Invariant Manifolds, 3-Body Problem & Petit Grand Tour of Jovian Moons

Gómez, Koon, Lo, Marsden, Masemont and Ross

Wang Sang Koon
Control and Dynamical Systems, Caltech

koon@cds.caltech.edu

Acknowledgements

- **H. Poincaré, J. Moser**
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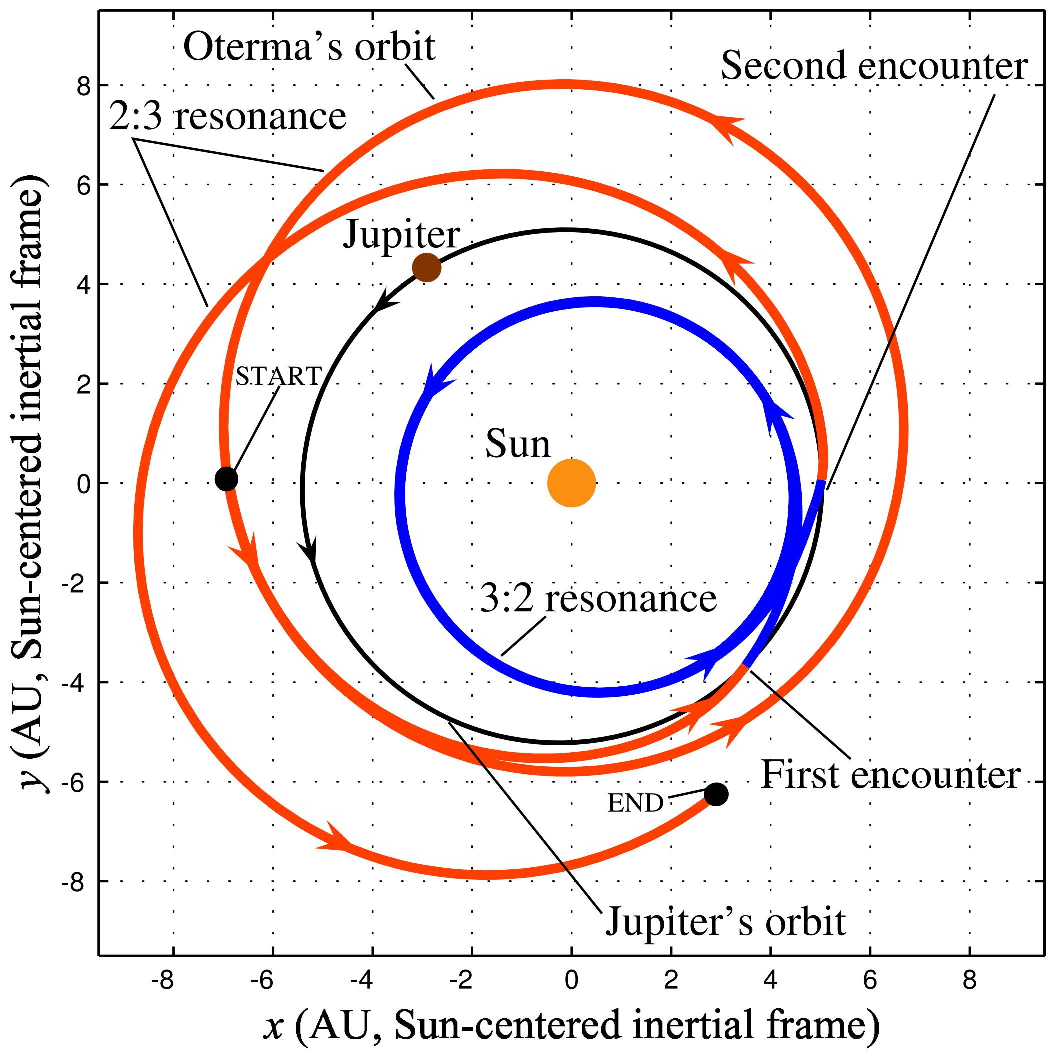
- **E. Belbruno, B. Marsden, J. Miller**
- **K. Howell, B. Barden, R. Wilson**

Outline of Presentation

- **Main Theme**
  - how to use invariant manifold of 3-body problem in space mission design.
- **Background** and **Motivation**
- **Major Results** and **Some Technical Details**.
- **Low Energy Transfer between Jovian Moons.**
- **Conclusion and Ongoing Work**.

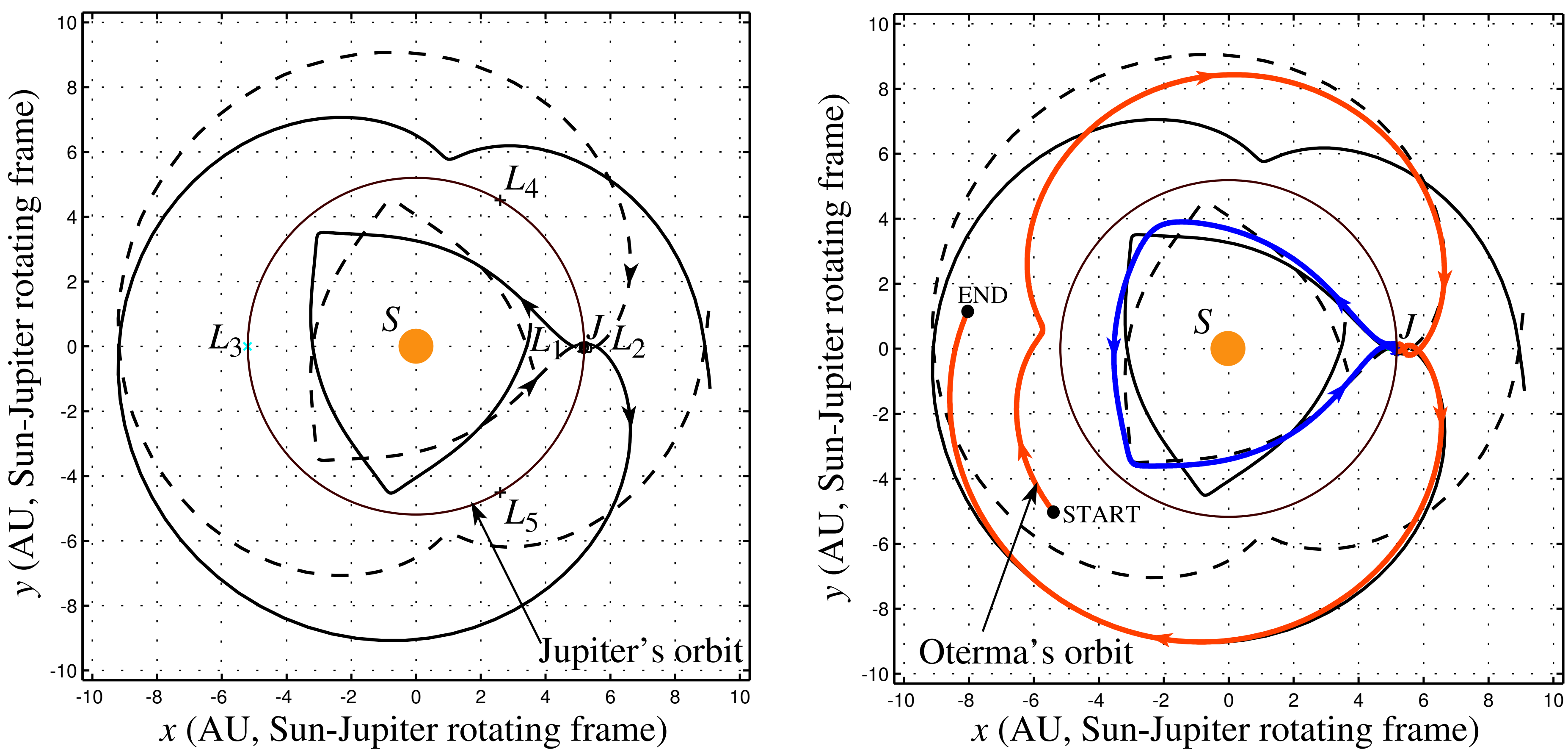
Jupiter Comets

- **Rapid transition** from **outside** to **inside** Jupiter's orbit.
- **Captured temporarily** by Jupiter during transition.
- **Exterior** (**2:3 resonance**). **Interior** (**3:2 resonance**).



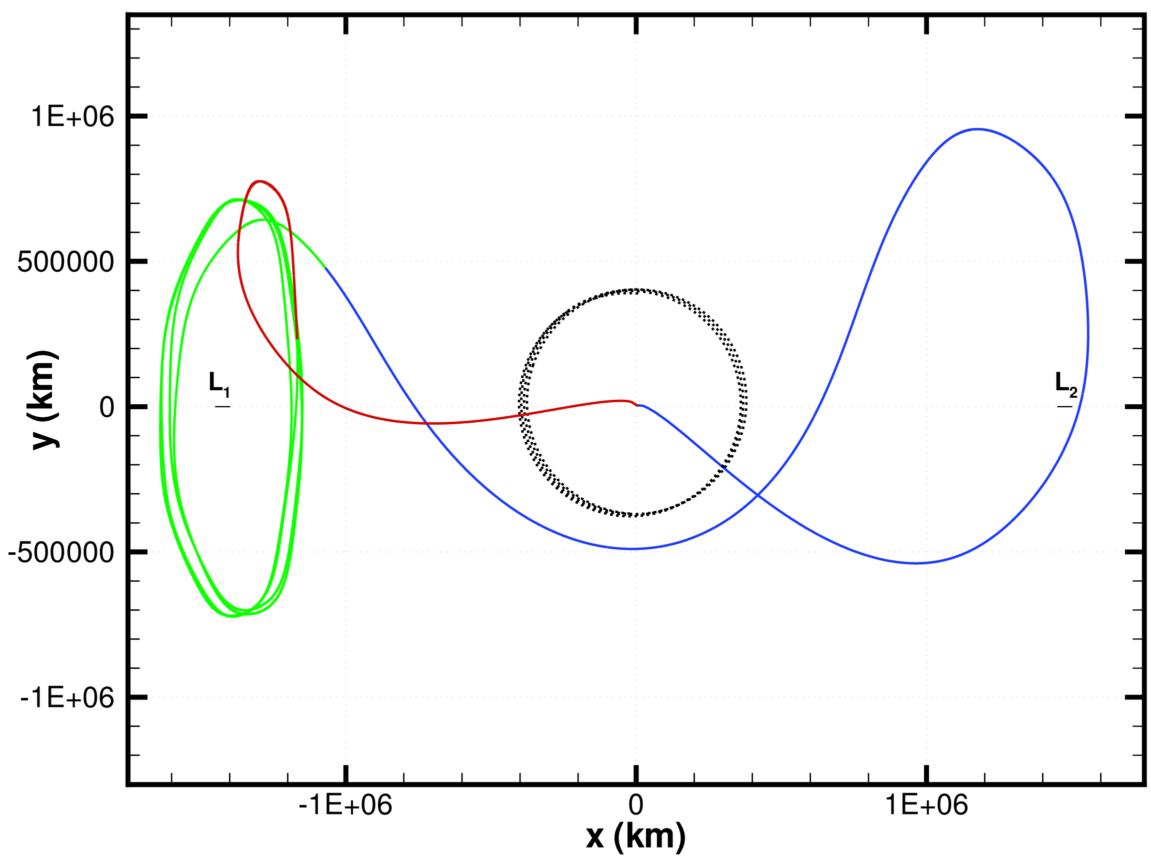
Jupiter Comets

- Belbruno and B. Marsden [1997]
- Lo and Ross [1997]
  - Comet in **rotating frame** follows **invariant manifolds**.
- Moser, Conley and McGehee. LLibre, Martinez and Simó [1985].



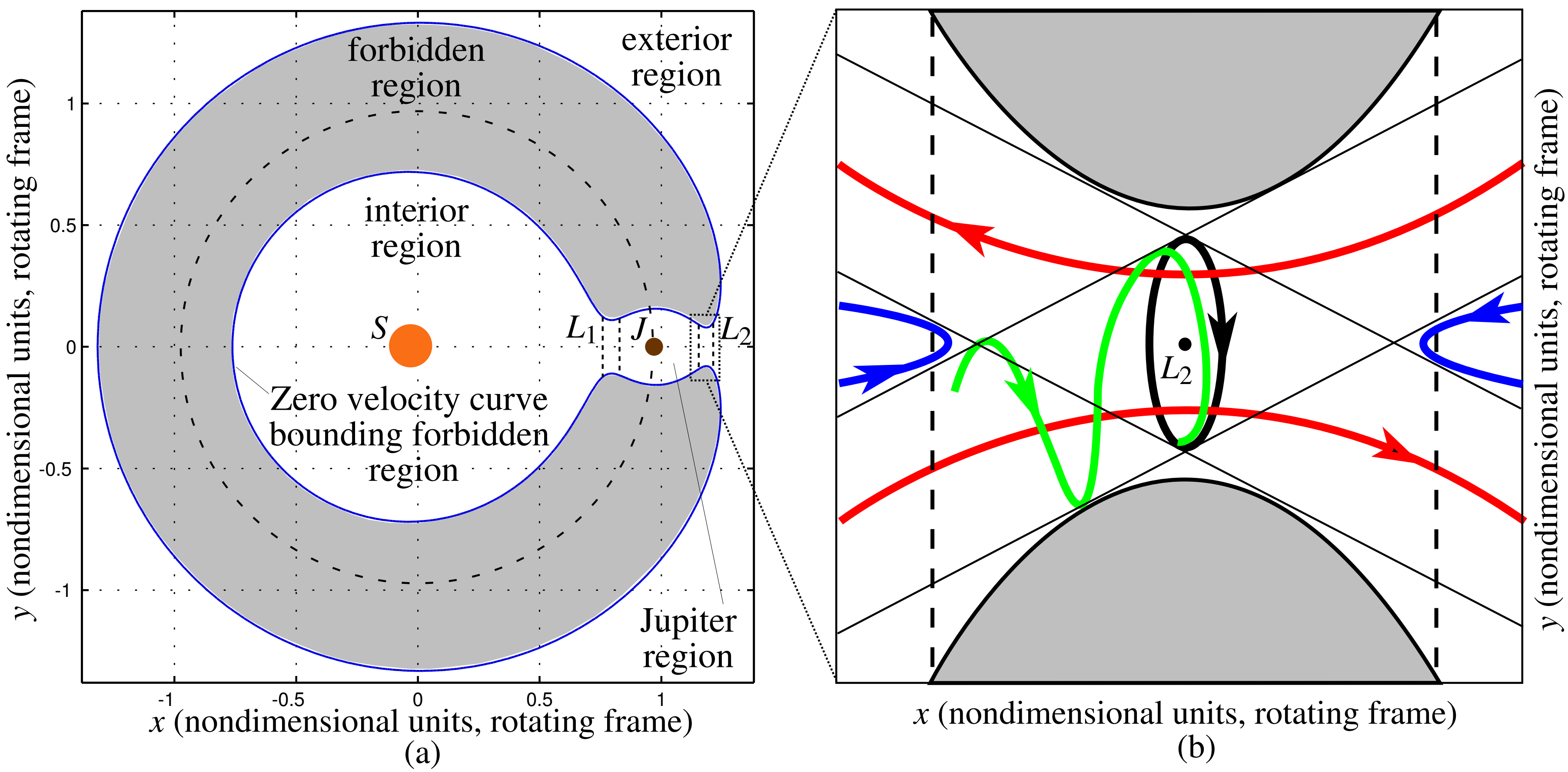
Genesis Discovery Mission

- To understand better **heteroclinic dynamics** used by **Genesis Mission** in returning solar wind sample to Earth.
  - Howell, Lo, Barden and Wilson.
- **Halo orbit**, **transfer**/ **return** trajectories in rotating frame.



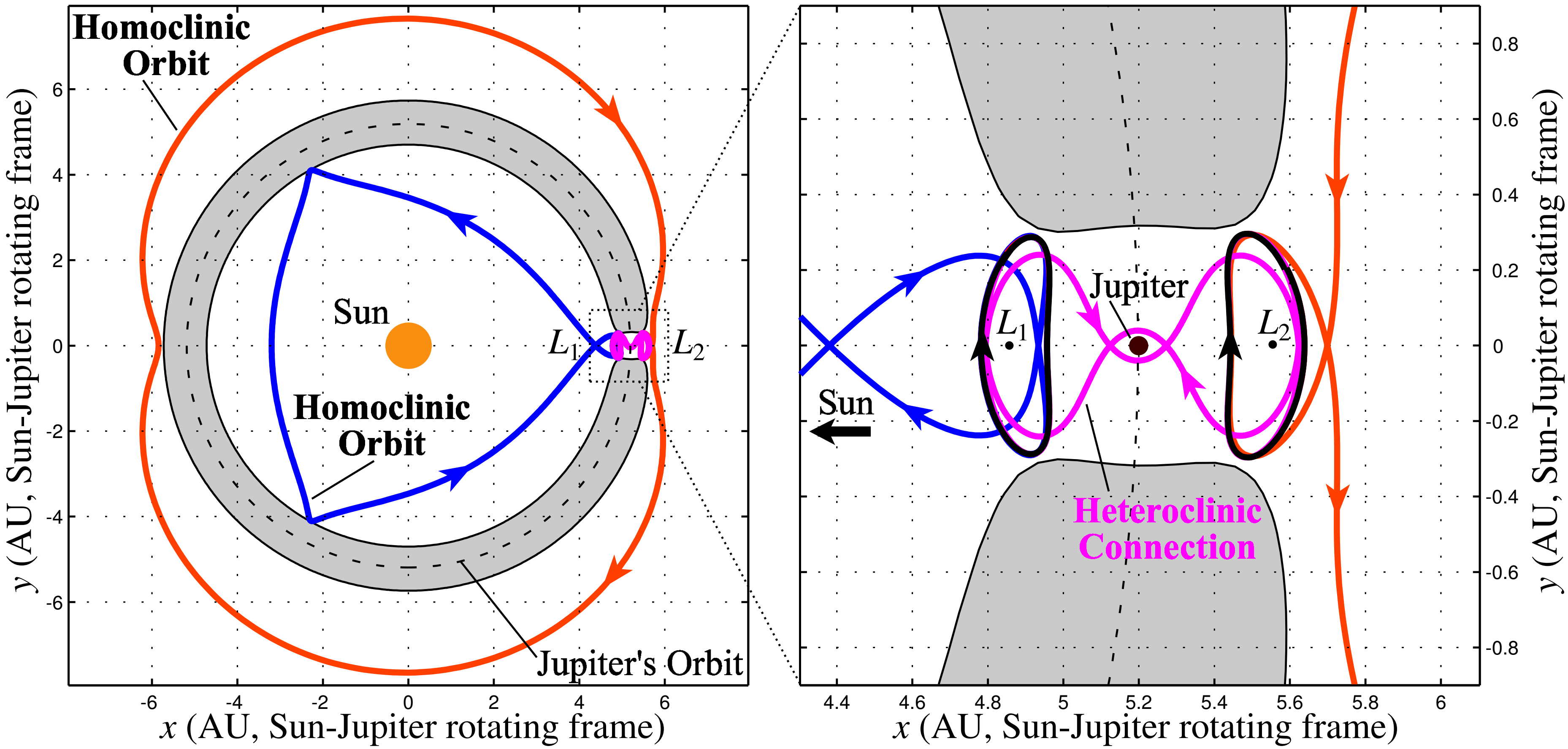
The Flow near L_1 and L_2

- For **energy value** just above that of L_2, **Hill's region** contains a **"neck"** about L_1 & L_2.
- Comet can make **transition** through these equilibrium regions.
- 4 types of orbits: **periodic**, **asymptotic**, **transit** & **nontransit**.



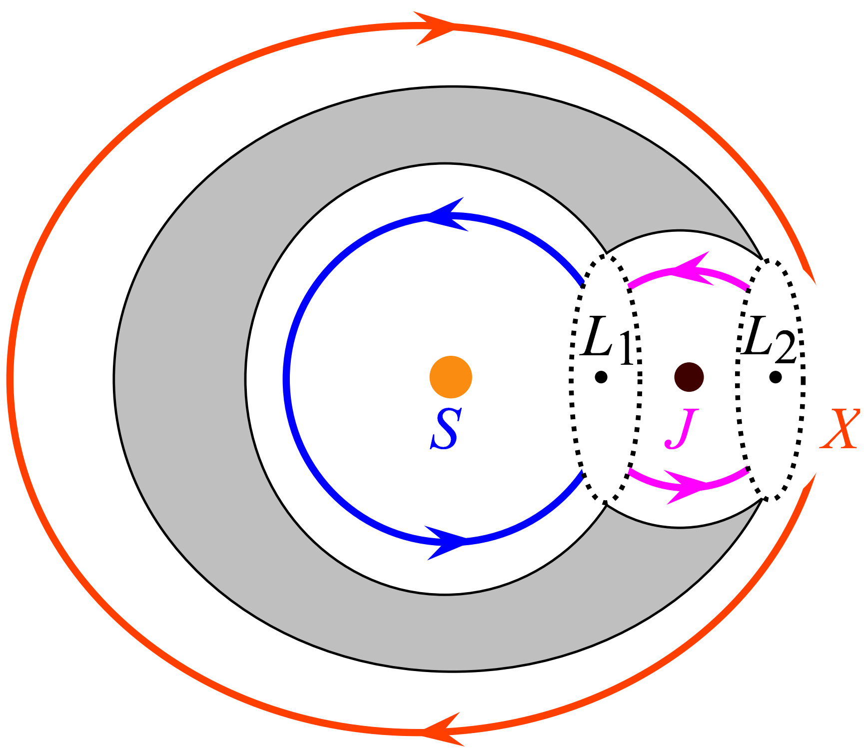
Major Result (A): Heteroclinic Connection

- Found **heteroclinic connection** between pair of periodic orbits.
- Found a large class of **orbits** near this (homo/heteroclinic) ***chain***.
- Comet can follow these ***channels*** in rapid transition.



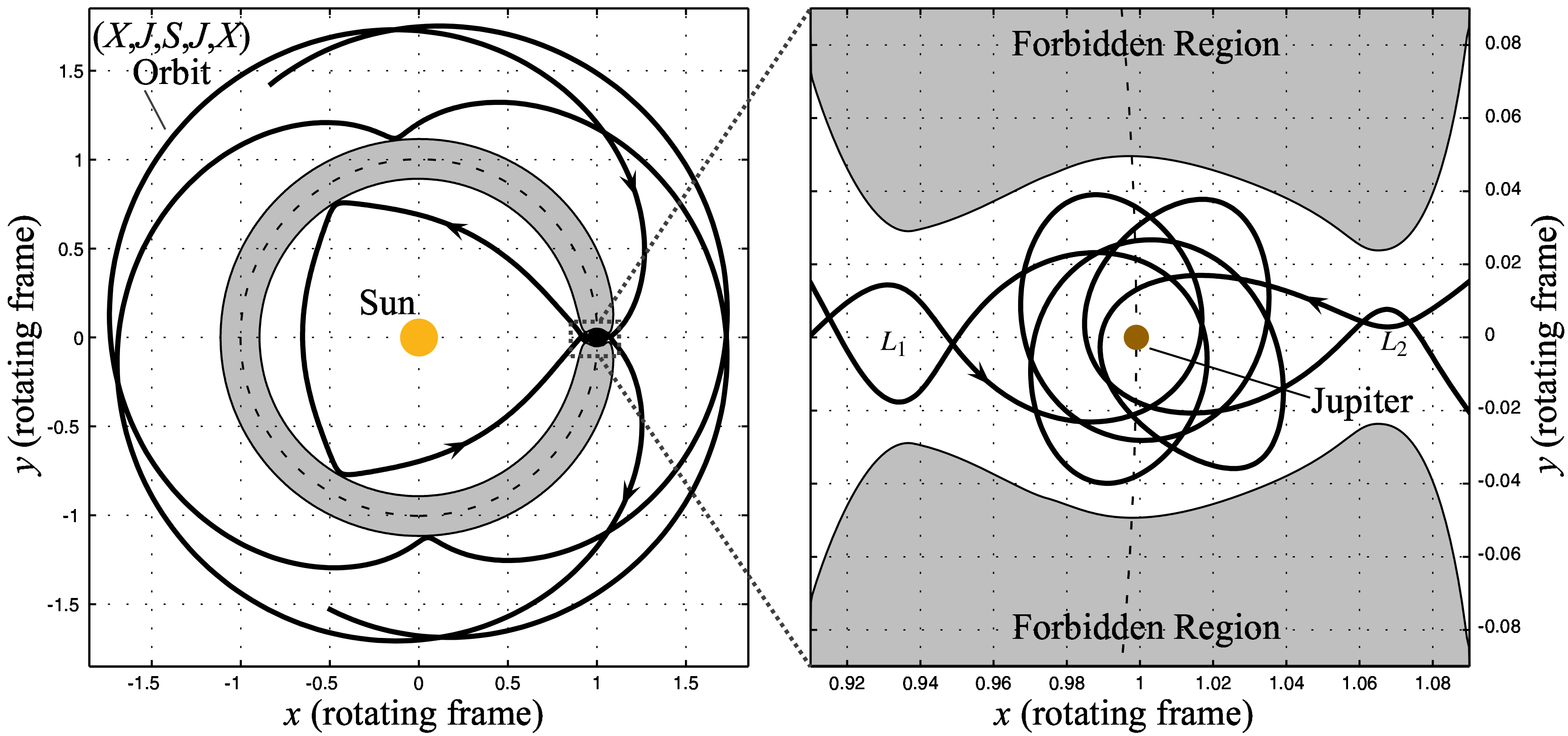
Major Result (B): Existence of Transitional Orbits

- **Symbolic sequence** used to label itinerary of each comet orbit.
- ***Main Theorem***: For any admissible **itinerary**, e.g., $(\ldots, \mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X}, \ldots)$, there exists an orbit whose **whereabouts** matches this **itinerary**.
- Can even specify **number of revolutions** the comet makes around Sun & Jupiter (plus L_1 & L_2).



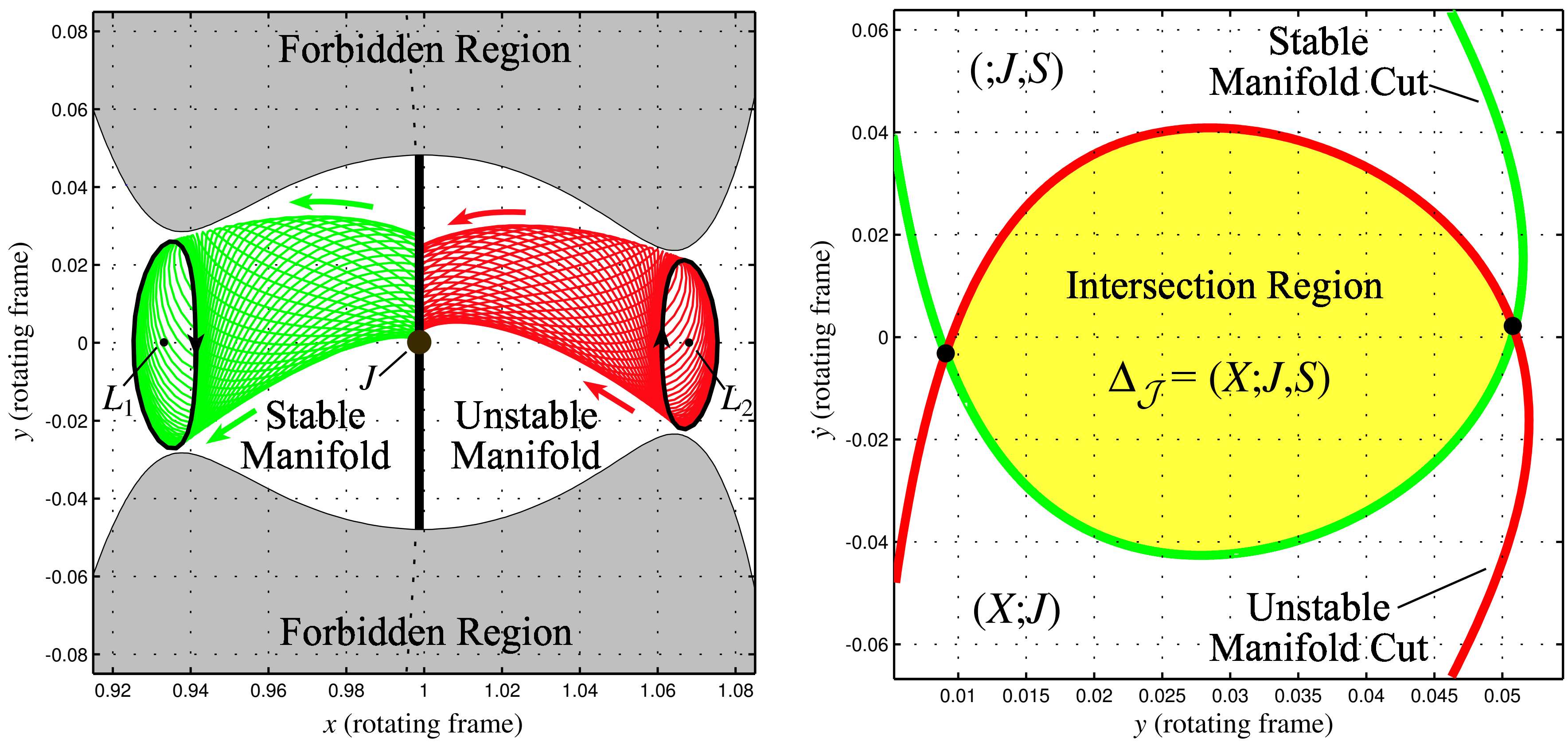
Major Result (C): Numerical Construction of Orbits

- Developed procedure to construct orbit with **prescribed itinerary**.
- Example: An orbit with itinerary $(\mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X})$.



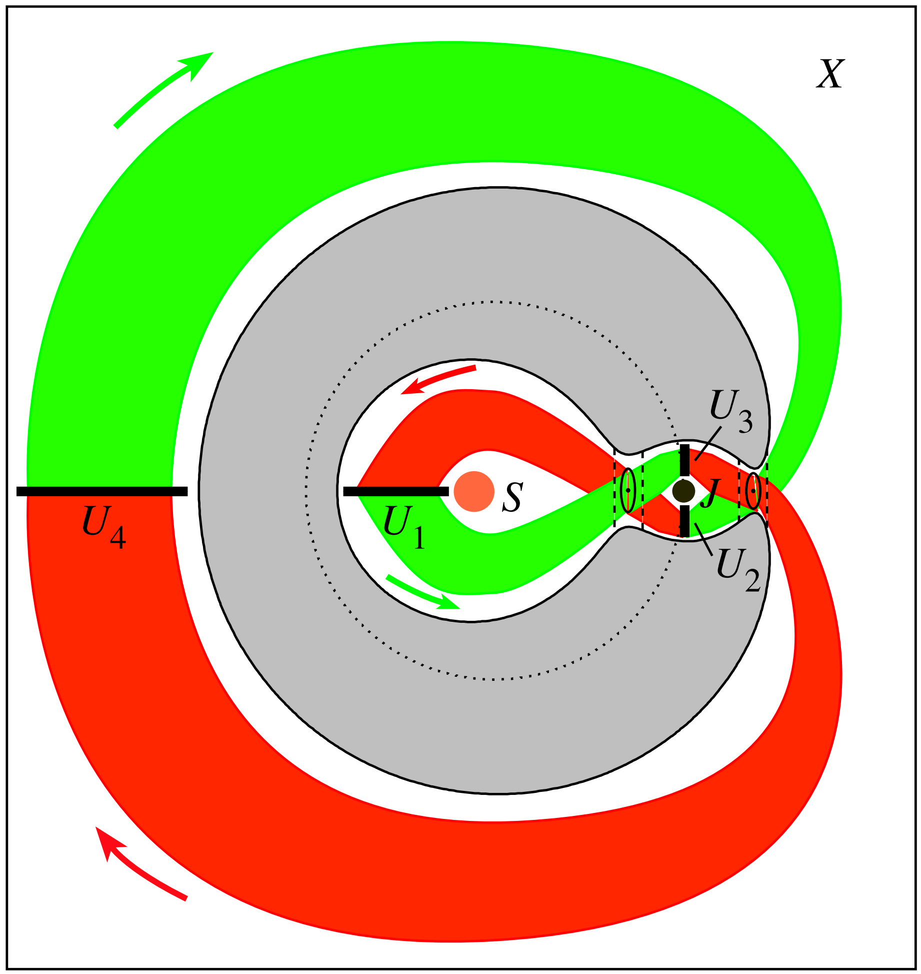
Details: Construction of $(\mathbf{J}, \mathbf{X}; \mathbf{J}, \mathbf{S}, \mathbf{J})$ Orbits

- Invariant manifold **tubes** separate transit from nontransit orbits.
- **Green curve** (Poincaré cut of L_1 **stable manifold**).
  Red curve (cut of L_2 **unstable manifold**).
- Any point inside the intersection region Δ_J is a $(\mathbf{X}; \mathbf{J}, \mathbf{S})$ orbit.



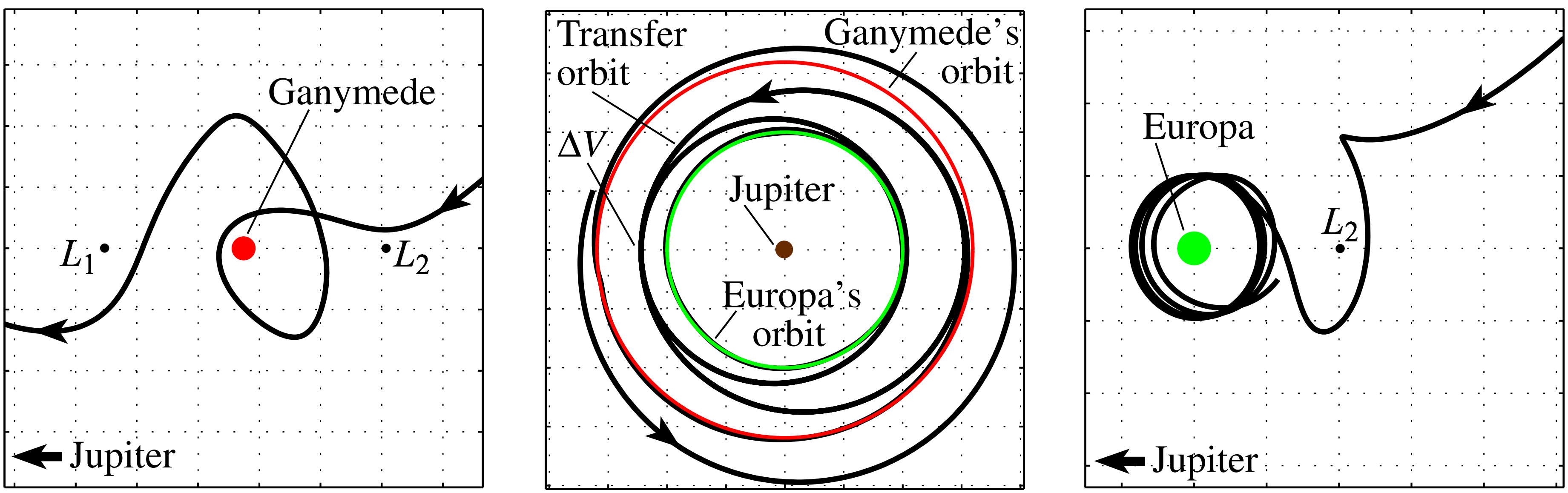
Details: Construction of $(\mathbf{J}, \mathbf{X}; \mathbf{J}, \mathbf{S}, \mathbf{J})$ Orbits

- The desired orbit can be constructed by
  - Choosing appropriate **Poincaré sections** and
  - linking invariant **manifold tubes** in right order.



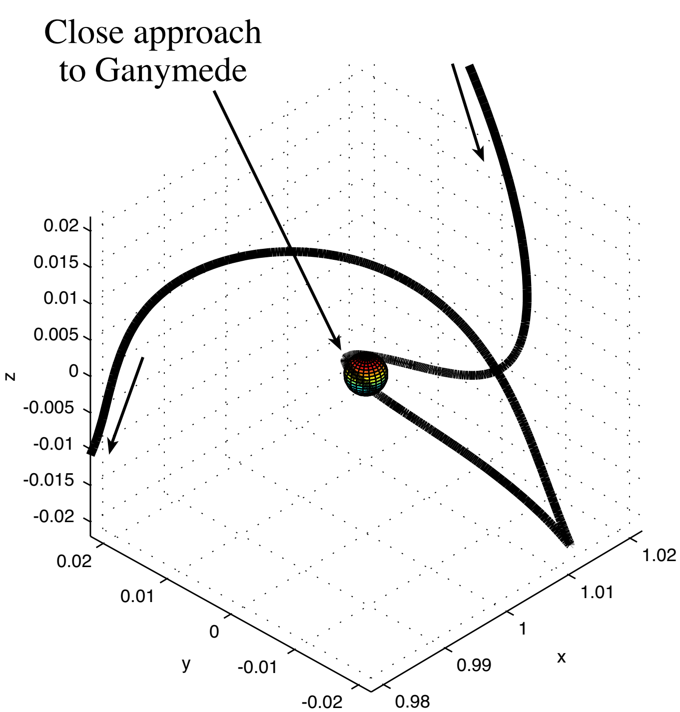
Petit Grand Tour of Jupiter's Moons (Planar Model)

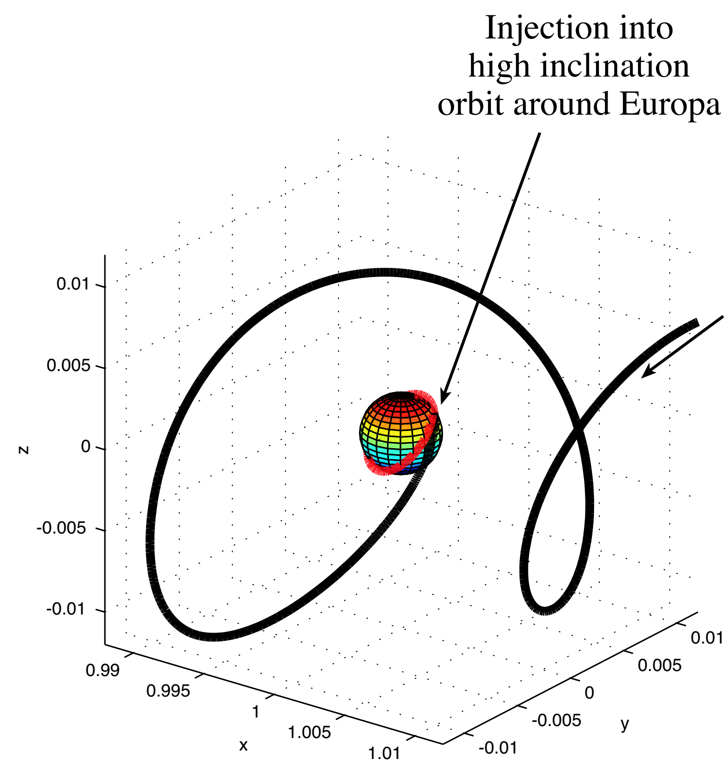
- Used **invariant manifolds** to construct trajectories with interesting characteristics:
  - Petit Grand Tour of Jupiter's moons. 1 orbit around **Ganymede**. 4 orbits around **Europa**.
  - A ΔV nudges the SC from **Jupiter-Ganymede** system to **Jupiter-Europa** system.
- Instead of **flybys**, can orbit several moons for **any duration**.



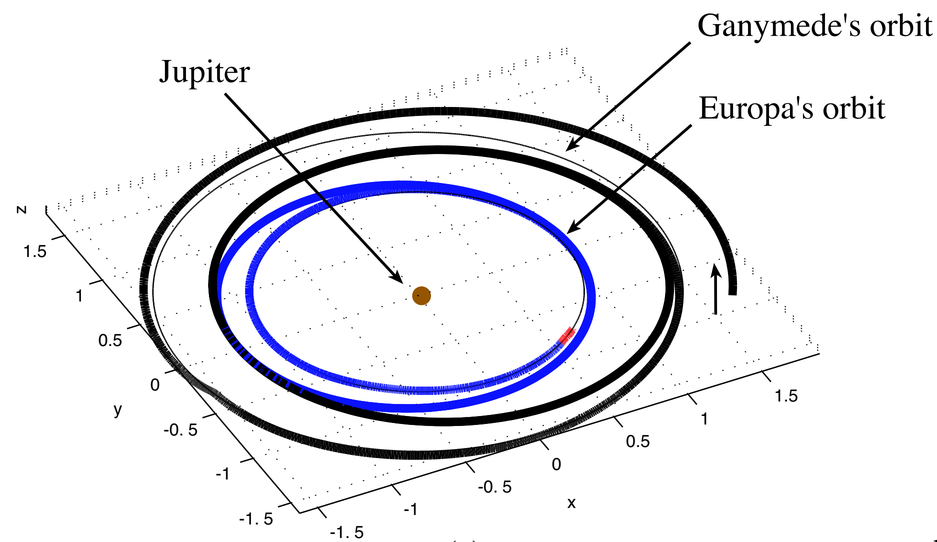
Extend from Planar Model to Spatial Model

- Previous work based on **planar** 3-body problem.
- Future missions will require **3D** capabilities.
  - Europa Orbiter mission needs a capture into a **high inclination orbit** around Europa.
- Current study has extended from **planar** to **spatial model**.

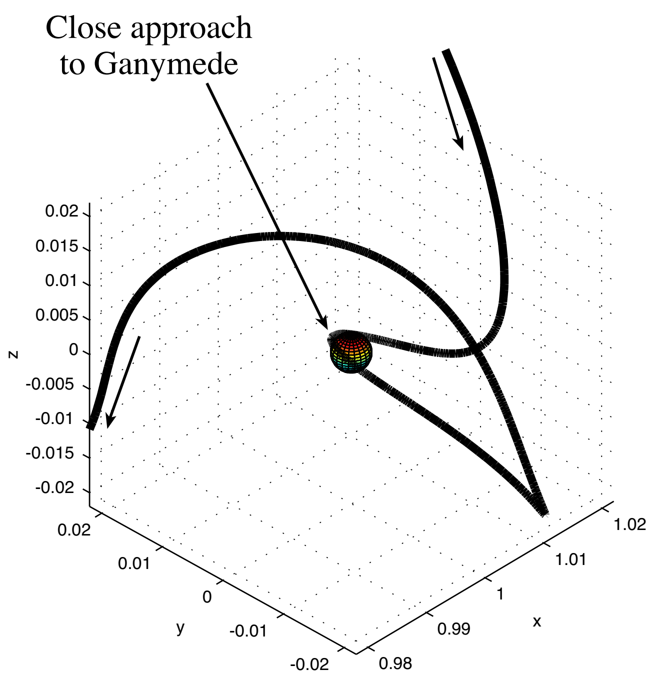




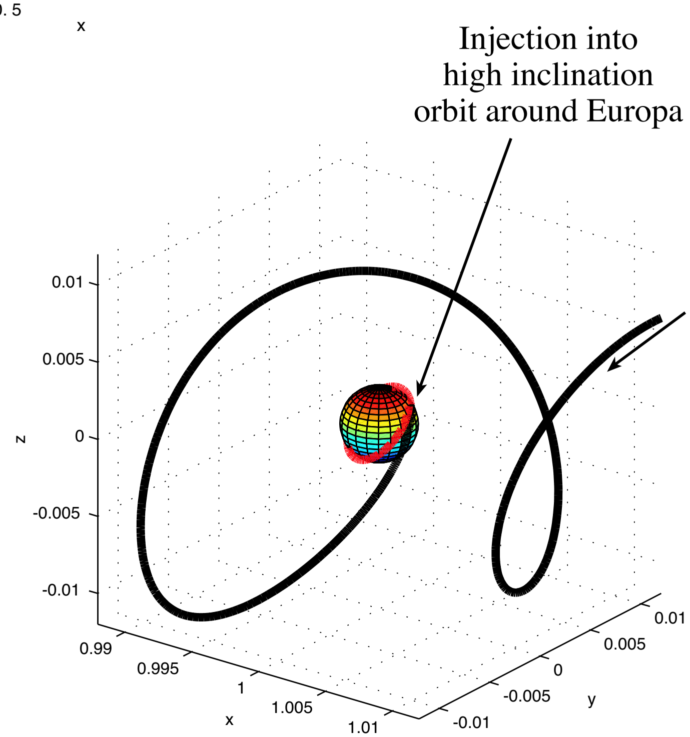
Extend from Planar Model to Spatial Model



(a)



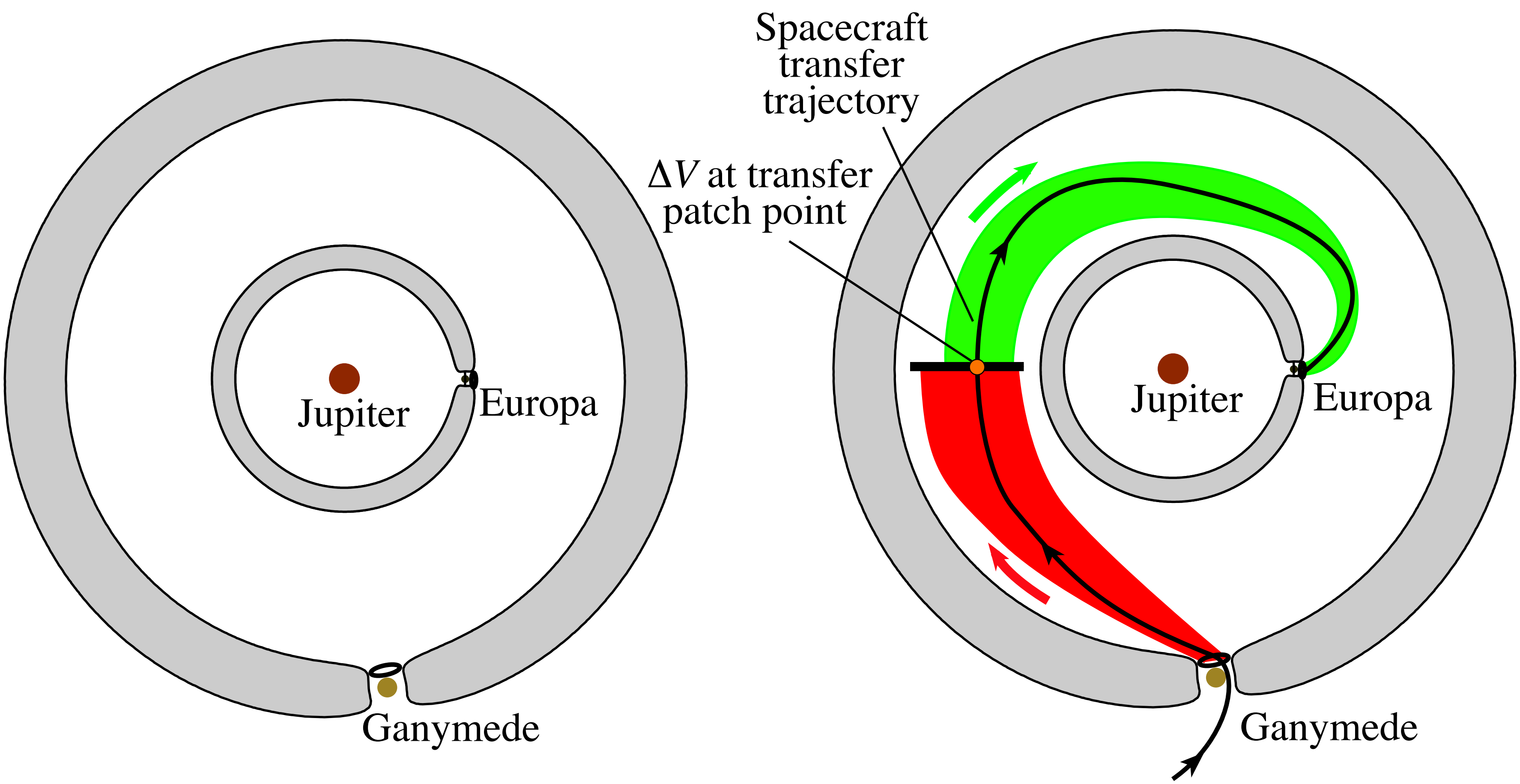
(b)



(c)

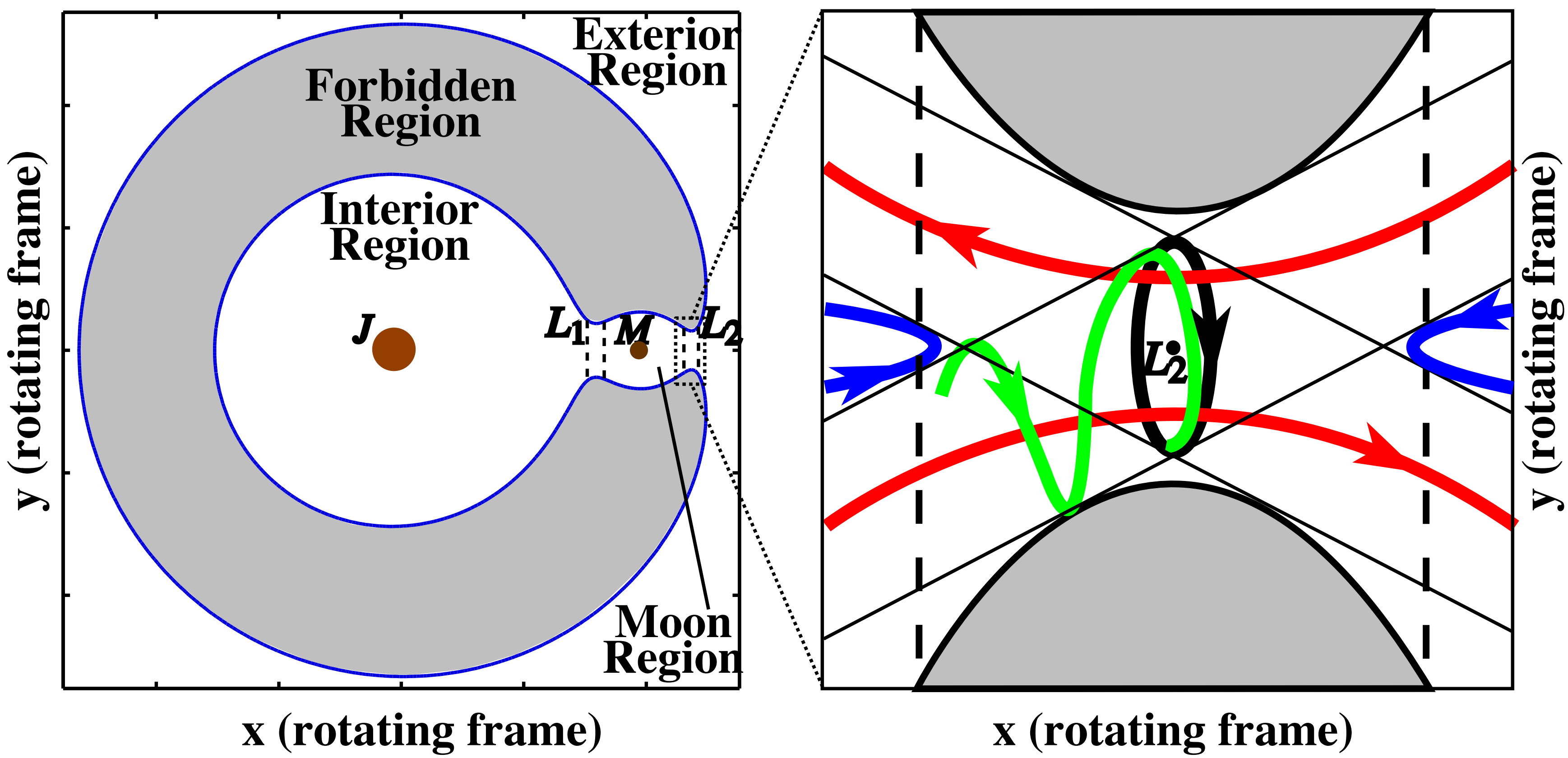
Petit Grand Tour of Jupiter's Moons

- Jupiter-Ganymede-Europa-SC 4-body system approximated as 2 **coupled 3-body systems**
- **Invariant manifold tubes** of **spatial** 3-body systems are linked in right order to construct orbit with desired itinerary.
- Initial solution refined in **4-body model**.



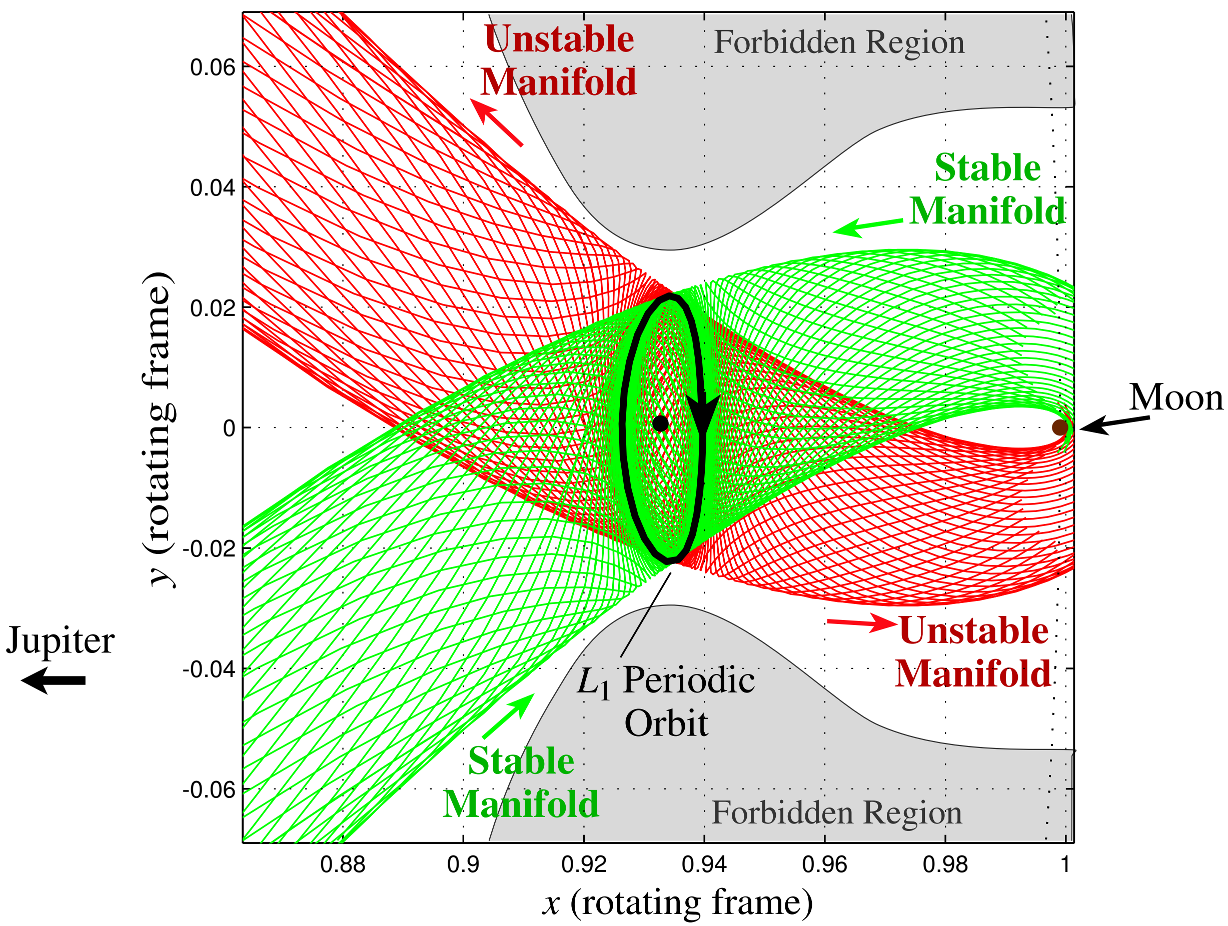
Planar Restricted 3-Body Problem

- **Recall:** for **energy value** just above that of L_2, **Hill's region** contains a **"neck"** about L_1 & L_2.
- Dynamics in each equilibrium region: **saddle x center**.
- 4 types of orbits: **periodic**, **asymptotic**, **transit** & **nontransit**.



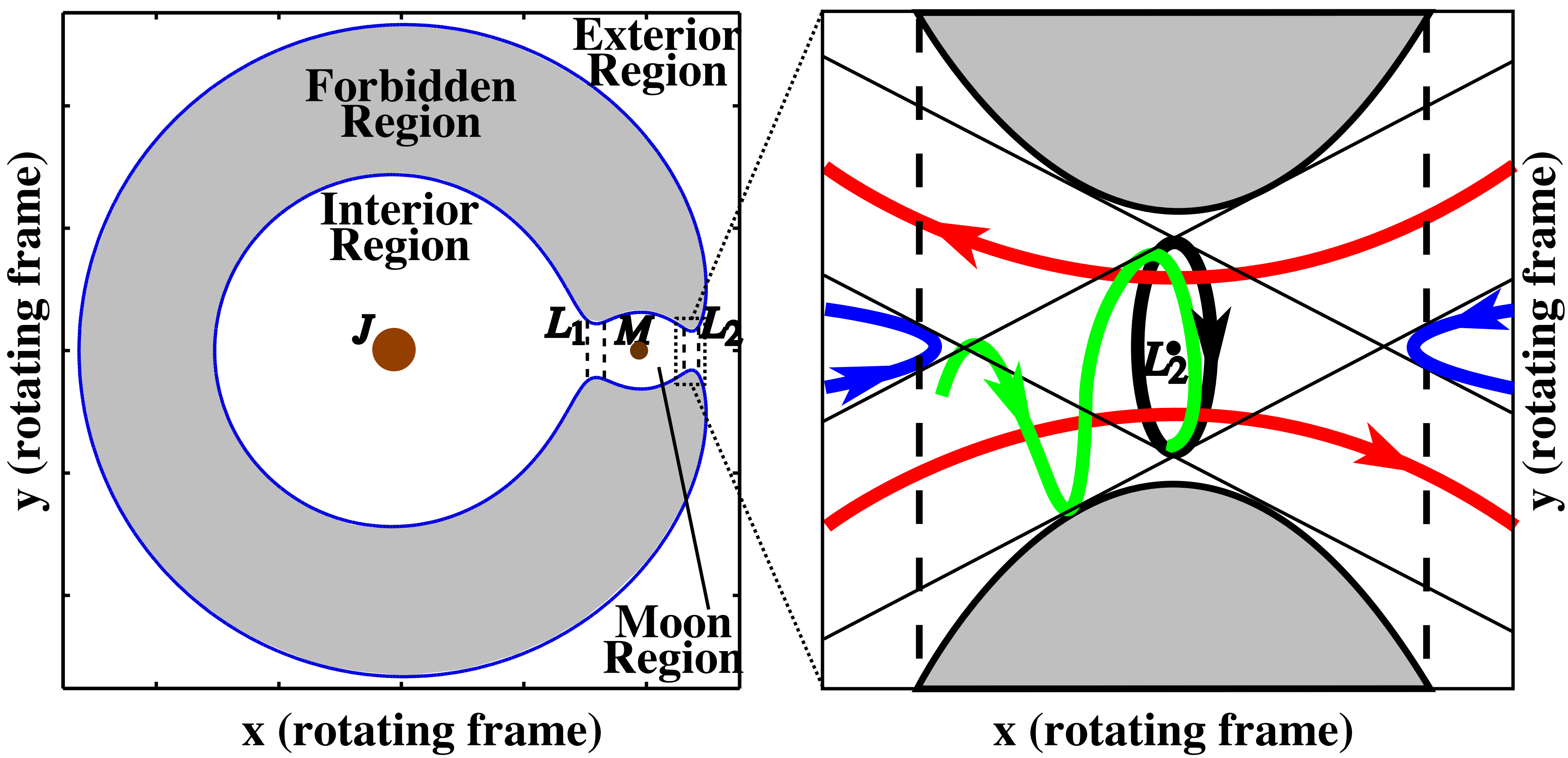
Planar: Invariant Manifold as Separatrix

- Asymptotic orbits form **2D invariant manifold tubes** in **3D energy surface**.
- They separate transit and non-transit orbits:
  - **Transit orbits** are those inside the tubes.
  - **Non-transit orbits** are those outside the tubes.



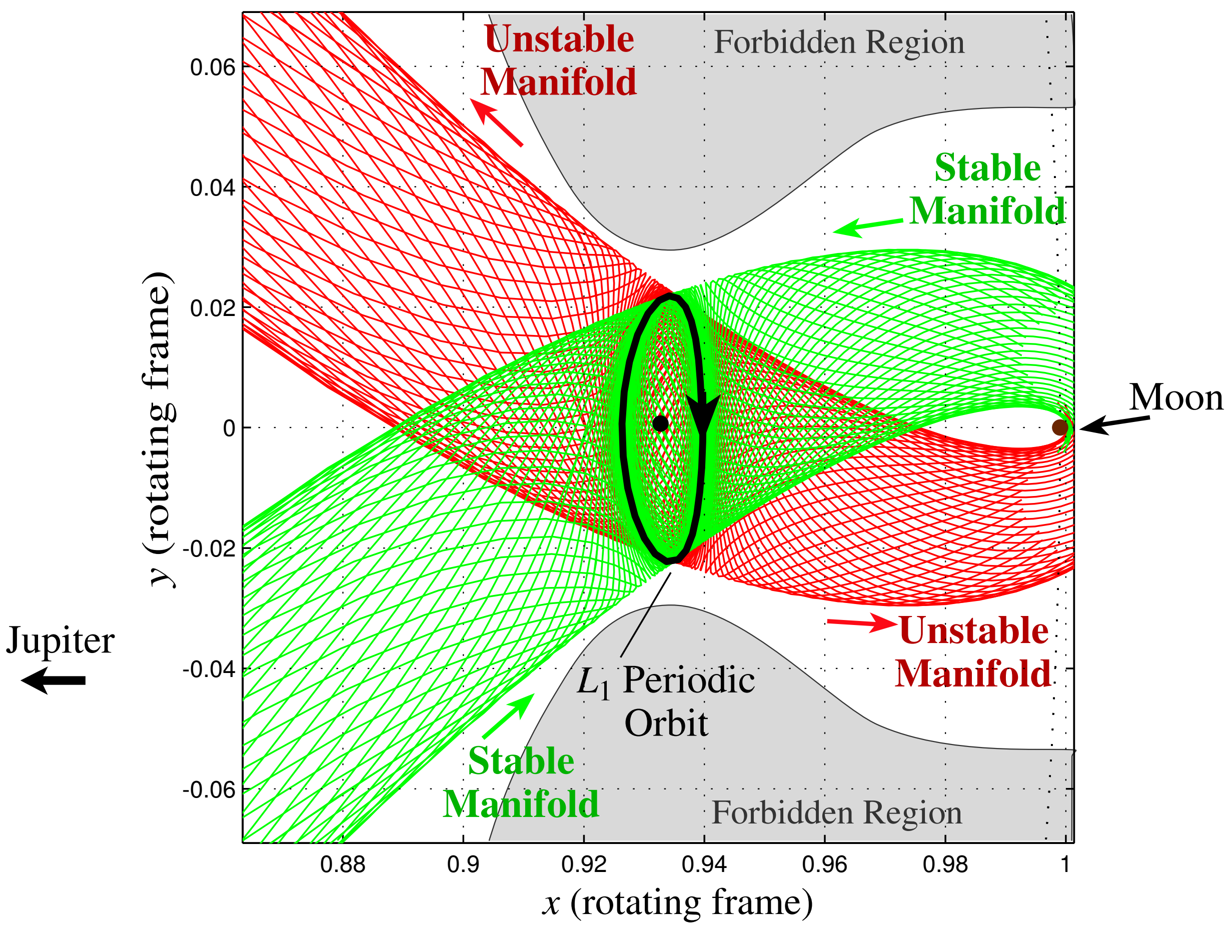
Spatial Restricted 3-Body Problem (CR3BP)

- Dynamics near equilibrium point: **saddle x center x center**.
  - **bounded orbits** (periodic/quasi-periodic): S^3 (3-sphere)
  - **asymptotic orbits** to 3-sphere: $S^3 \times I$ (**"tubes"**)
  - **transit** and **nontransit** orbits.



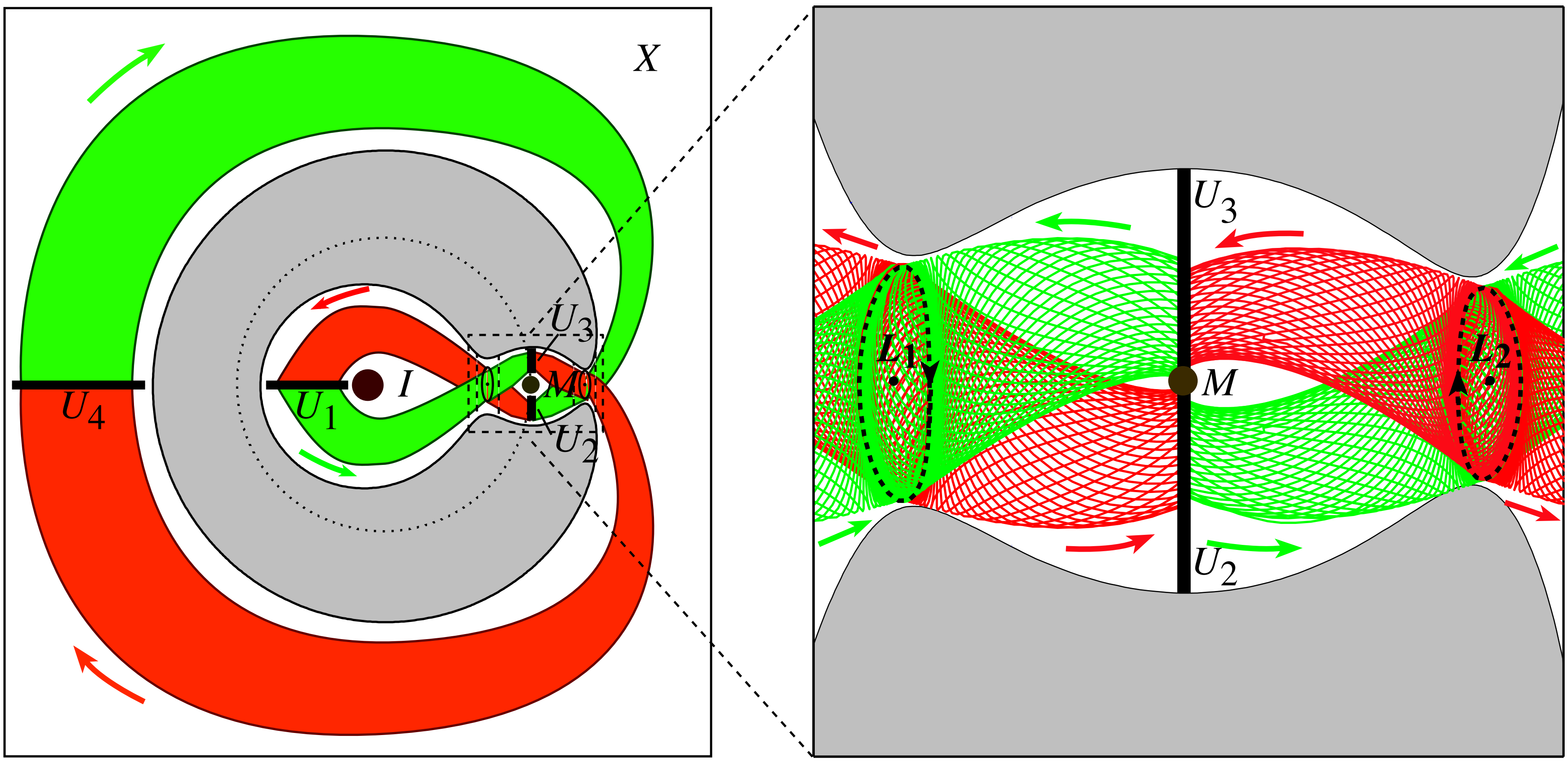
Spatial: Invariant Manifold as Separatrix

- Asymptotic orbits form **4D invariant manifold "tubes"** $(S^3 \times I)$ in **5D energy surface**.
- They separate transit and non-transit orbits:
  - **Transit orbits** are those inside the "tubes".
  - **Non-transit orbits** are those outside the "tubes".



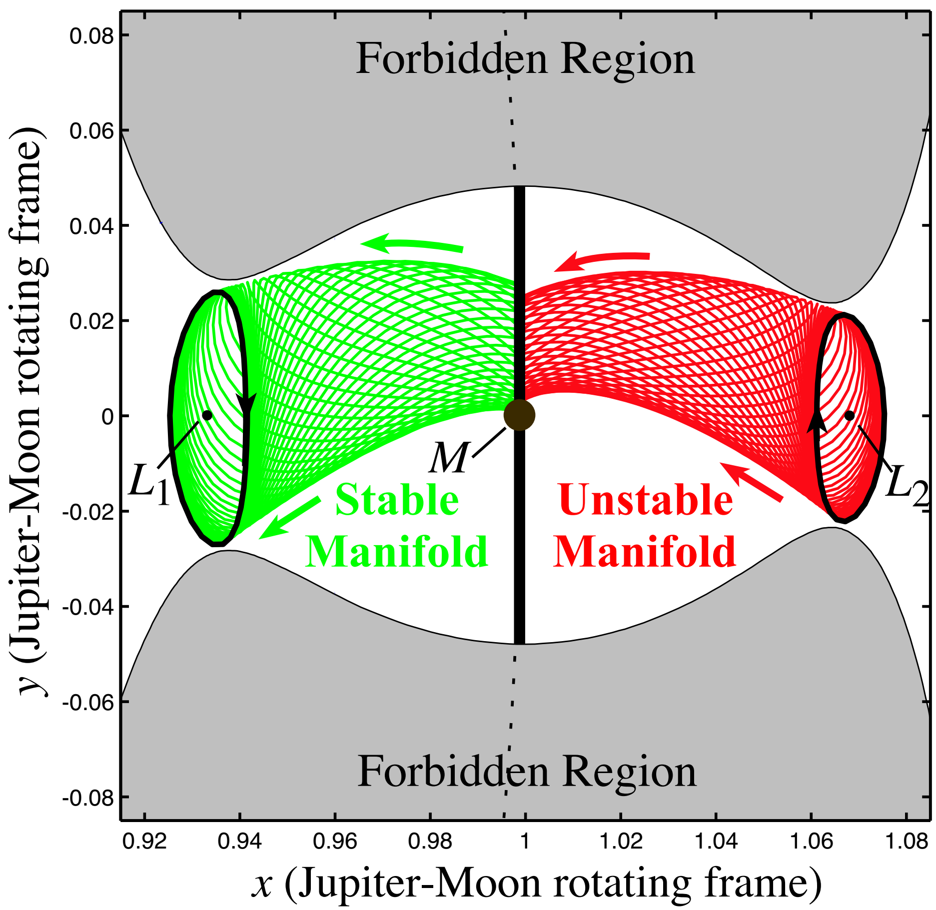
Construct Orbit with Desired Itinerary

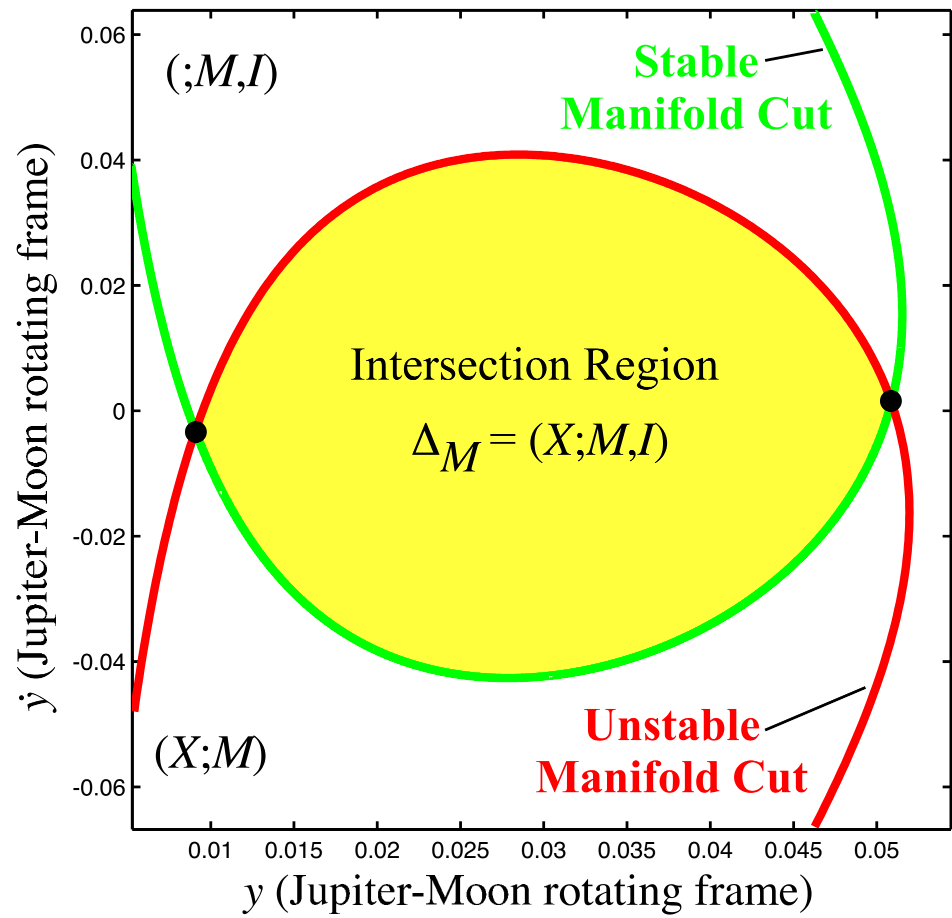
- How to **link** invariant manifold tubes to construct orbit with **desired itinerary**.
- Construction of $(X; M, I)$ orbit.



Planer: Construction of $(X; M, I)$ Orbits

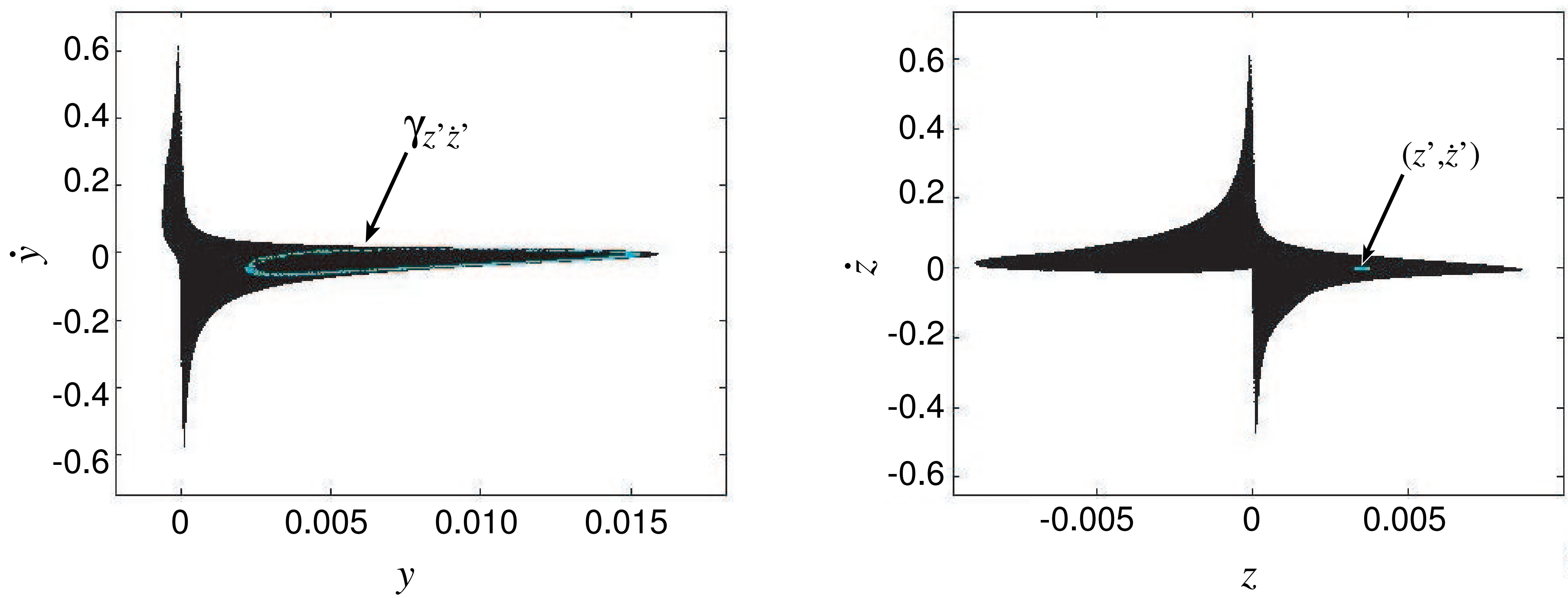
- Invariant mfd. **tubes** $(S \times I)$ separate transit/nontransit orbits.
- **Red curve** (S^1) (Poincaré cut of L_2 **unstable manifold**).
  Green curve (S^1) (cut of L_1 **stable manifold**).
- Any point inside the intersection region Δ_M is a $(X; M, I)$ orbit.





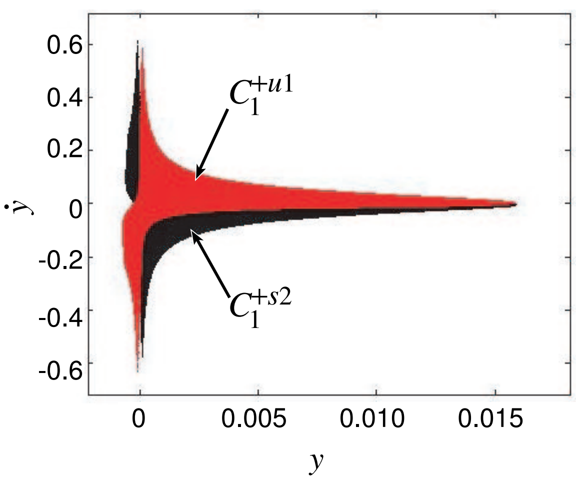
Spatial: Construction of $(X; M, I)$ orbits

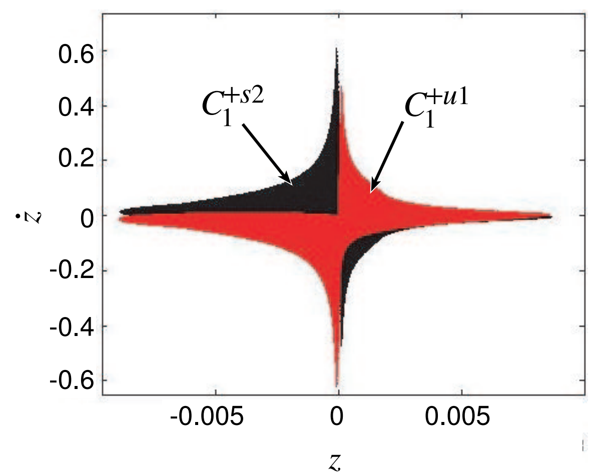
- Invariant manifold **tubes**: $(S^3 \times I)$.
- Poincaré **cut** is a topological **3-sphere** S^3 in $\mathbb{R}^4$.
  - S^3 looks like **disk x disk**: $\xi^2 + \dot{\xi}^2 + \eta^2 + \dot{\eta}^2 = r^2 = r_\xi^2 + r_\eta^2$
- If $z = c, \dot{z} = 0$, its **projection** on $(y, \dot{y})$ **plane** is a **curve**.
- Any point inside this **curve** is a $(X; M)$ orbit.

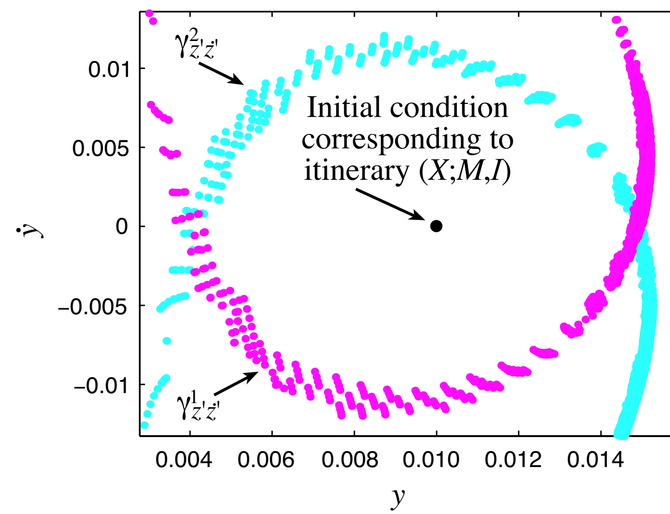


Spatial: Construction of $(\mathrm{X};\mathrm{M},\mathrm{I})$ orbits

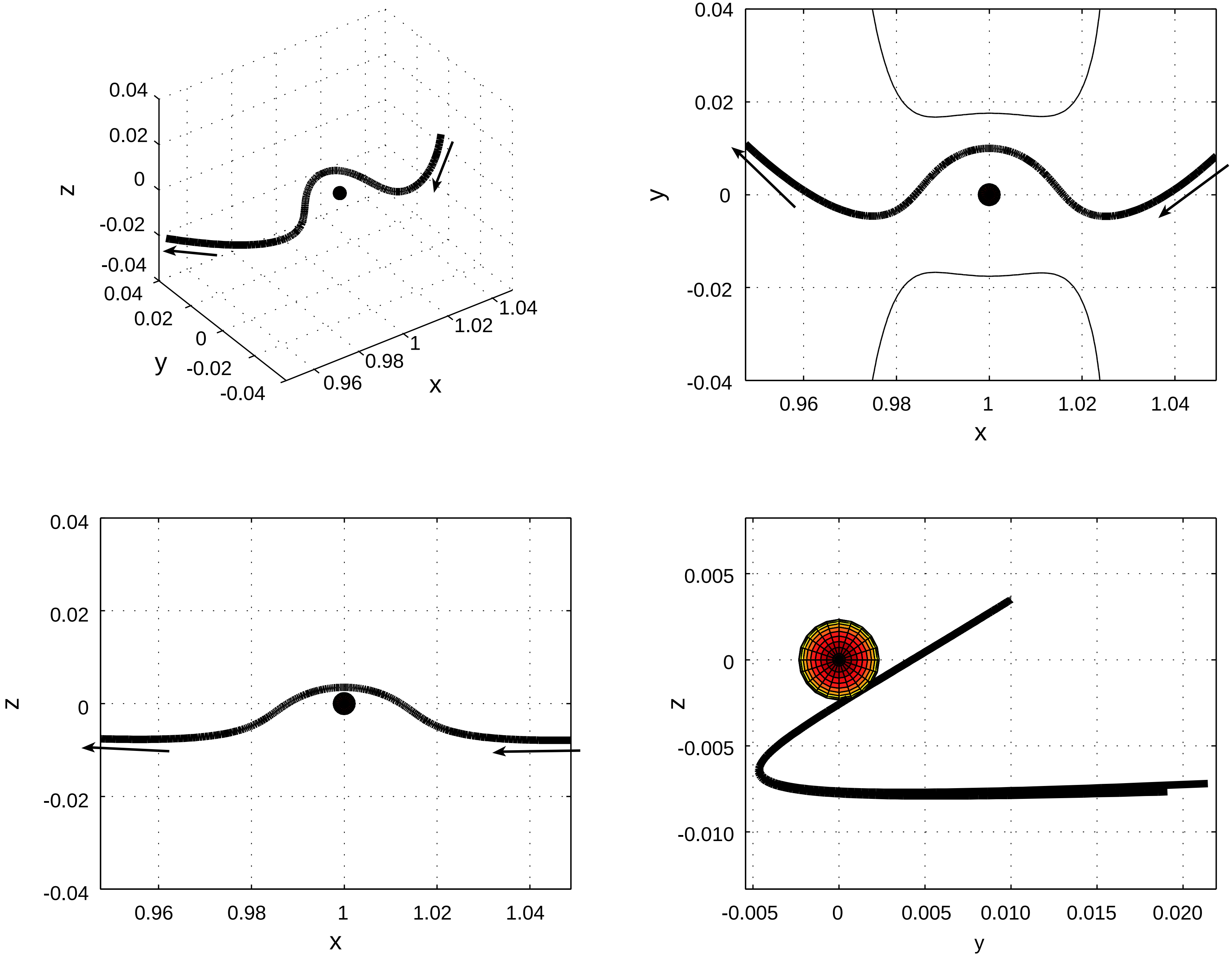
- Similarly, while **cut** of stable manifold **“tube”** is a S^3, its **projection** on $(y, \dot{y})$ **plane** is a **curve** for $z = c, \dot{z} = 0$.
- Any point inside this **curve** is a (M, I) orbit.
- Hence, any point inside the **intersection region** Δ_M is a $(X; M, I)$ orbit.





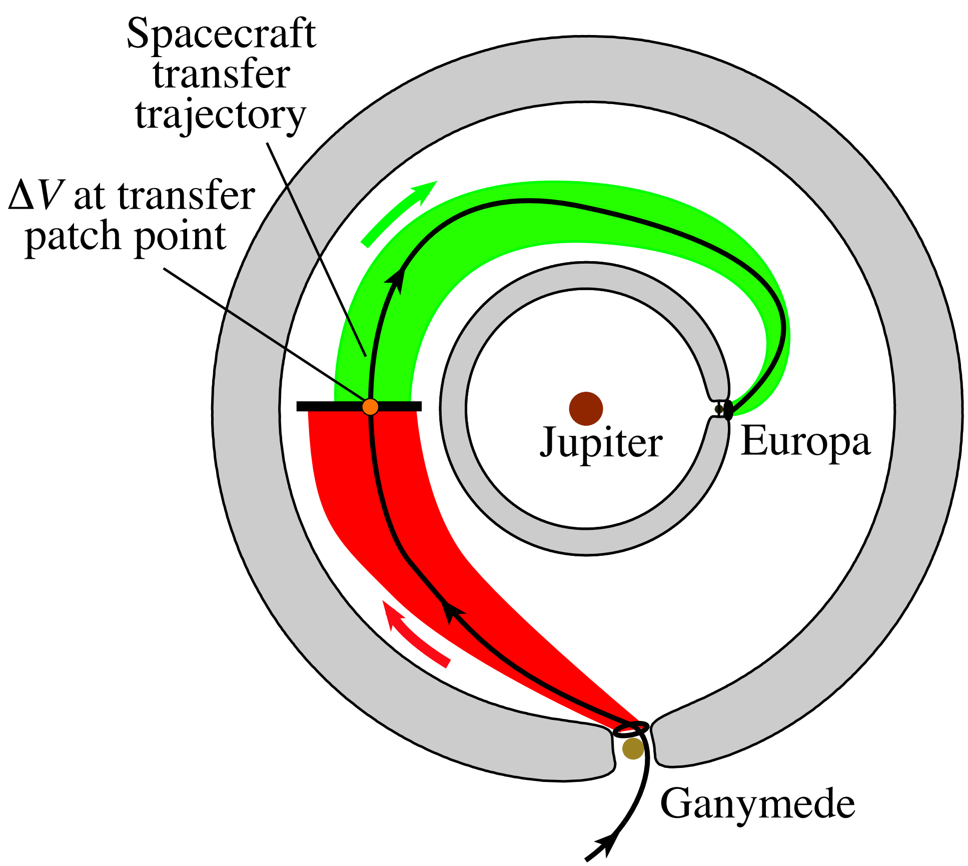


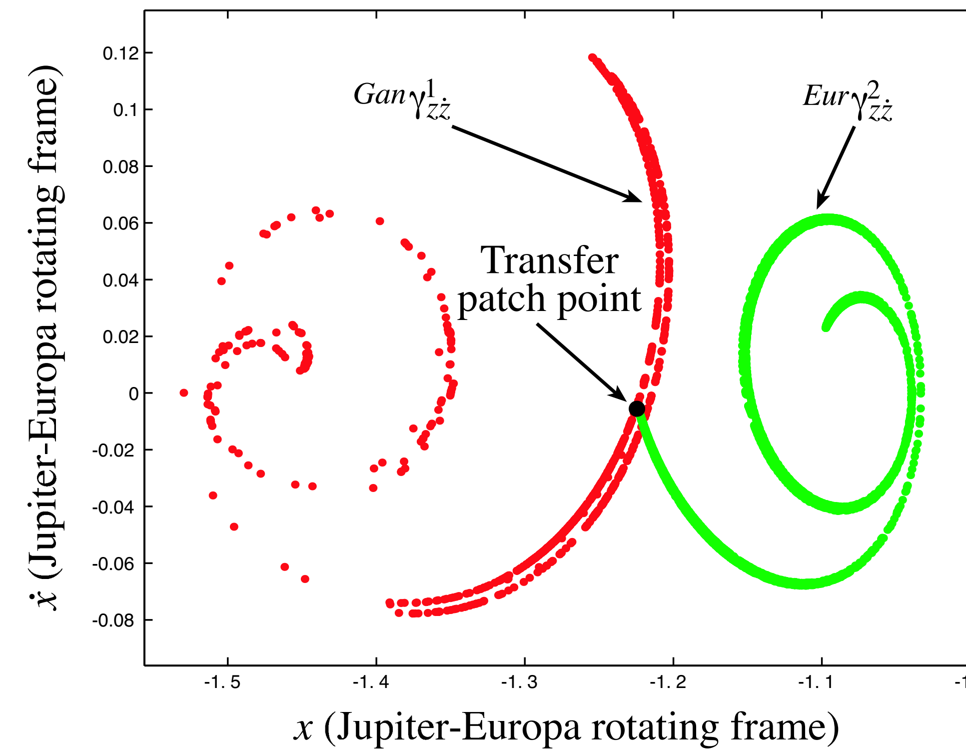
Spatial: Construction of $(X; M, I)$ orbits



Petit Grand Tour of Jupiter's Moon

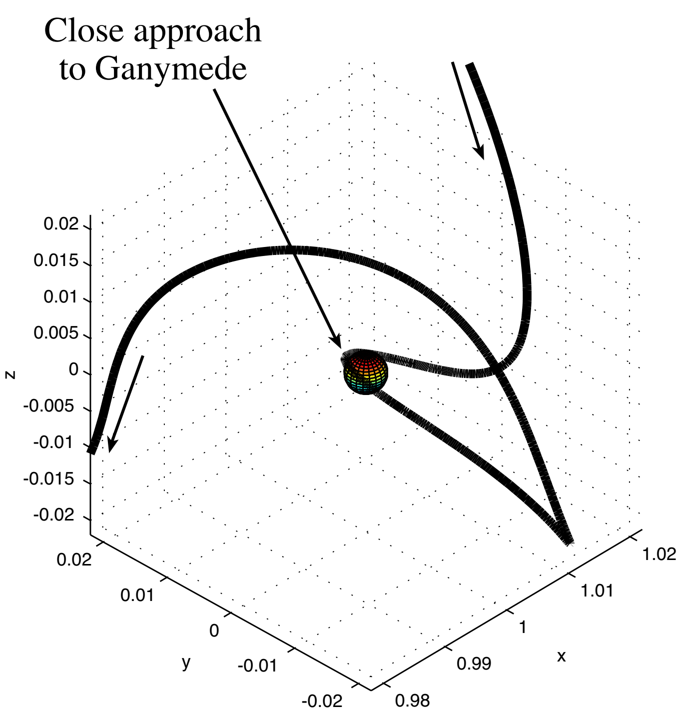
- Petit Grand Tour can be constructed similarly
  - Approximate 4-body system as 2 nested **3-body systems**.
  - Choose appropriate **Poinaré section**.
  - Link invariant **manifold tubes** in right order.
  - Refine initial solution in **4-body model**.

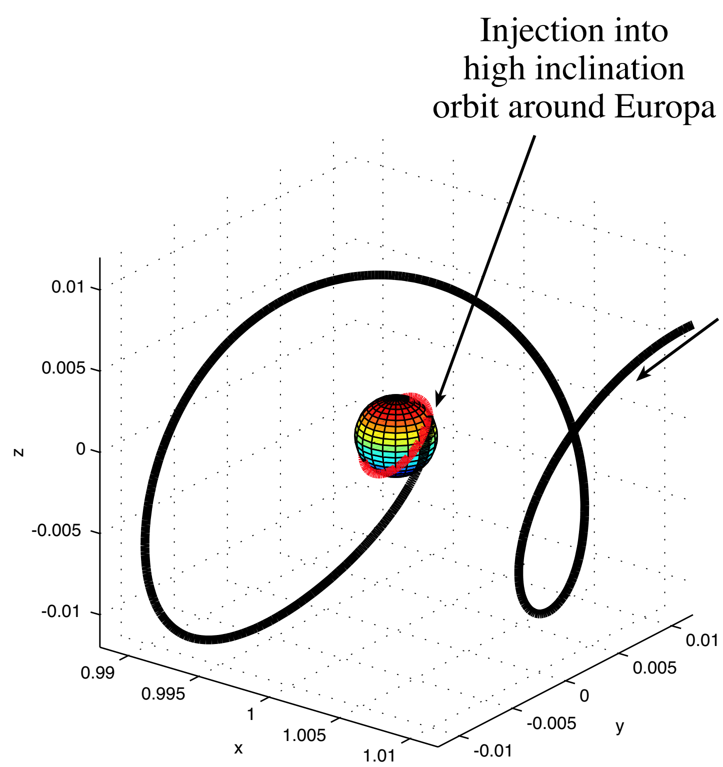




Conclusion and Future Work

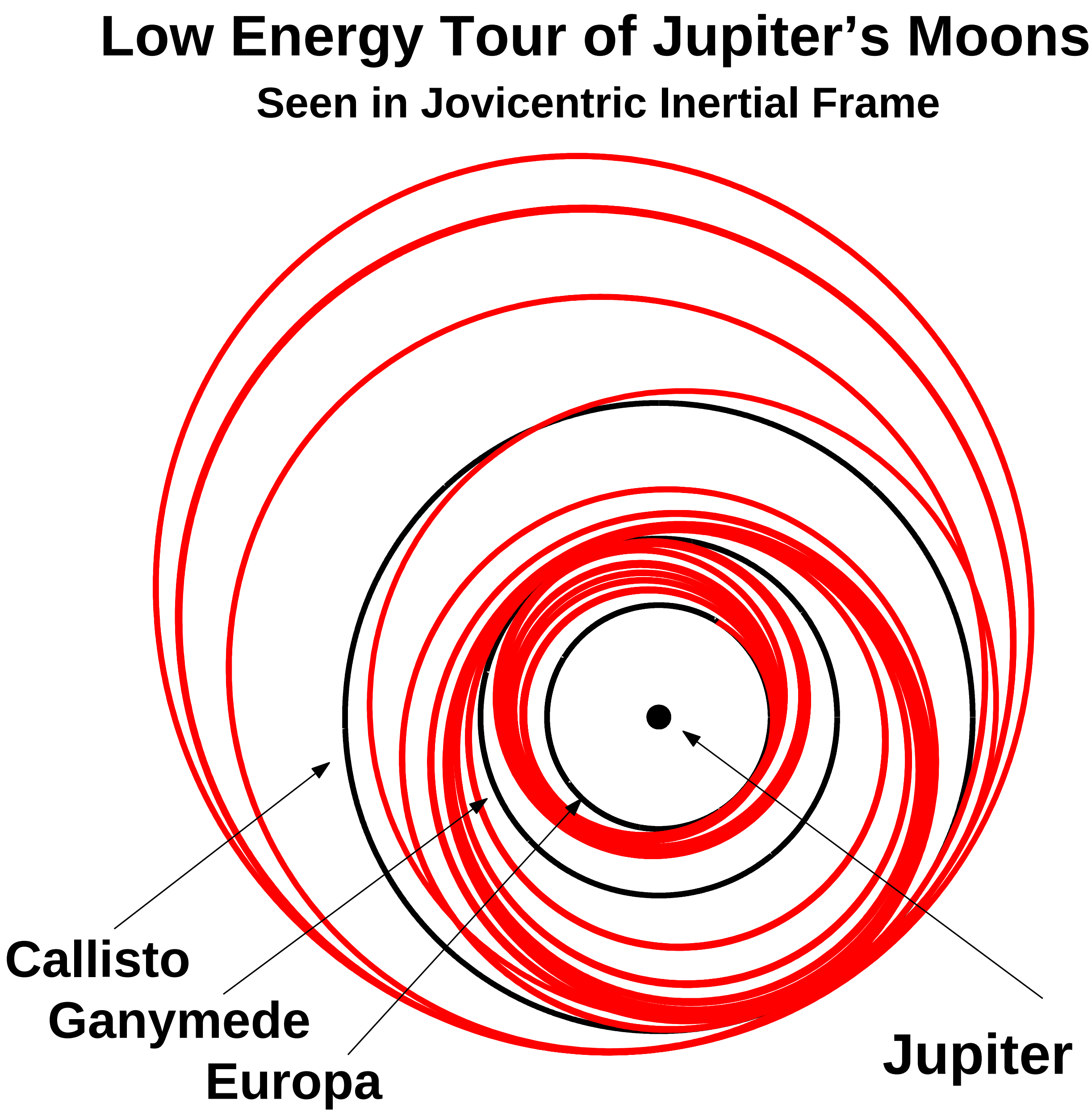
- In our study of **spatial** CR3BP:
  - Invariant manifolds still act as **separtrices**.
  - Construct orbit with **prescribed itinerary**.
  - Construct **Petit Grand Tour** of Jupiter's moons that ends in a **high inclination** orbit around Europa.
- Will construct useful trajectories in **Sun-Earth-Moon** system.





Future Work: Pump Down via Resonance Encounters

- For this new tour: $\Delta V = 20m/s$.



References and Other Informations

- **Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross**
  Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics,
  Chaos, vol. 10, (2000) pp. 427-469.
  - **http://www.cds.caltech.edu/~koon/**
  - **http://ojps.aip.org/chaos/**

- **Gómez, G., W.S. Koon, M.W. Lo, J.E. Marsden, J. Masdemont and S.D. Ross**
  Invariant Manifolds, The Spatial Three-Body Problem and Space Mission Design.

- Email: **koon@cds.caltech.edu**