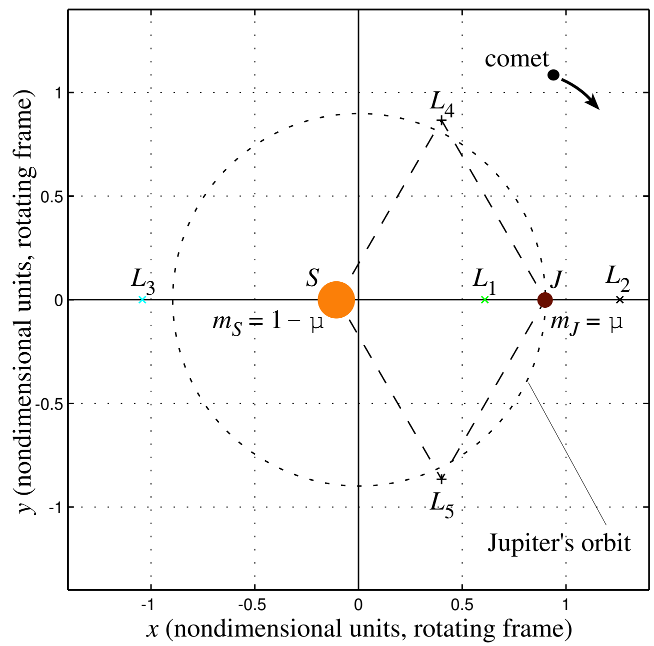
Equilibrium Points

- Equations of motion for SC are

$$\ddot{x} - 2\dot{y} = -\frac{\partial U}{\partial x}, \qquad \ddot{y} + 2\dot{x} = -\frac{\partial U}{\partial y},$$

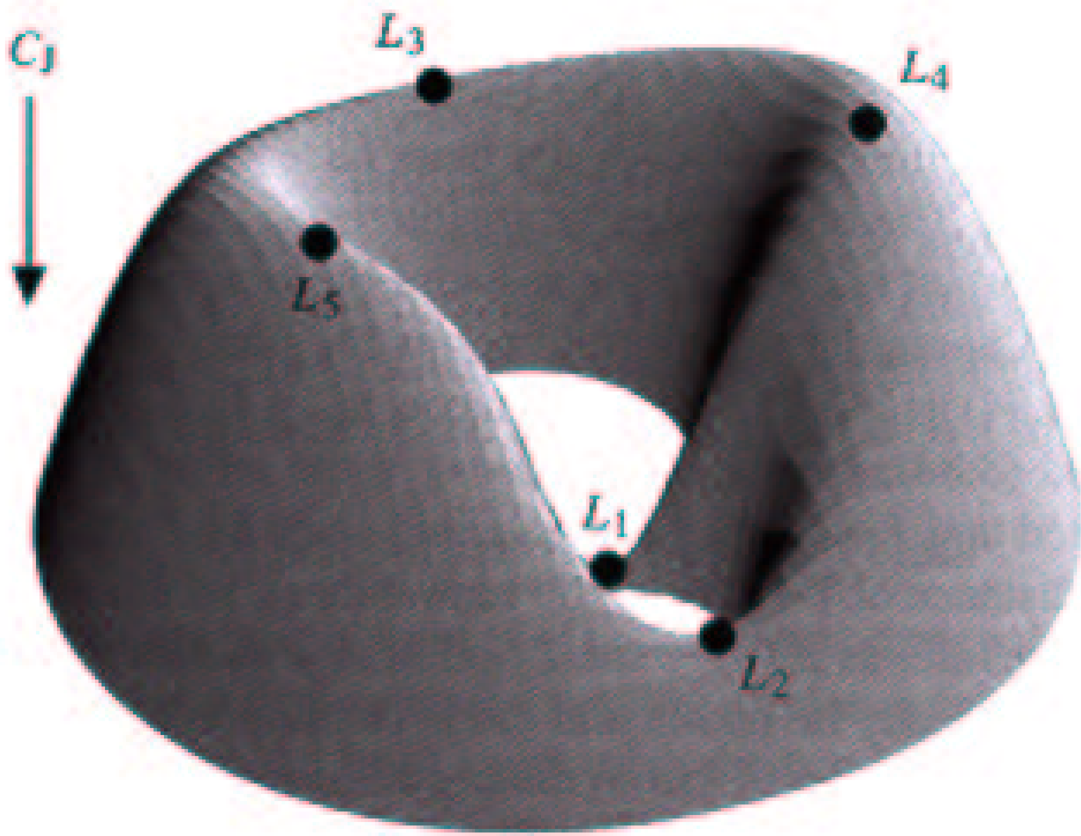
where $U(x,y) = -\frac{x^2+y^2}{2} - \frac{1-\mu}{r_s} - \frac{\mu}{r_j}$.

- Five equilibrium points:
  - 3 **unstable** collinear equilbrium points, L_1, L_2, L_3.
  - 2 equilateral equilibrium points, L_4, L_5.



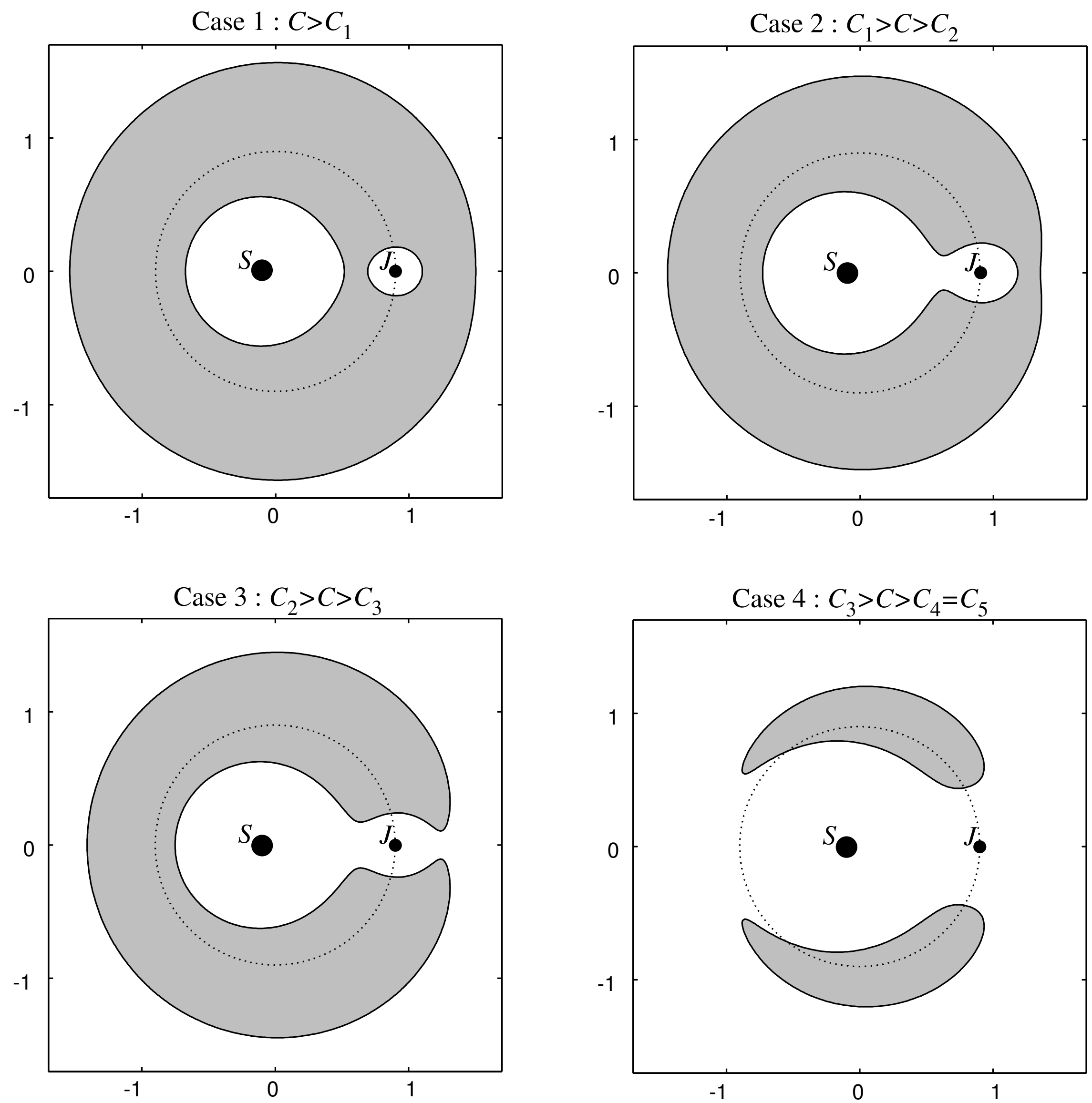
Hill's Realm

- **Energy integral**: $E(x, y, \dot{x}, \dot{y}) = (\dot{x}^2 + \dot{y}^2)/2 + U(x, y)$.
- E can be used to determine (**Hill's**) **realm** in position space where SC is energetically permitted to move.
- **Effective potential**: $U(x, y) = -\frac{x^2+y^2}{2} - \frac{1-\mu}{r_s} - \frac{\mu}{r_j}$.



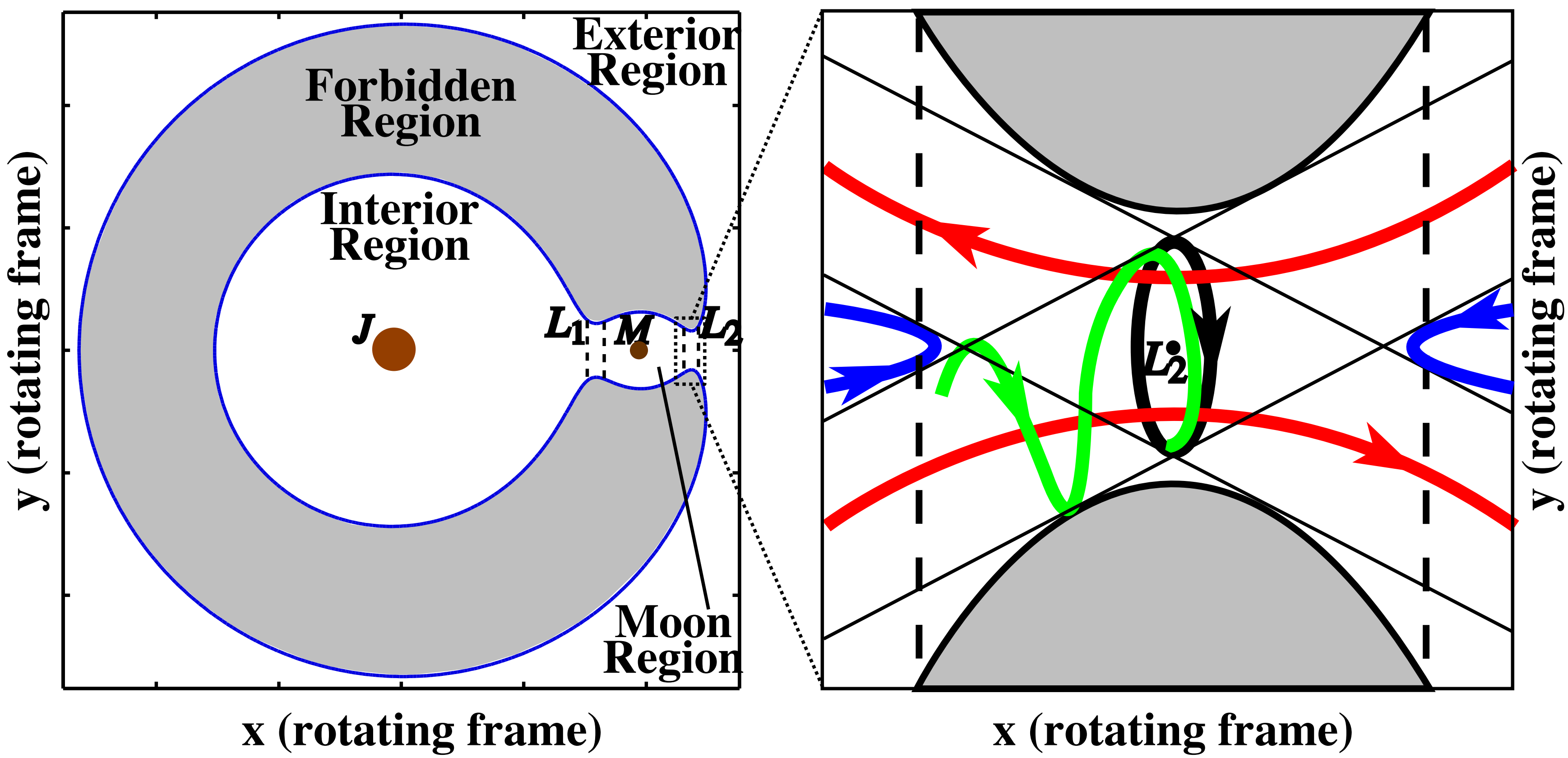
Hill's Realm

- To fix **energy value** E is to fix **height** of plot of $U(x, y)$. **Contour plots** give **5** cases of Hill's realm.



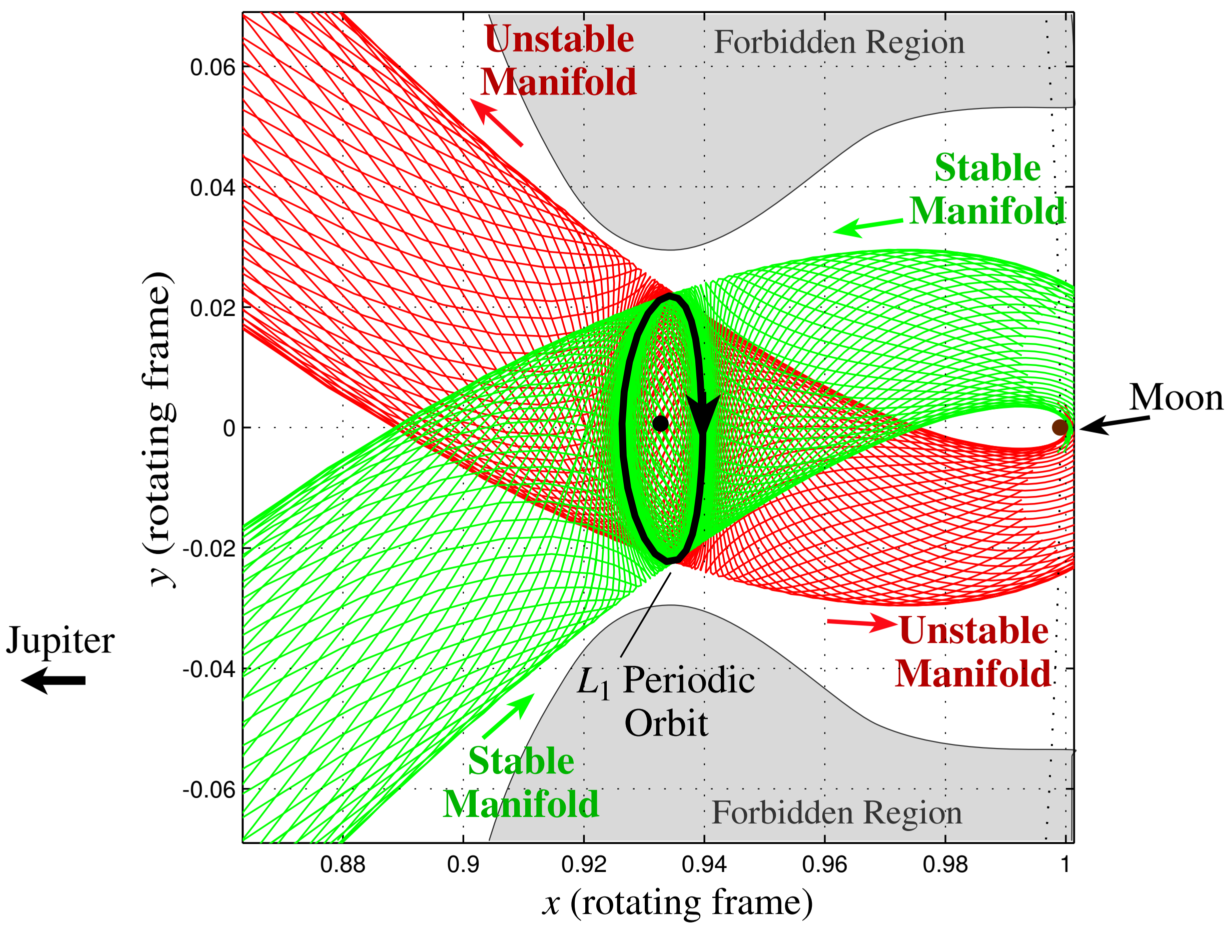
The Flow near L_1 and L_2

- For **energy value** just above that of L_2,
  Hill's realm contains a **"neck"** about L_1 & L_2.
- SC can make **transition** through these equilibrium realms.
- 4 types of orbits:
  periodic, **asymptotic**, **transit** & **nontransit**.



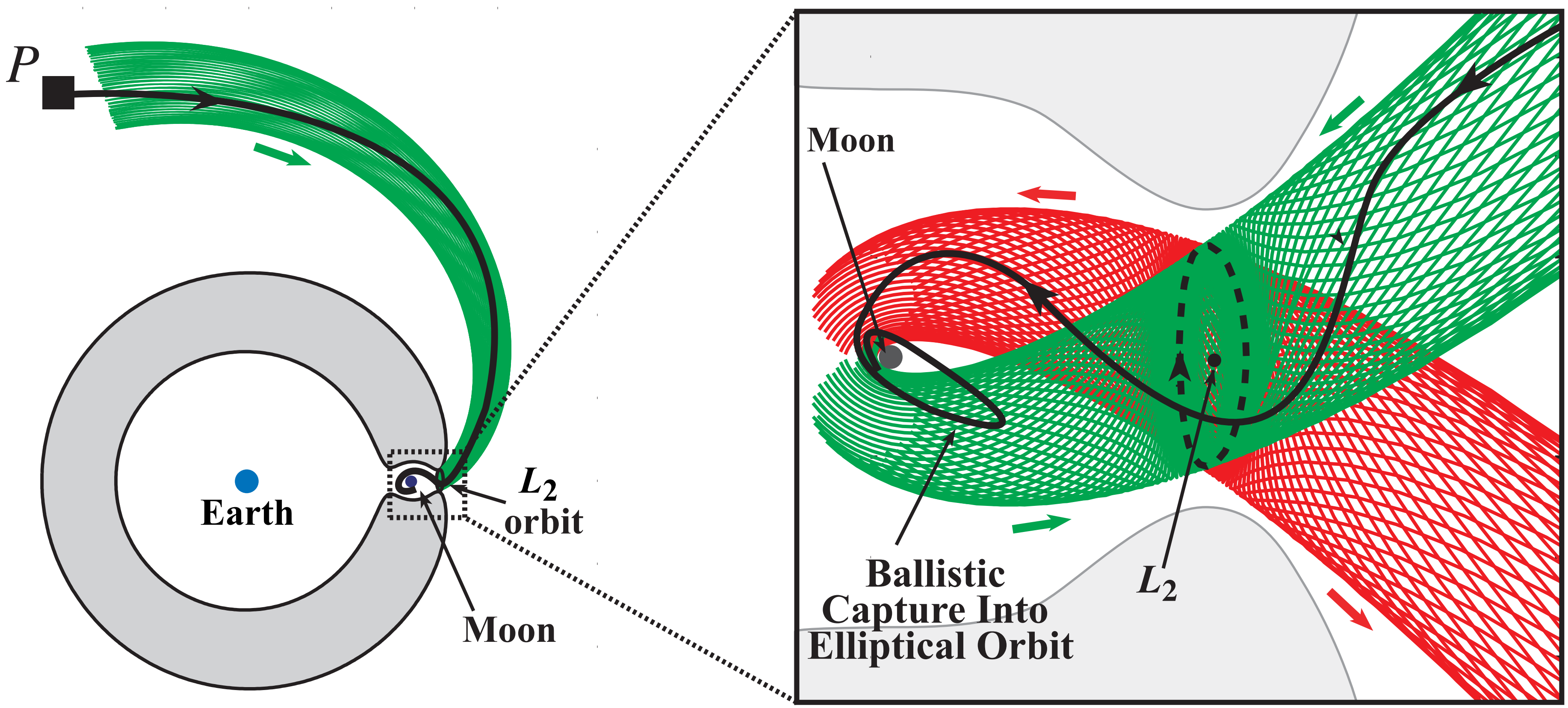
Invariant Manifold as Separatrix

- Asymptotic orbits form **2D invariant manifold tubes** in **3D energy surface**.
- They separate transit and non-transit orbits:
  - **Transit orbits** are those inside the tubes.
  - **Non-transit orbits** are those outside the tubes.



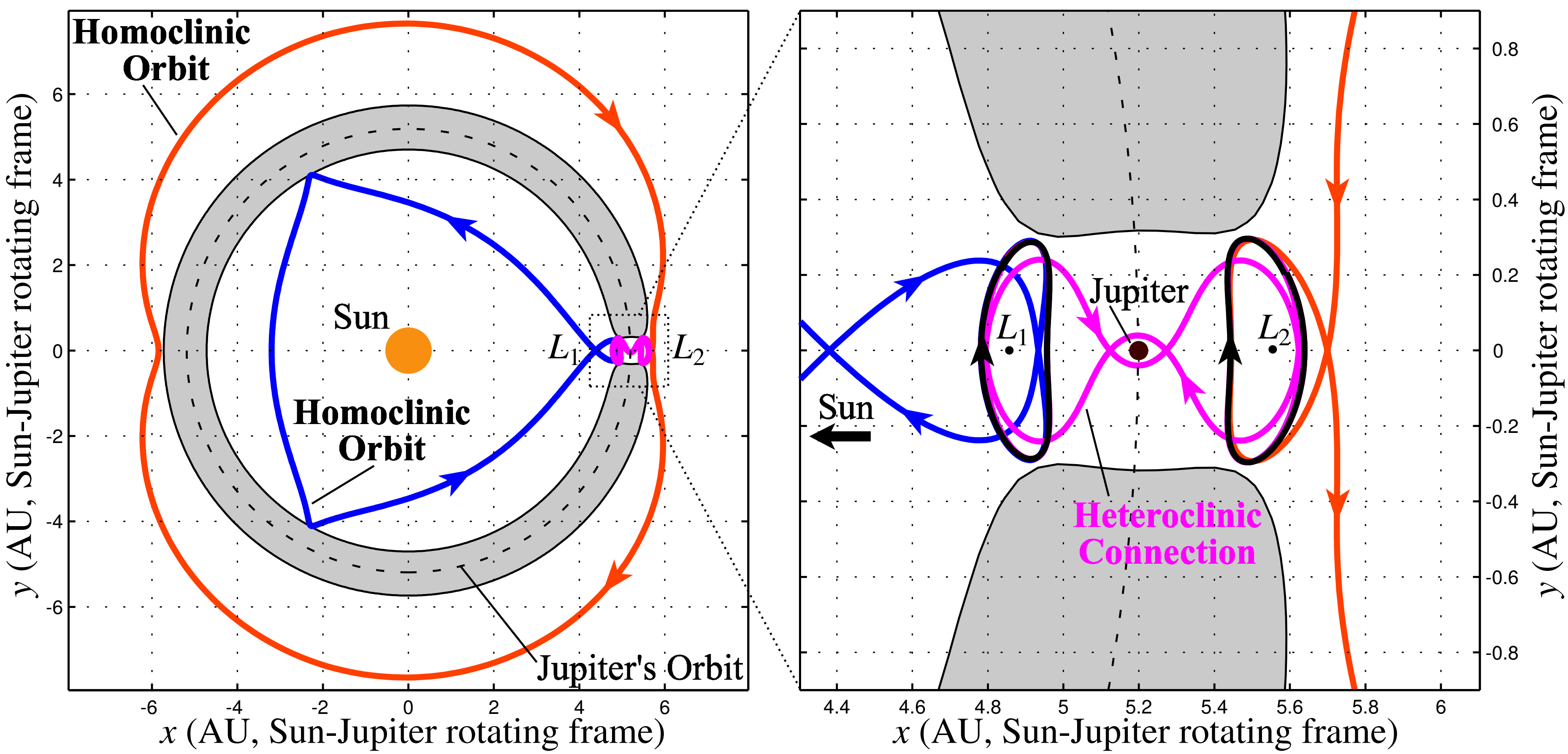
Invariant Manifold as Separatrix

- **Invariant Manifold Tubes** associated with periodic orbits about L_1, L_2 control **ballistic capture** and **escape**.



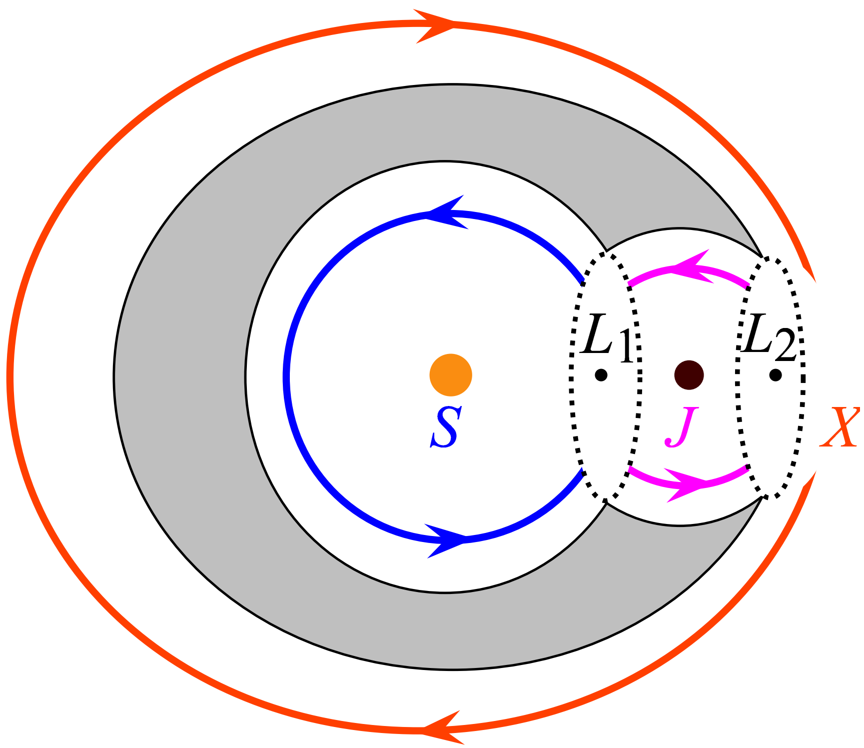
Heteroclinic Connection

- Found **heteroclinic connection** between pair of periodic orbits.
- Found a large class of **orbits** near this (homo/heteroclinic) ***chain***.
- SC can follow these ***channels*** in rapid transition.



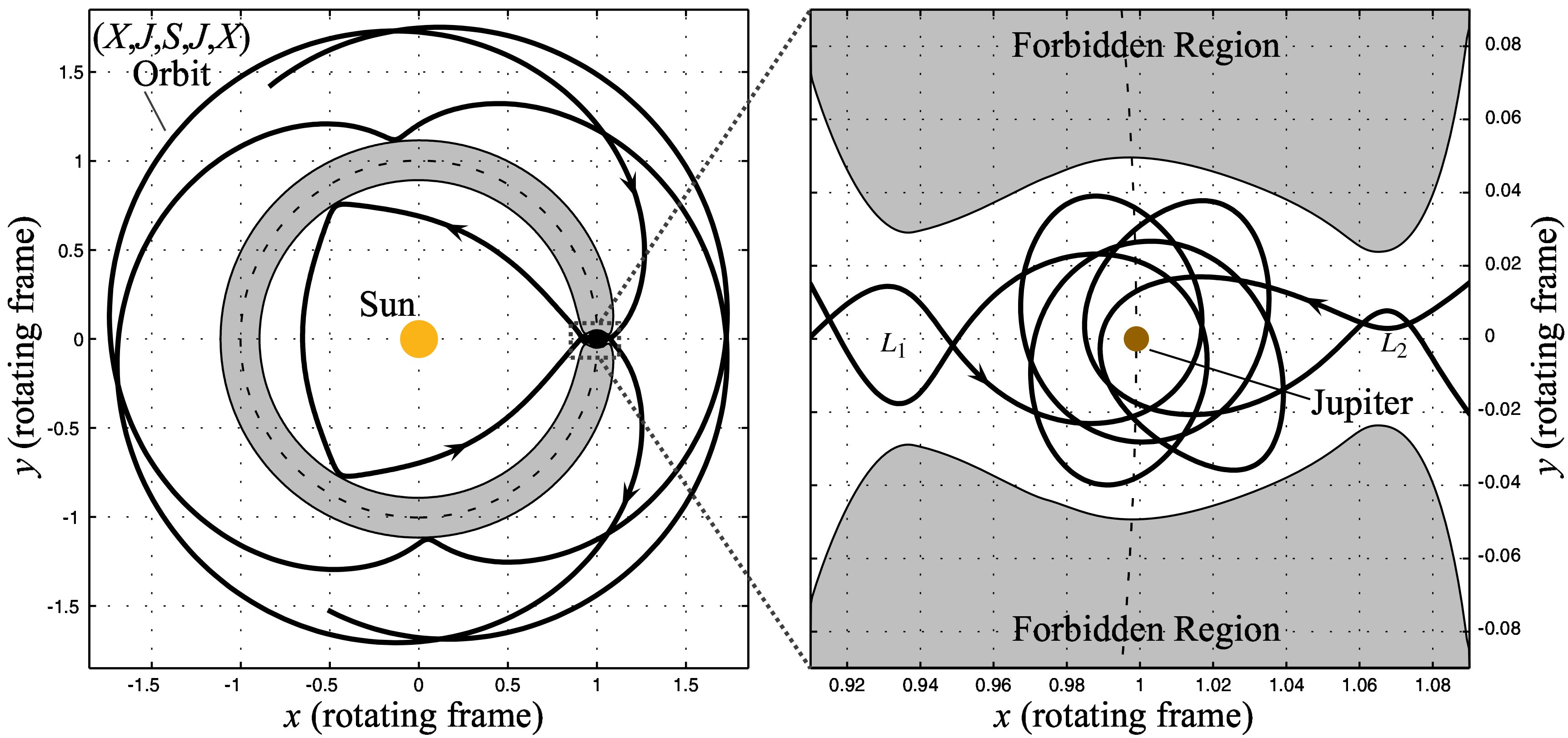
Existence of Transitional Orbits

- ***Main Theorem***: For any admissible **itinerary**, e.g., (. . . , **X**, **J**; **S**, **J**, **X**, . . .), there exists an orbit whose **whereabouts** matches this **itinerary**.
- Can even specify **number of revolutions** the comet makes around Sun & Jupiter (plus L_1 & L_2).
- **3-Body** trajectories much richer than **2-body** trajectories.



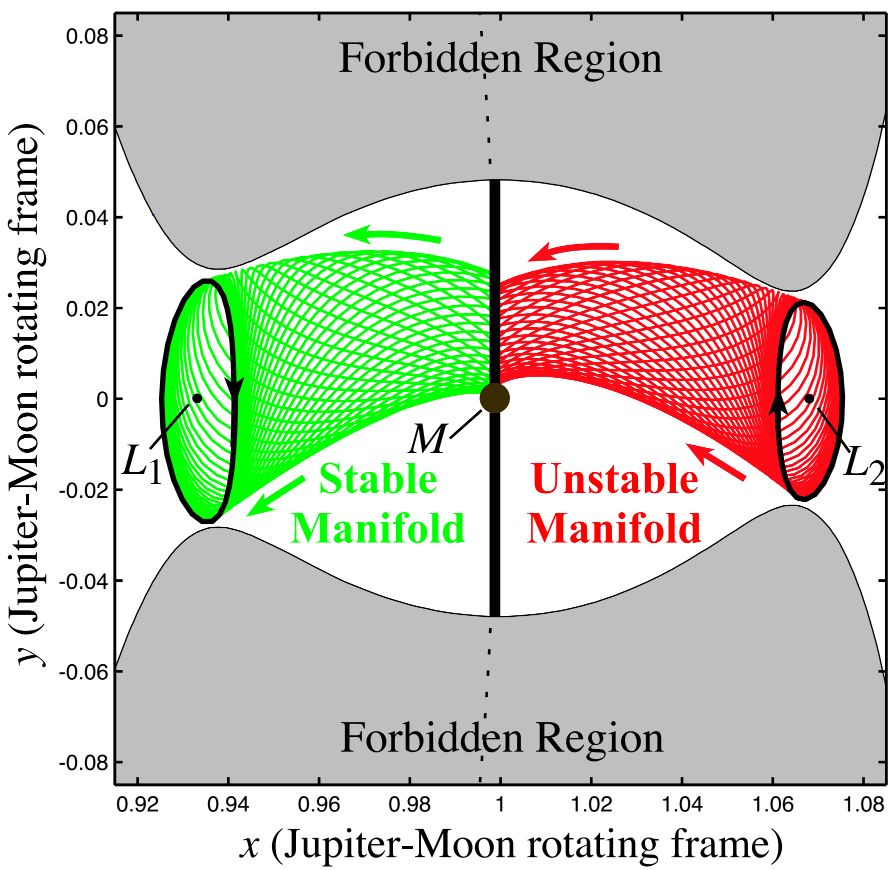
Numerical Construction of Orbits

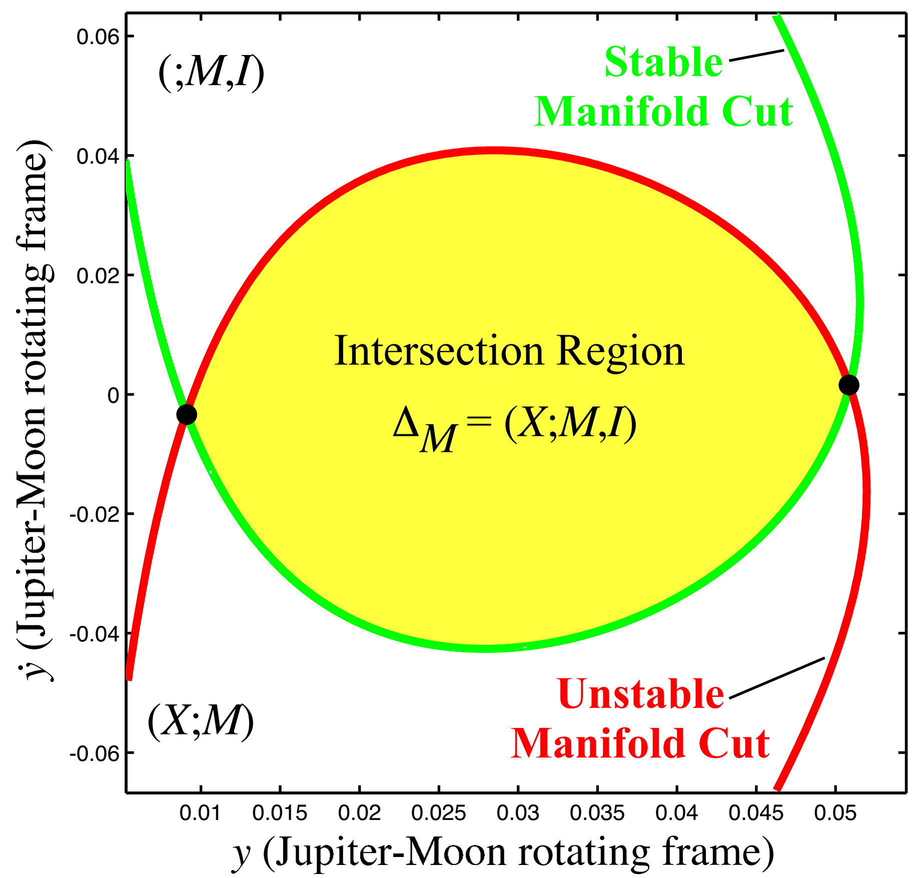
- Developed procedure to construct orbit with **prescribed itinerary**.
- Example: An orbit with itinerary $(\mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X})$.



Construction of (M, X; M, I, M) Orbits

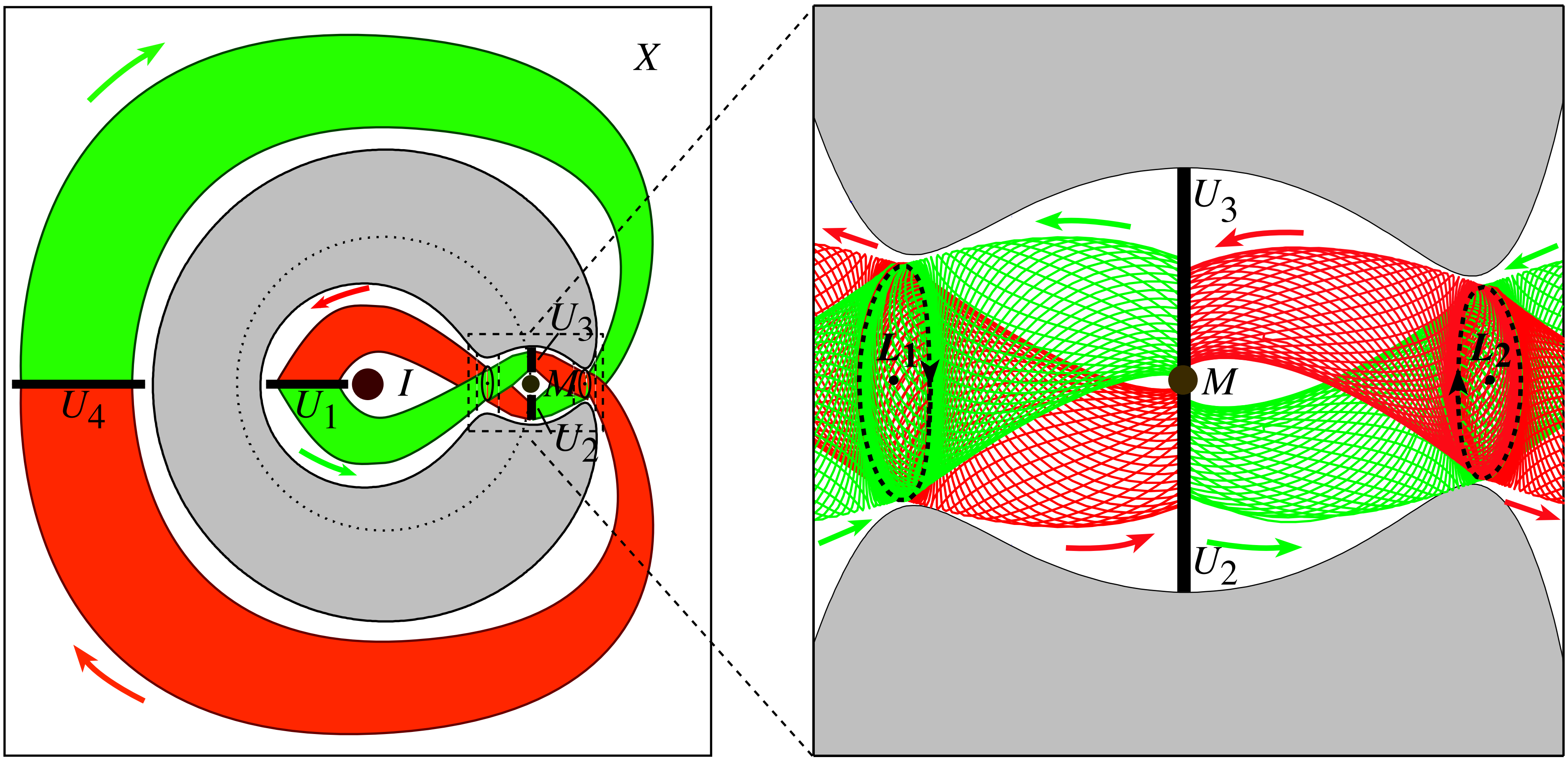
- Invariant mfd. **tubes** ($S \times I$) separate transit/nontransit orbits.
- **Red curve (S^1)** (Poincaré cut of L_2 **unstable manifold**).
  Green curve (S^1) (cut of L_1 **stable manifold**).
- Any point inside the intersection region Δ_M is a $(X; M, I)$ orbit.





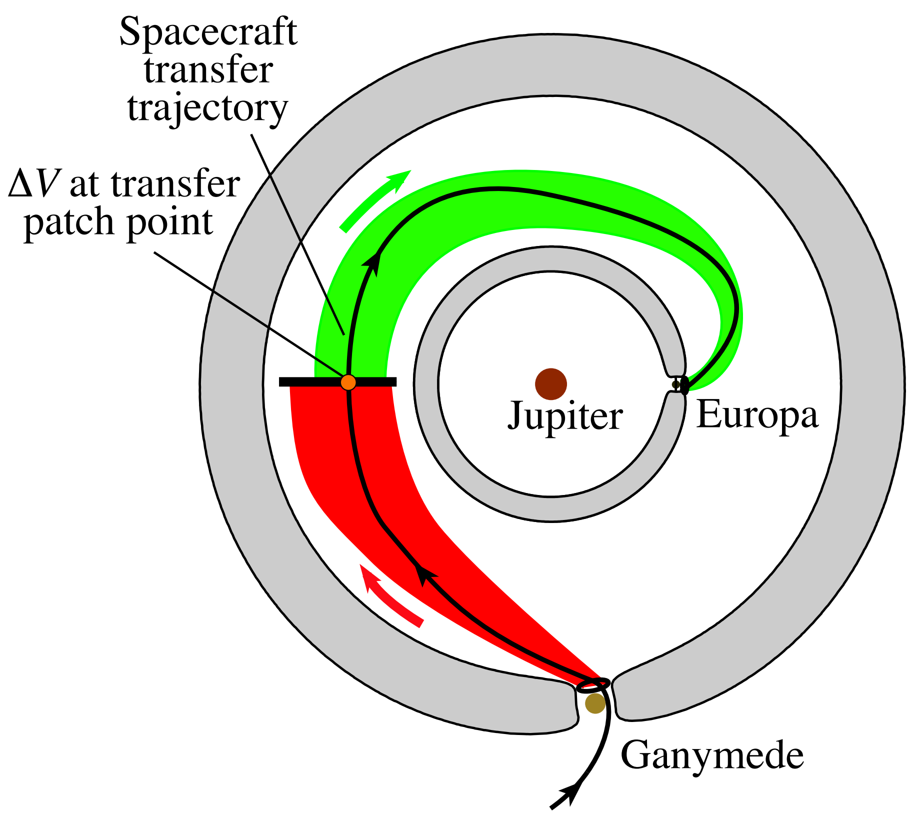
Construction of (M,X;M,I,M) Orbits

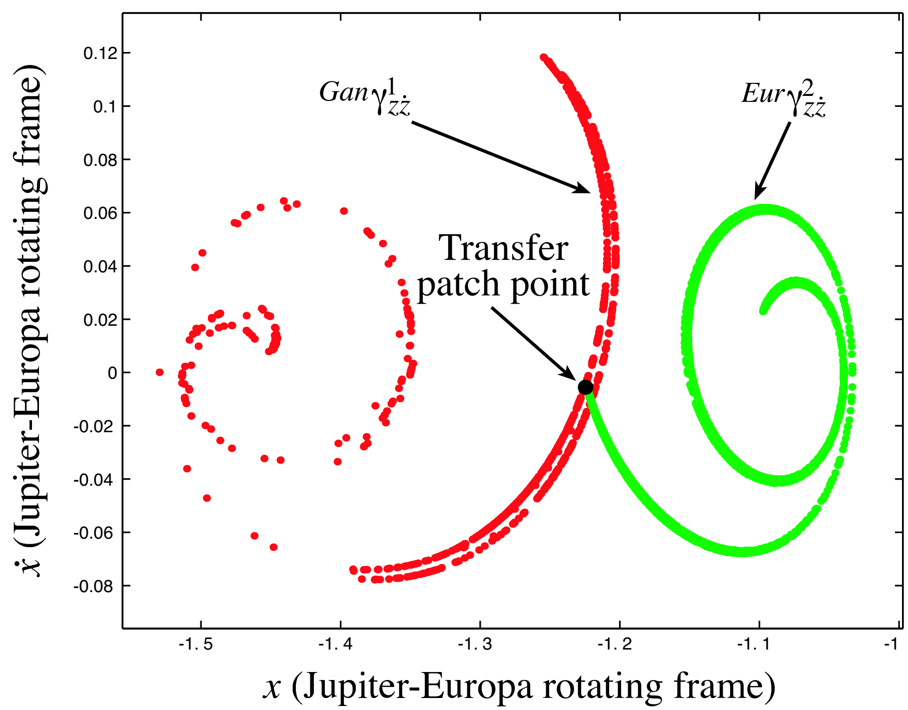
- The desired orbit can be constructed by
  - Choosing appropriate **Poincaré sections** and
  - linking invariant **manifold tubes** in right order.



Petit Grand Tour of Jupiter's Moon

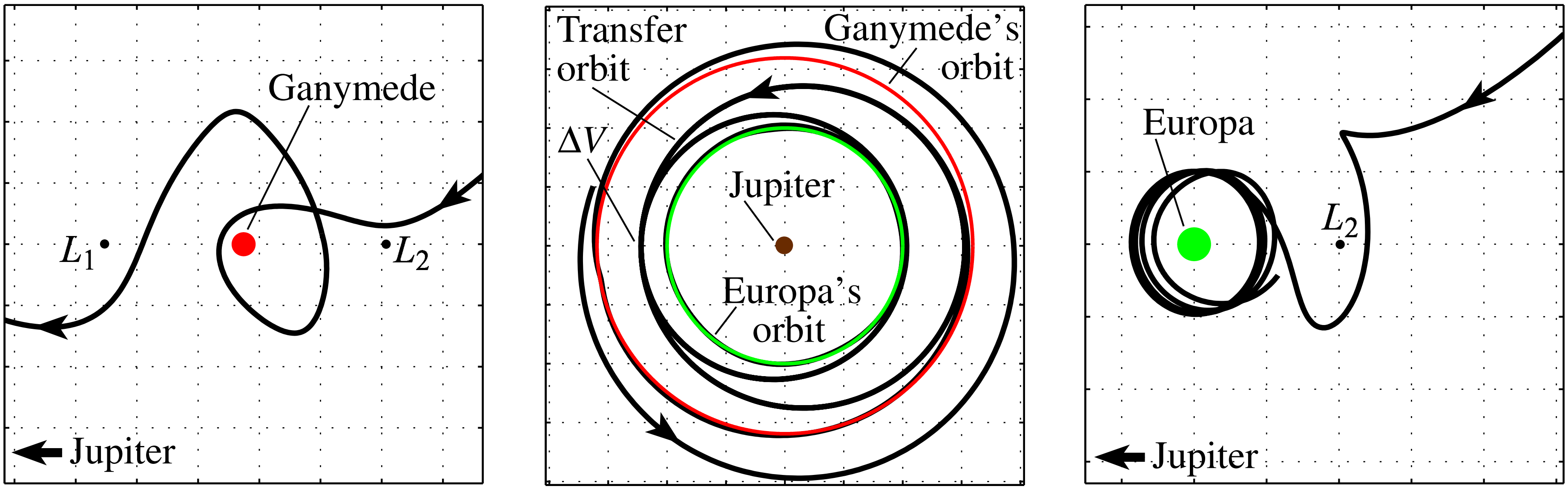
- Petit Grand Tour can be constructed similarly
  - Approximate 4-body system as 2 nested **3-body systems**.
  - Choose appropriate **Poinaré section**.
  - Link invariant **manifold tubes** in right order.
  - Refine initial solution in **4-body model**.



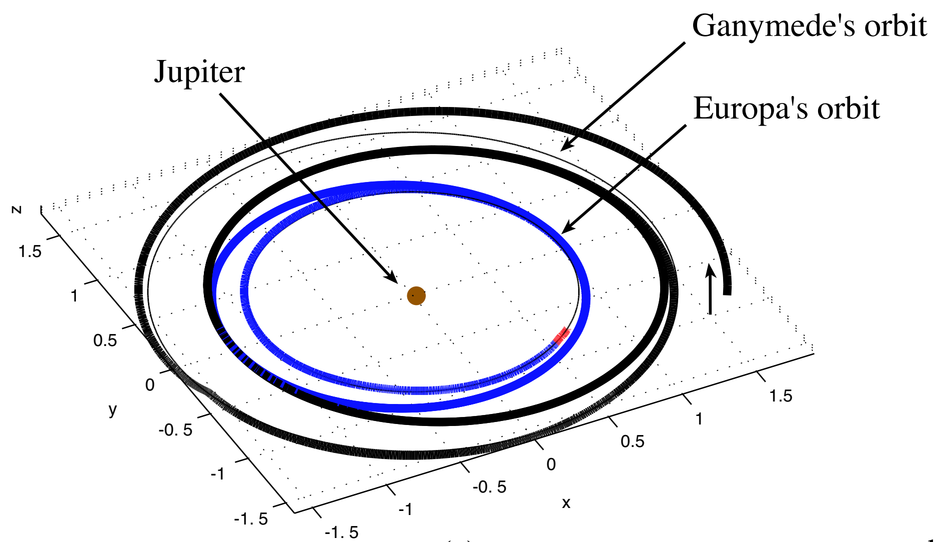


Petit Grand Tour of Jupiter's Moons (Planar Model)

- Used **invariant manifolds** to construct trajectories with interesting characteristics:
  - Petit Grand Tour of Jupiter's moons. 1 orbit around **Ganymede**. 4 orbits around **Europa**.
  - A ΔV nudges the SC from **Jupiter-Ganymede** system to **Jupiter-Europa** system.
- Instead of **flybys**, can orbit several moons for **any duration**.



Extend from Planar Model to Spatial Model



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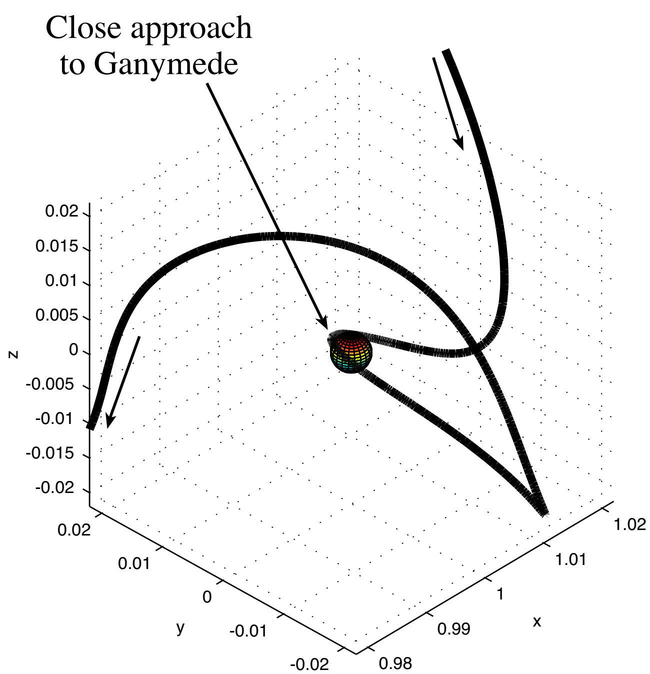

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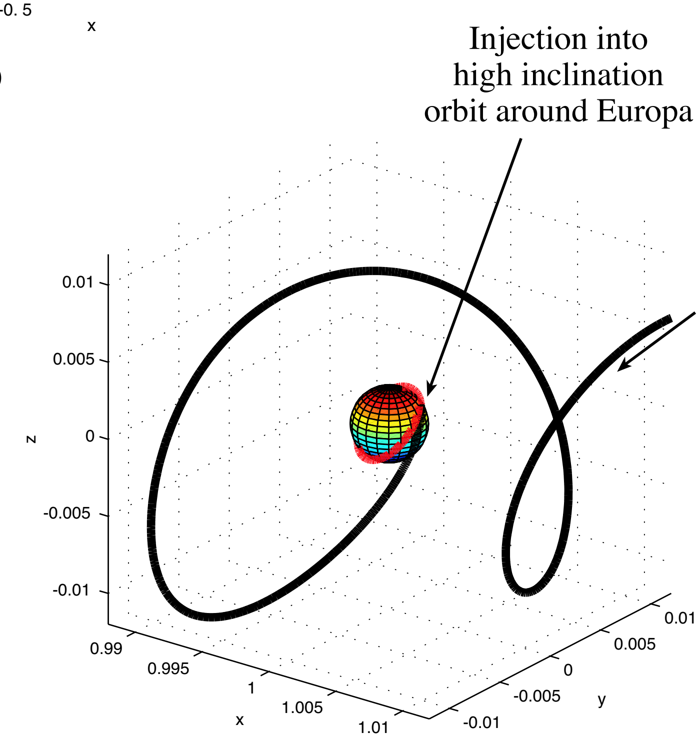
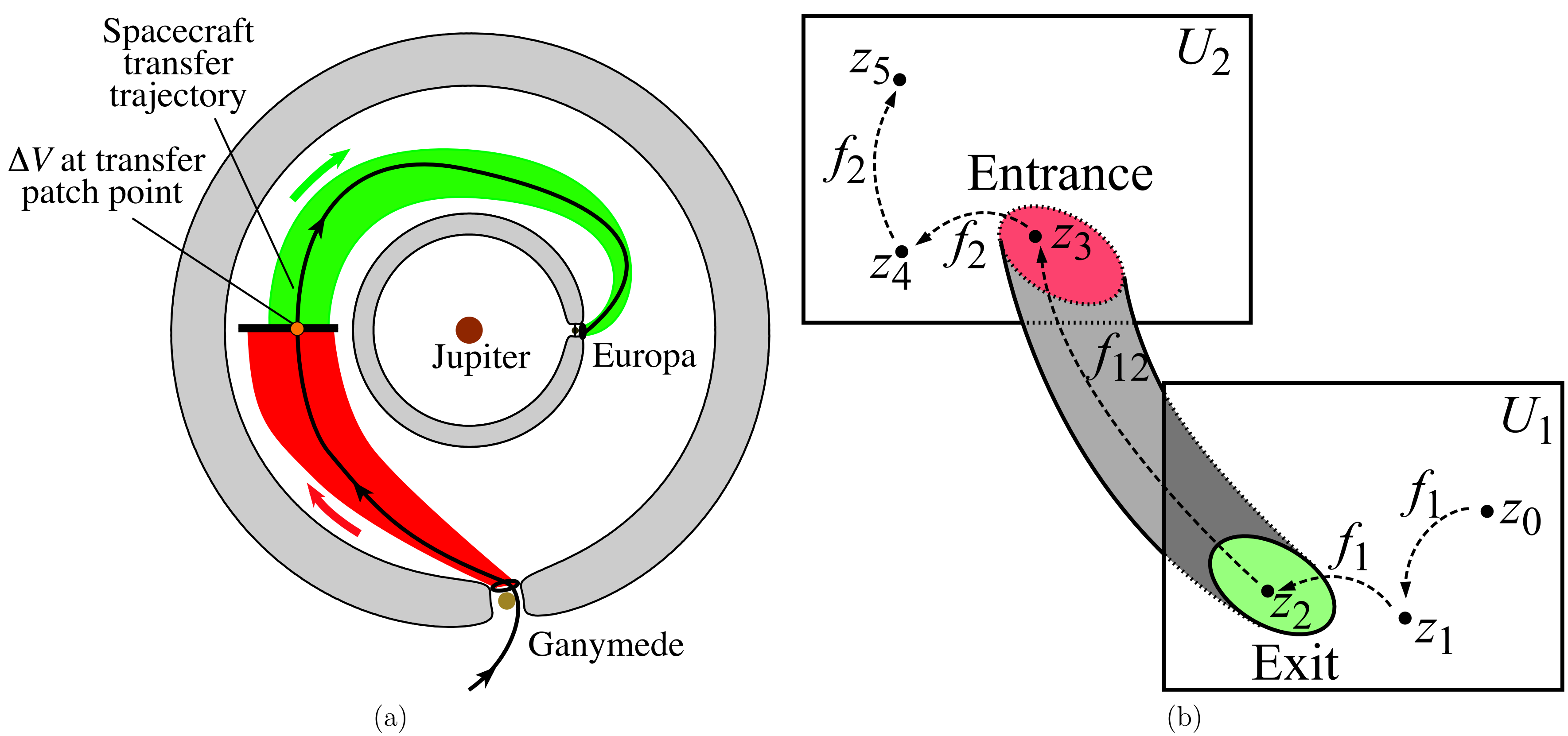


(c)

Look for Natural Pathways to Bridge the Gap

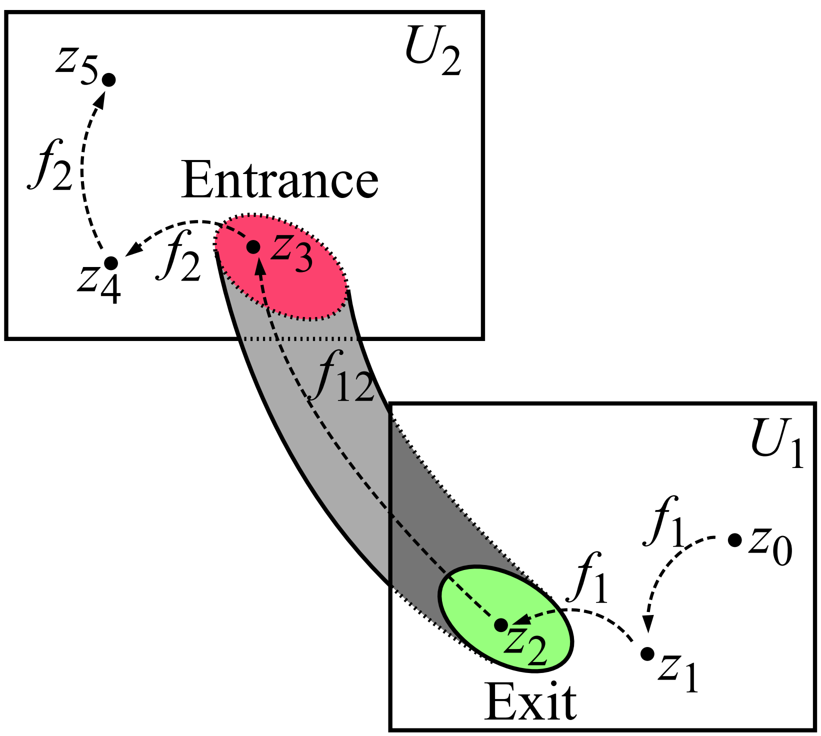
- **Tubes** of two 3-body systems **may not intersect** for awhile. May need large ΔV to "jump" from one tube to another.
- Look for **natural pathways** to bridge the gap
  - between z_0 where tube of one system (**Ganymede**) **enters** and z_2 where tube of another system **exits** (into Europa realm) by "hopping" through **phase space** (z_1).



Transport in Phase Space via Tube & Lobe Dynamics

- By using
  - **tubes** of rapid transition that connect realms
  - **lobe dynamics** to hop through phase space,

  New tour only needs $\Delta V = 20$m/s (50 times less).



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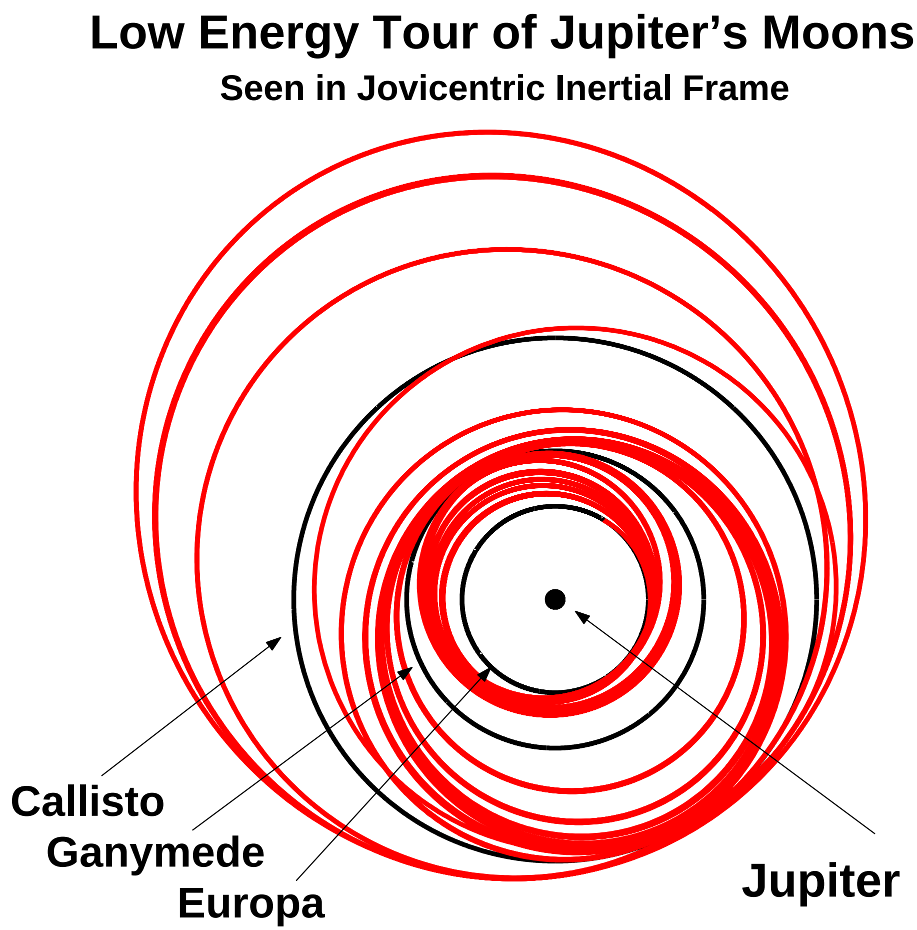
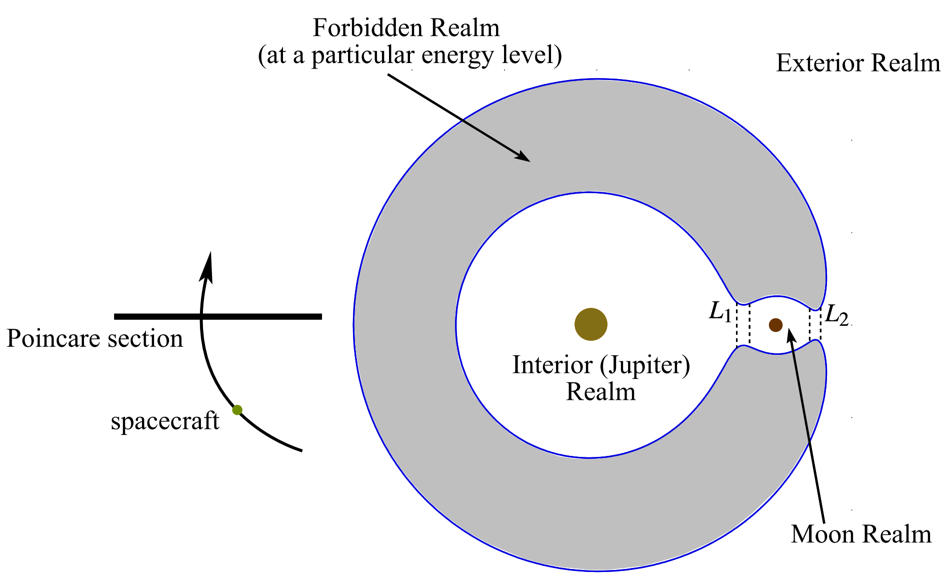


(b)

Lobe Dynamics: Mixed Phase Space

- Poincaré section reveals **mixed phase space**:
  - **resonance regions** and
  - "**chaotic sea**".



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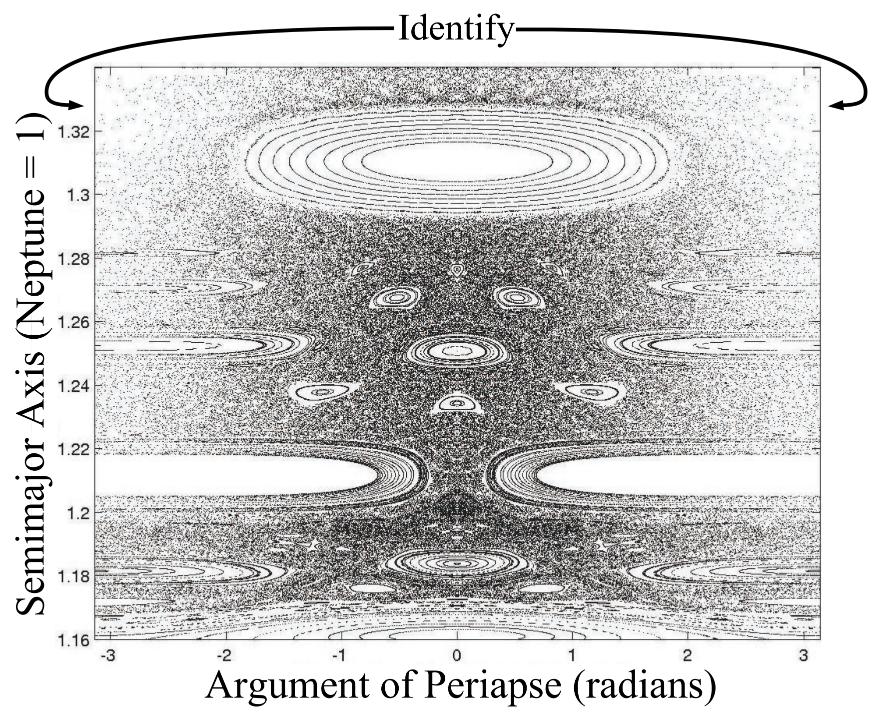
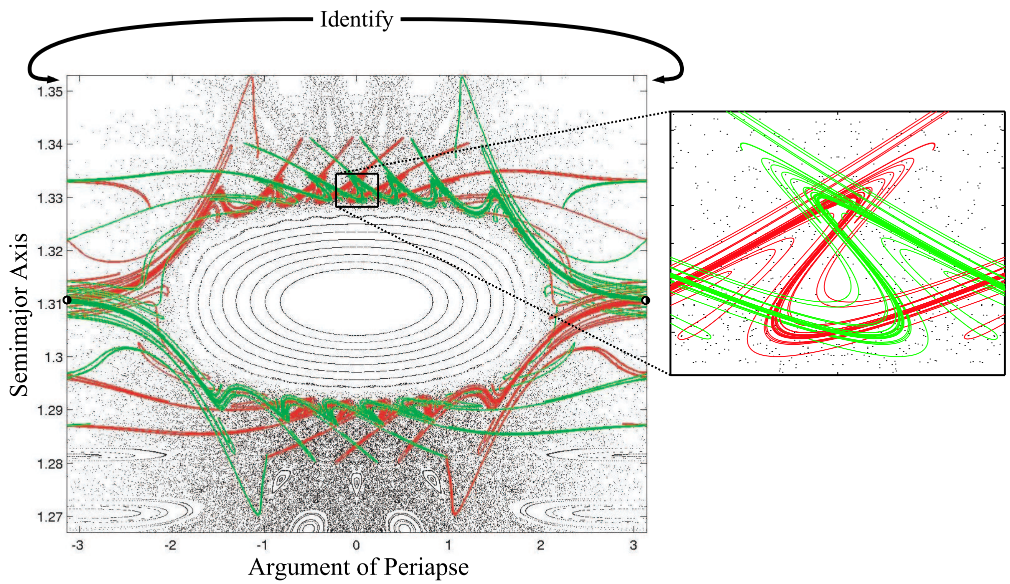
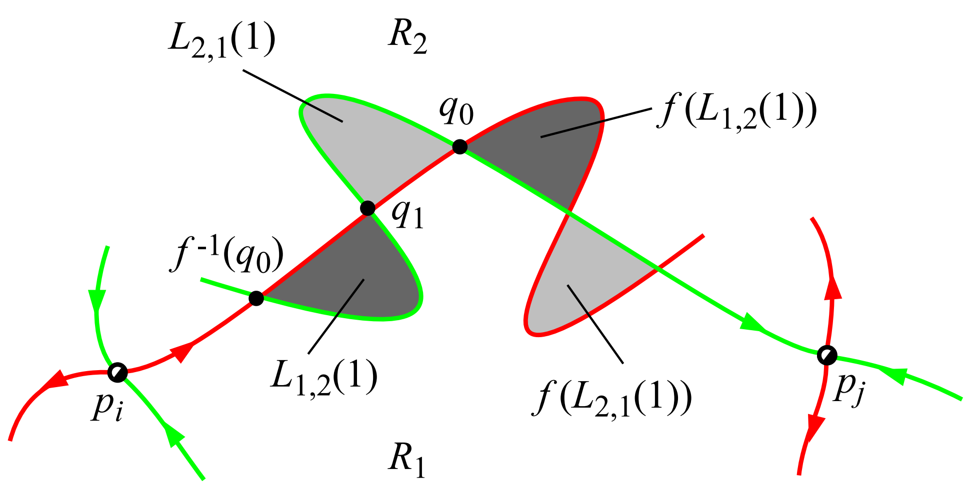


(b)

Transport between Regions via Lobe Dynamics

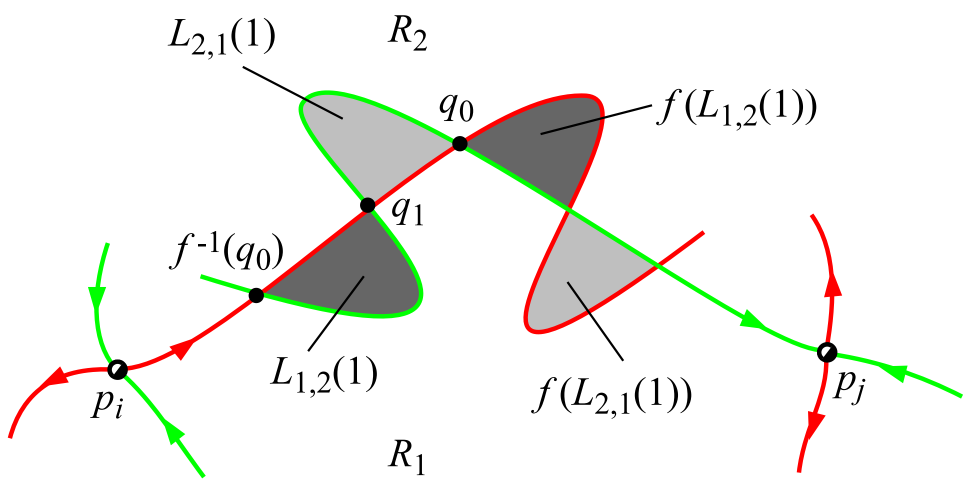
- **Invariant manifolds** divide phase space into resonance regions.
- Transport between regions can be studied via **lobe dynamics**.





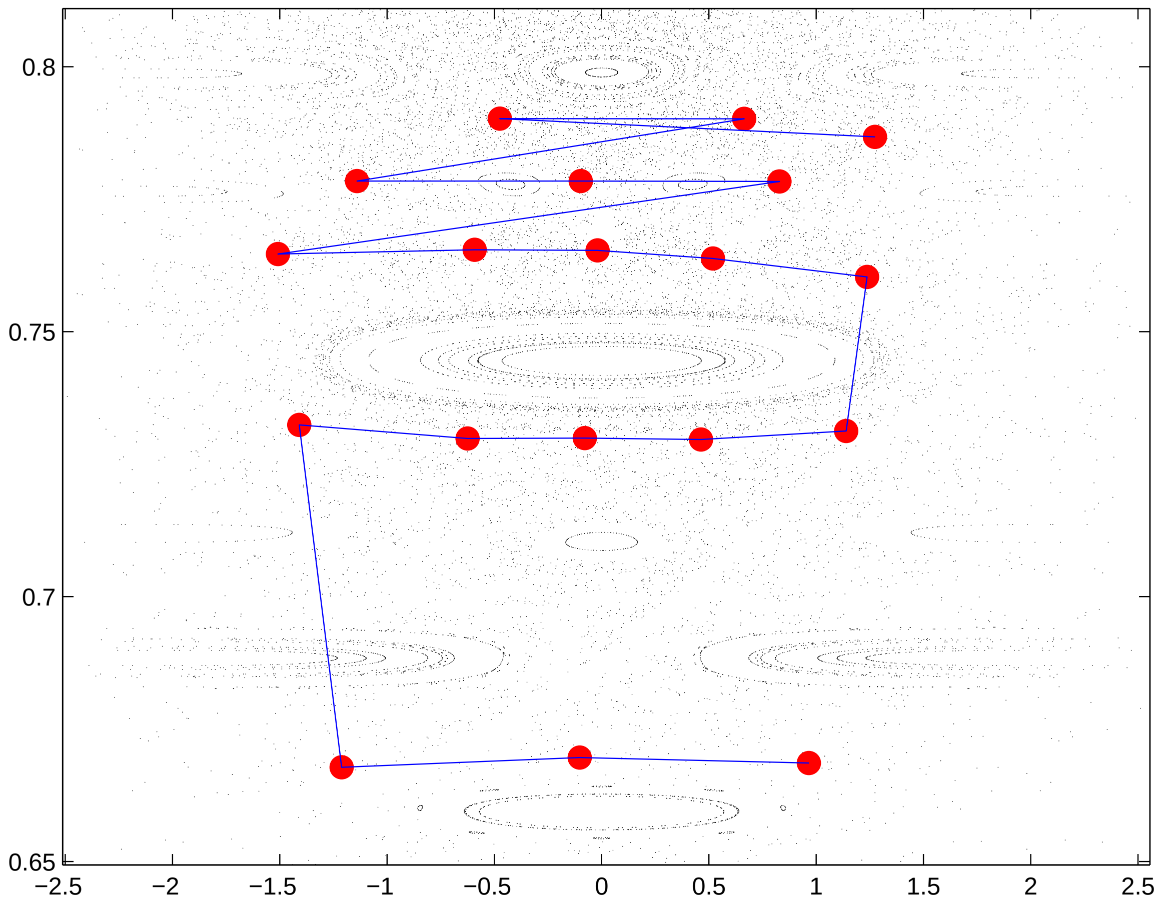
Transport between Regions via Lobe Dynamics

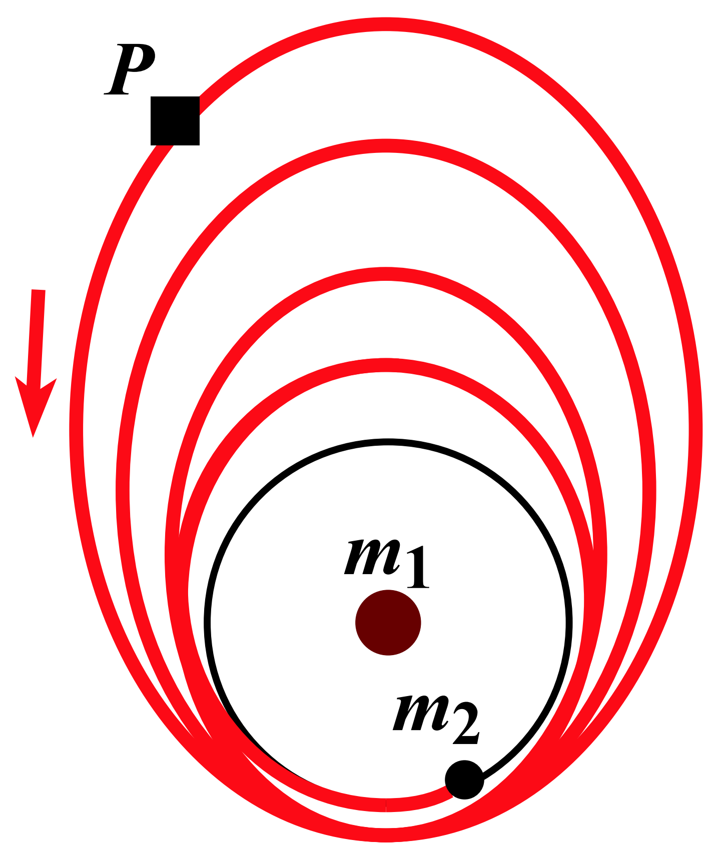
- Segments of **unstable** and **stable** manifolds form **partial barriers** between regions R_1 and R_2.
- $L_{1,2}(1), L_{2,1}(1)$ are **lobes**; they form a **turnstile**.
  - In one iteration, only points from R_1 to R_2 are in $L_{1,2}(1)$
  - only points from R_2 to R_1 are in $L_{2,1}(1)$.
- By studying pre-images of $L_{1,2}(1)$, one can find efficient way from R_1 to R_2.



Hopping through Resonaces in Low Energy Tour

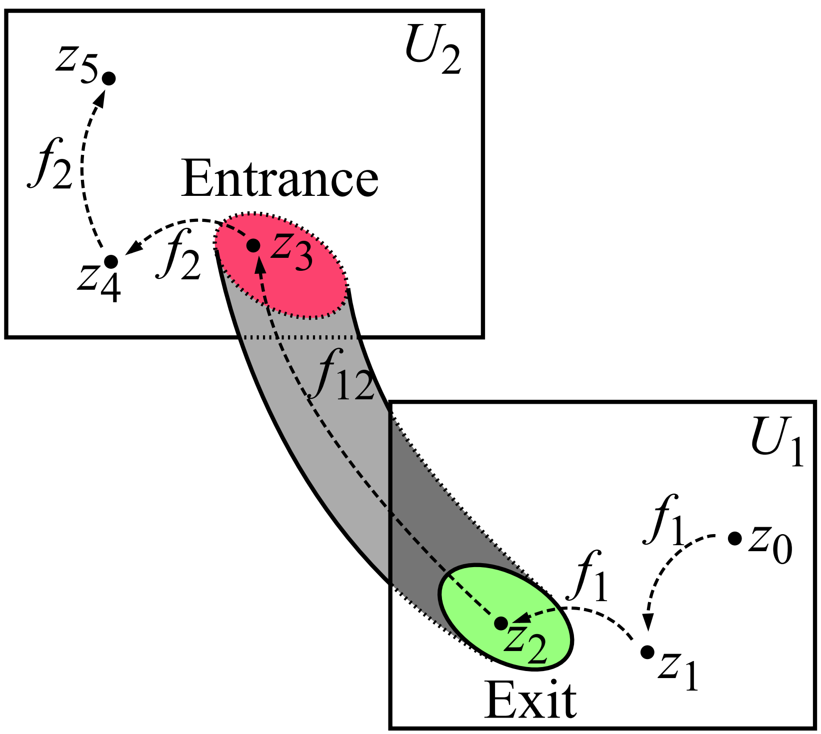
- Guided by lobe dynamics, **hopping** through resonances (essential for low energy tour) can be performed.
- To get SC captured by secondary (m_2), need to decrease **semi-major axis** passing through **resonances**.





Tube/Lobe Dynamics: Transport in Solar System

- To use **tube** dynamics/**lobe** dynamics of **spatial** 3-body problem to **systematically** design low-fuel trajectory.
- Part of our program to study transport in solar system using **tube** and **lobe dynamics**.



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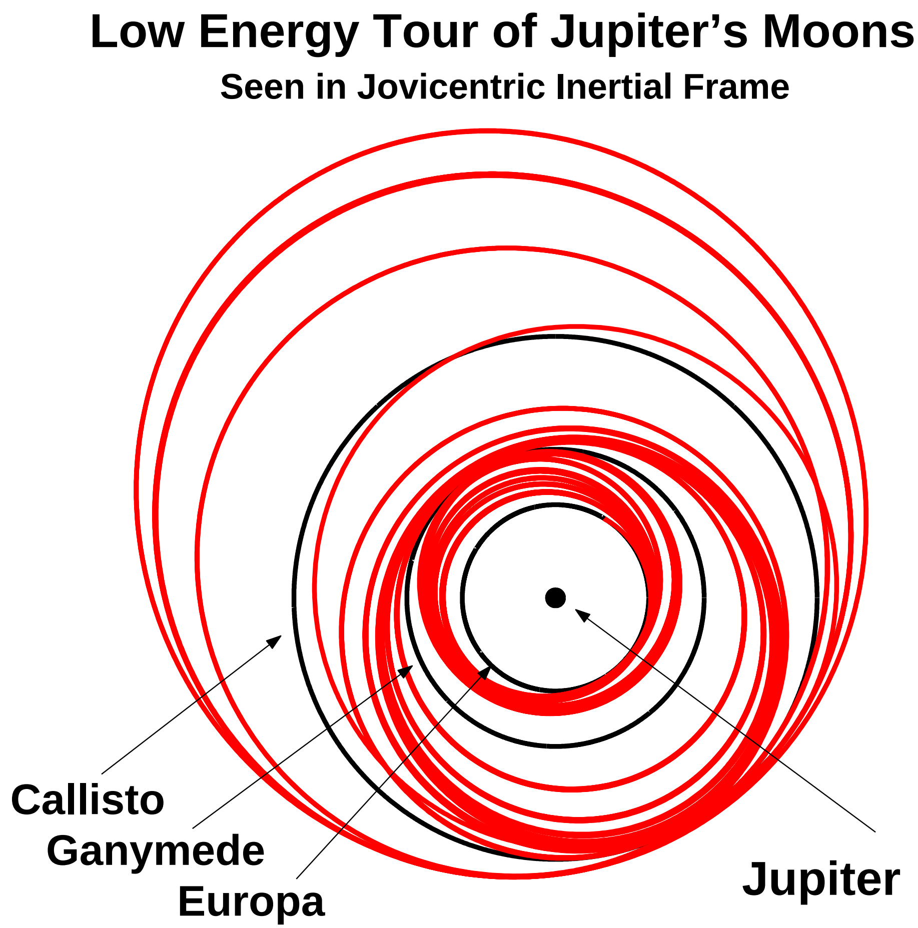

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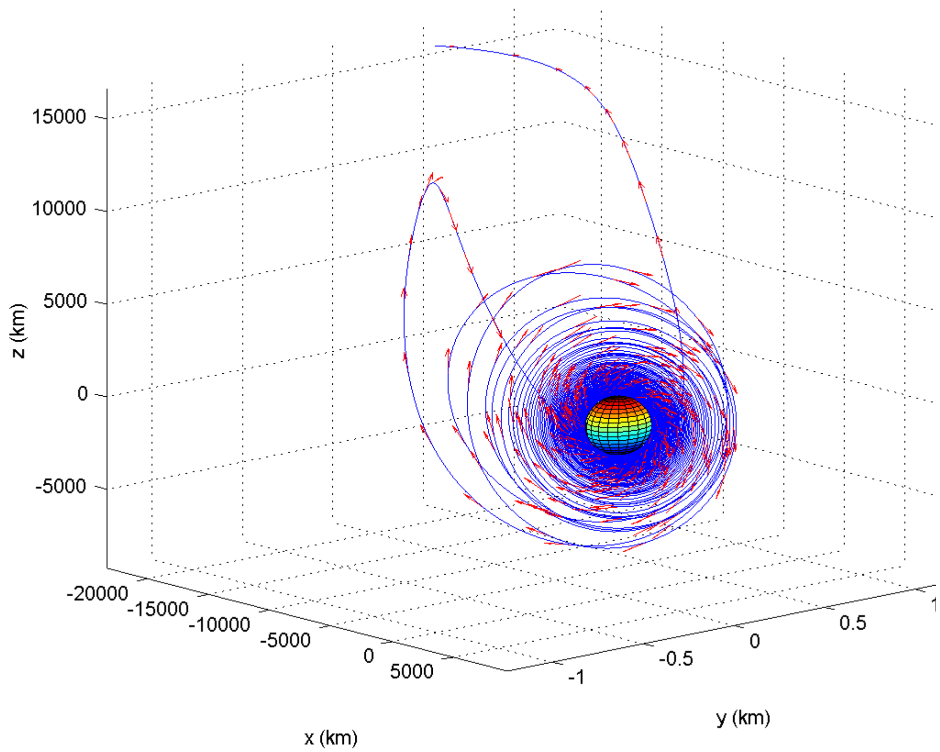
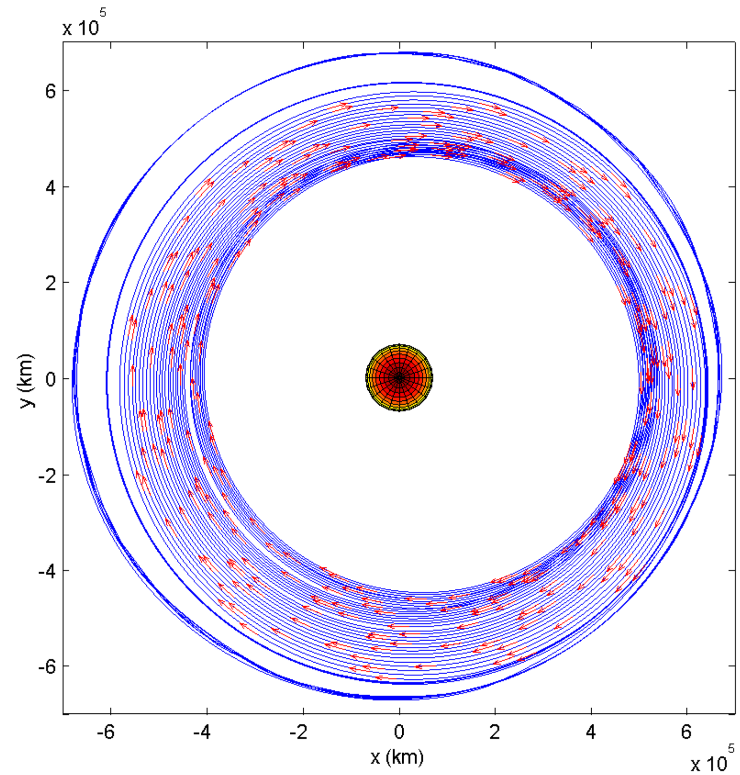
Low Thrust Trajectories in a Multi-Body Environment

- Incorporation of **low thrust**.
- Design to take best advantage of **natural dynamics**.
- See **Shane D. Ross**.



Spiral out from Europa



Europa to Io transfer

Parking a Satellite near an Asteroid Pair

- Find **stable periodic/quasi-periodic orbits** for **SC** to observe **binary** as it orbits the Sun.
  - Model for asteroid pair: **sphere** and **rigid body** (3 connected masses).
  - Model for SC motion: binary in **relative equilibrium**.
- See **Gabern, Koon, and Marsden [2004]**.