J2 Dynamics and Formation Flight of Micro-satellites

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Acknowledgments

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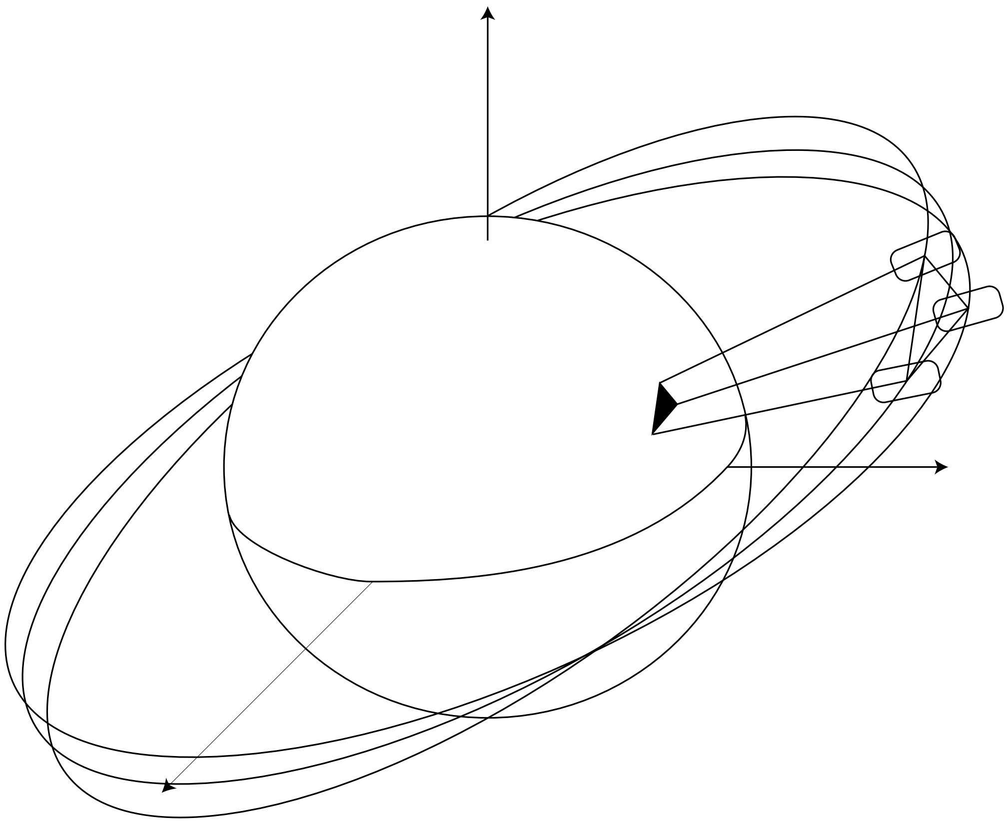
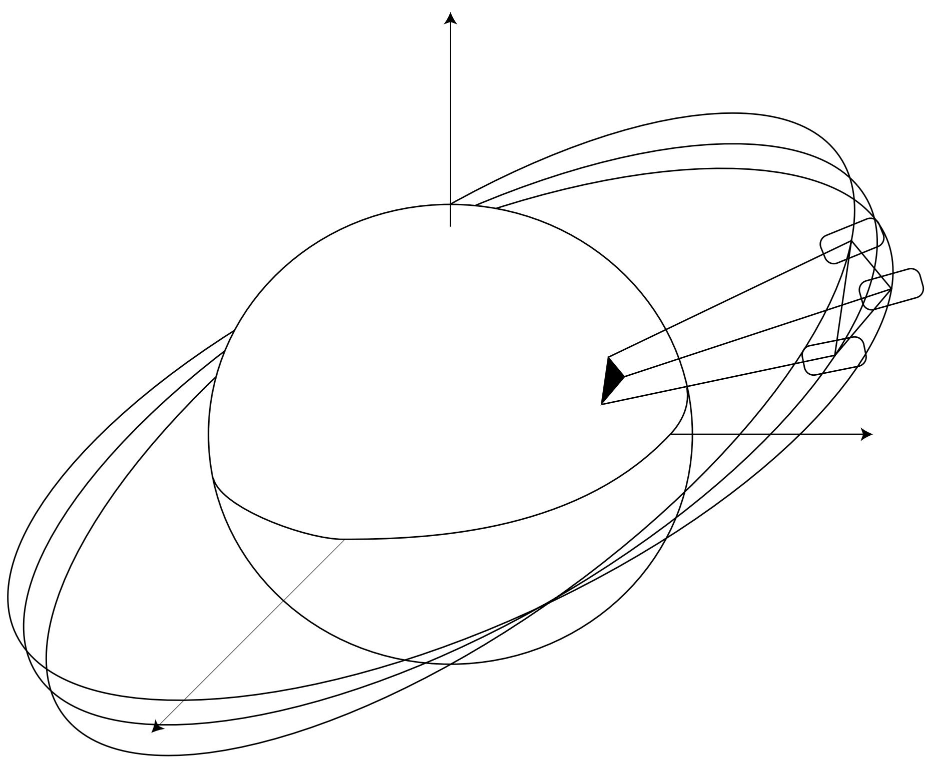
Motivation

- Distribute functions of **single large satellite** among several **small cooperative units**.
- Many potential applications: Earth observation.
- A **cluster** can synthesize **larger aperture** providing much **better image resolution** via interferometry.



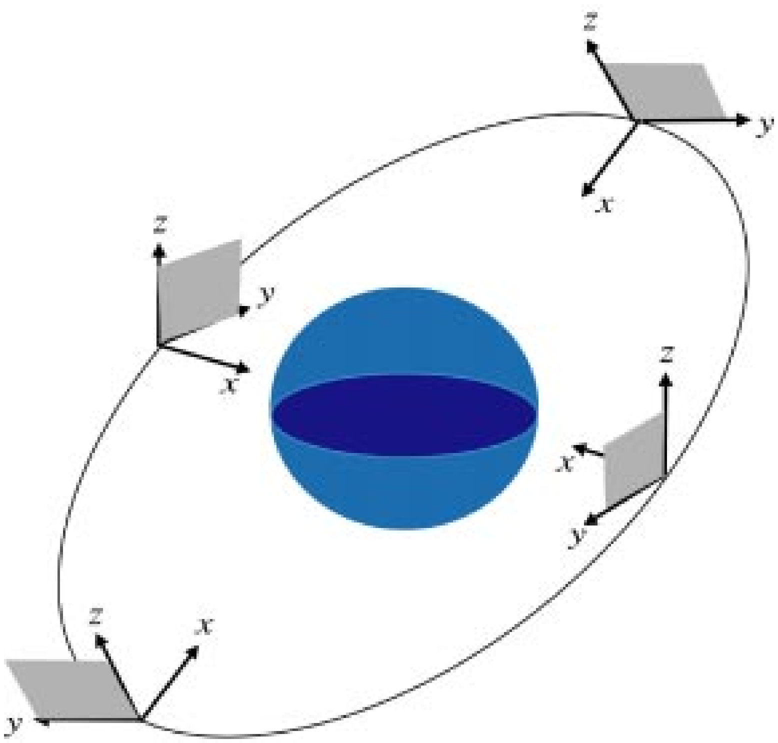
Main Results

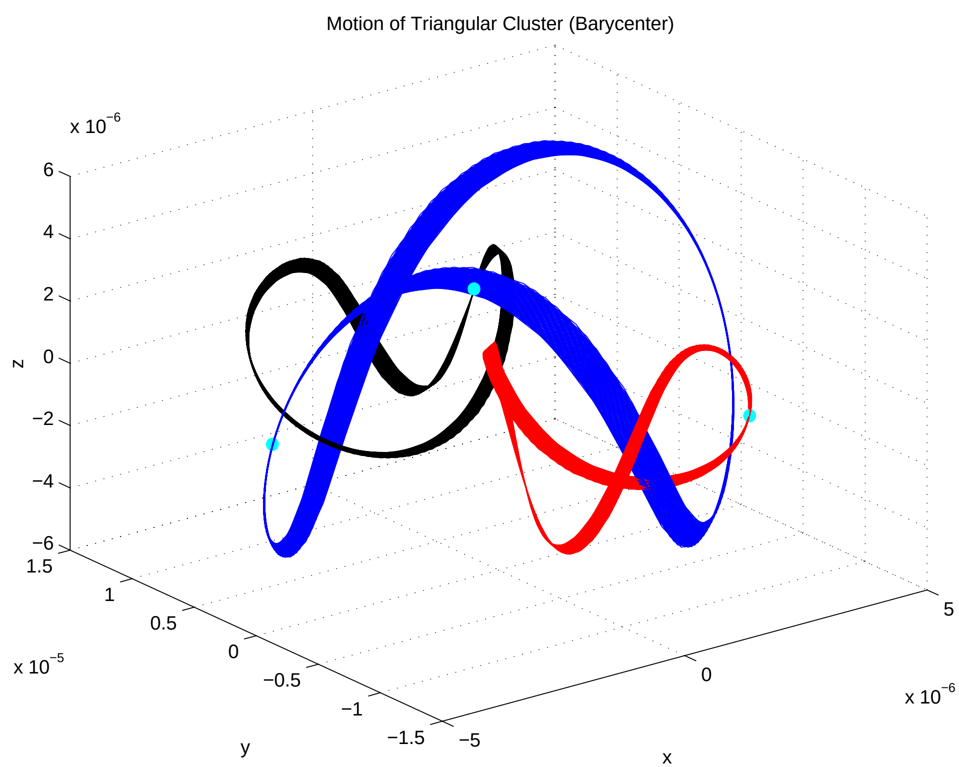
- Study dynamics of relative motion in Earth's gravitational field.
  - At 800 km, J_2 **effect** much larger than atmospheric drag, solar pressure, electro-magnetic effects.
- Use symmetry reduction/Poincaré section tools to find orbits
  - **cluster remains close for many years, with very little dispersion, even with no controls.**



Triangular Cluster

- Plot of **relative motions** of 3 satellites
  - in moving frame whose origin at instantaneous center of mass whose **yz-plane** is **local horizontal**
  - for **100 revolutions** around Earth (for a week).

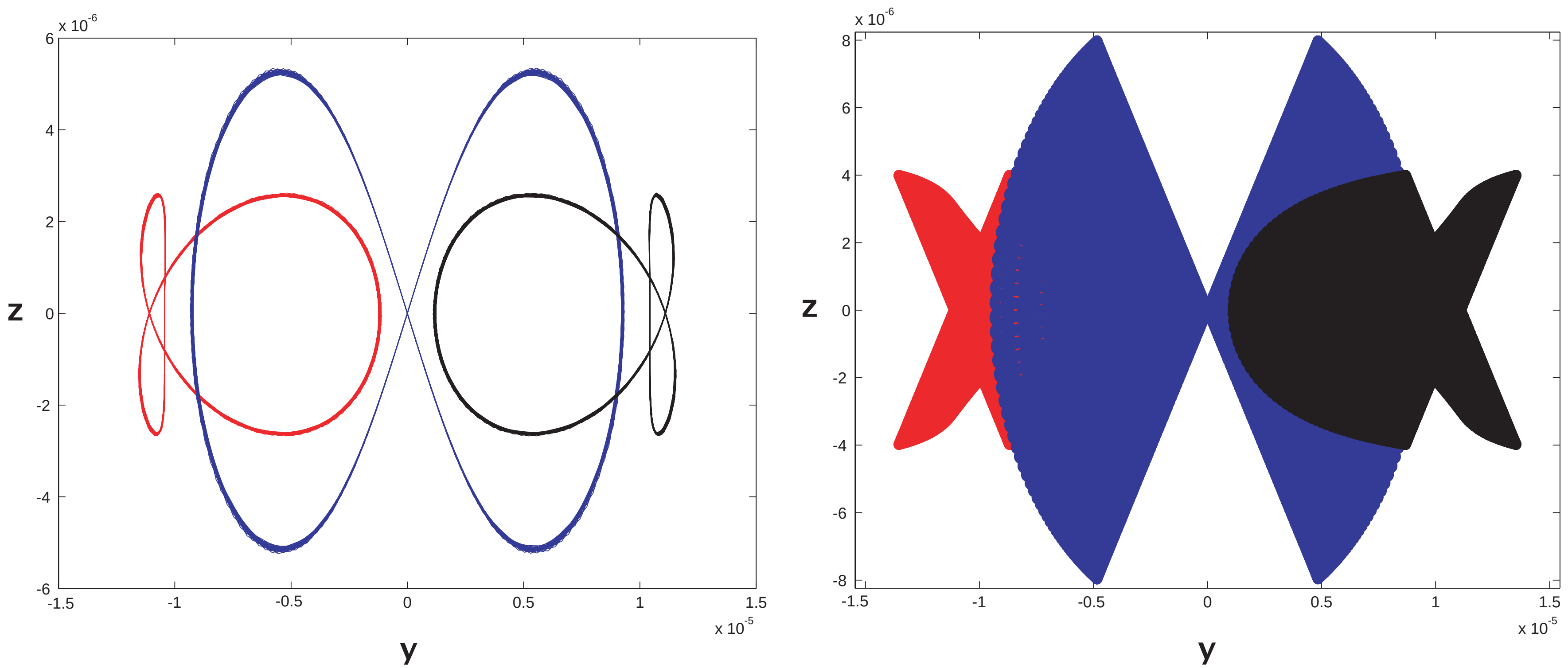




Triangular Cluster

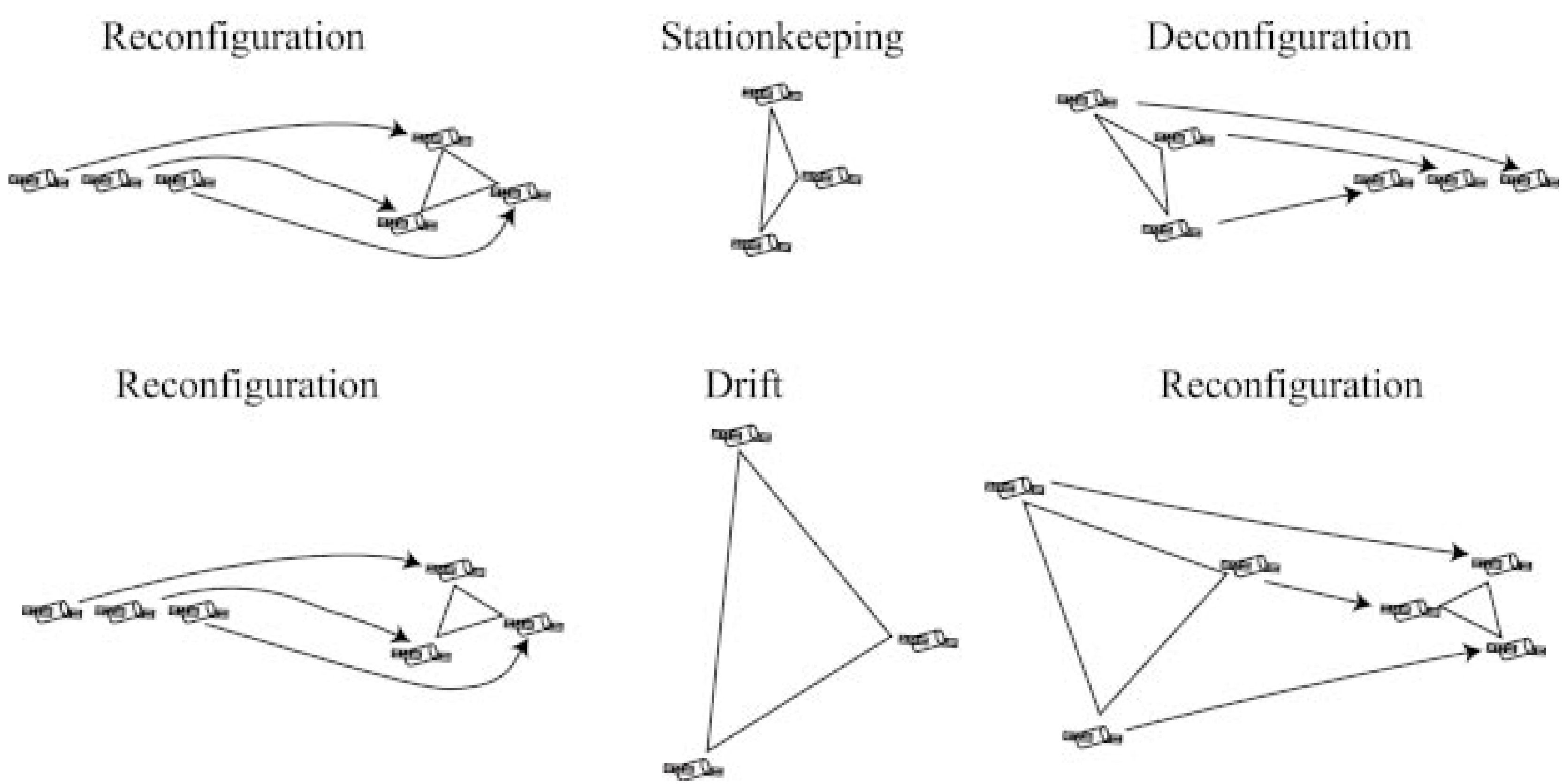
Triangular Cluster

- Plot of **relative motions** of 3 satellites in **local horizontal yz-plane**
  - for **100 revolutions** around Earth (for a week)
  - for **5000 revolutions** around Earth (for a year)
- Notice how small is the **dispersion** in a year (only in **meters**).



Main Results

- Instead of **orbital elements**, analysis done in **physical space**, make connection with **physical requirements** more direct.
- If coupled with **optimal control** techniques, will provide a **fuel-efficient** way to deal with
  - formation **maintenance**
  - formation **reconfiguration**

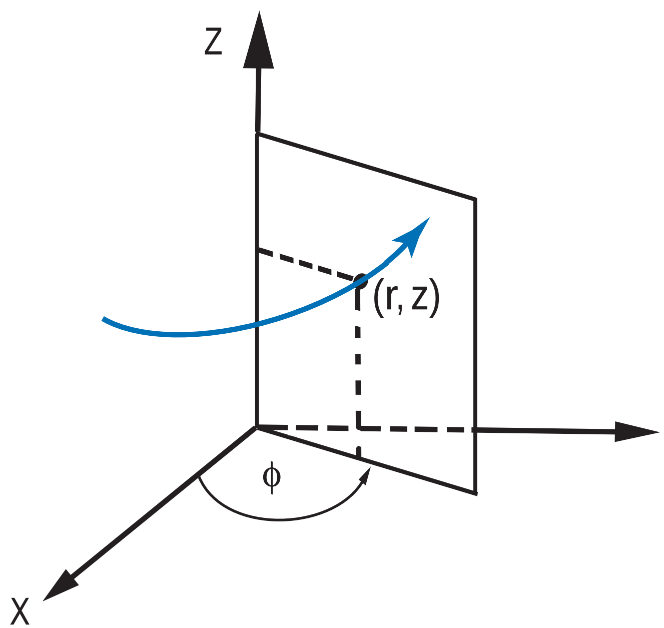


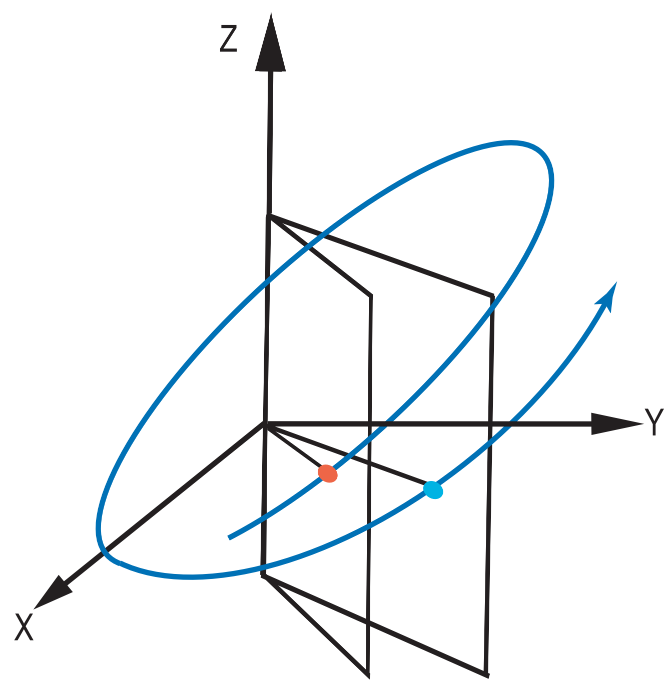
Find Initial Conditions for a Cluster

- Find **initial conditions** for a cluster that will stay together for long time under natural dynamics.
- In **meridian** coord. (r, z, ϕ),

$$U = -\frac{\mu}{\rho} + \frac{\mu R_e^2 J_2}{\rho^3}\left(\frac{3}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right), \qquad \rho = \sqrt{r^2 + z^2}$$

is **symmetric** w.r.t. z-axis (no ϕ).





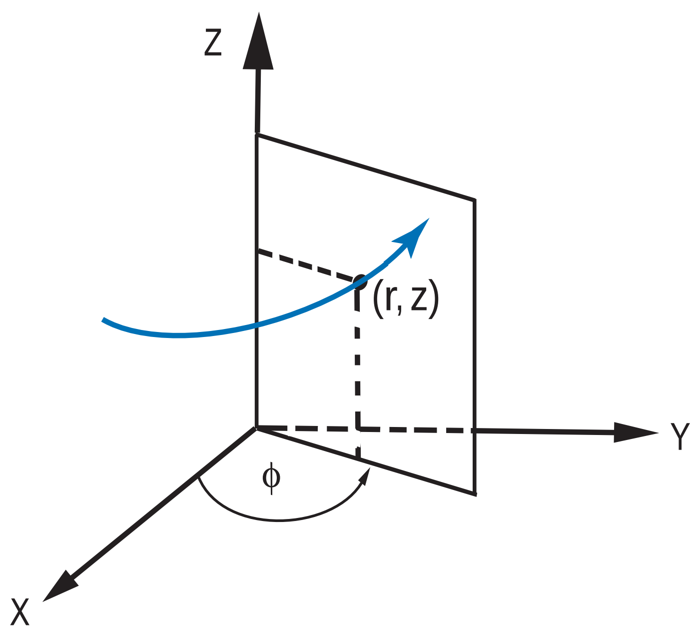
Symmetry Reduction

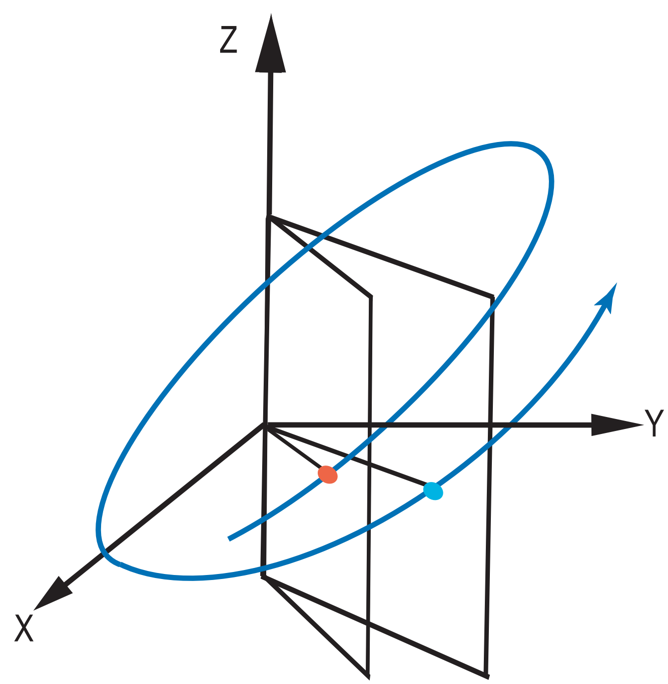
- Since no ϕ, equations of motion

$$\ddot{r} = f(r, z), \qquad \ddot{z} = g(r, z), \qquad \dot{\phi} = \frac{H_z}{r^2},$$

where H_z (z-component of angular momentum) is conserved.

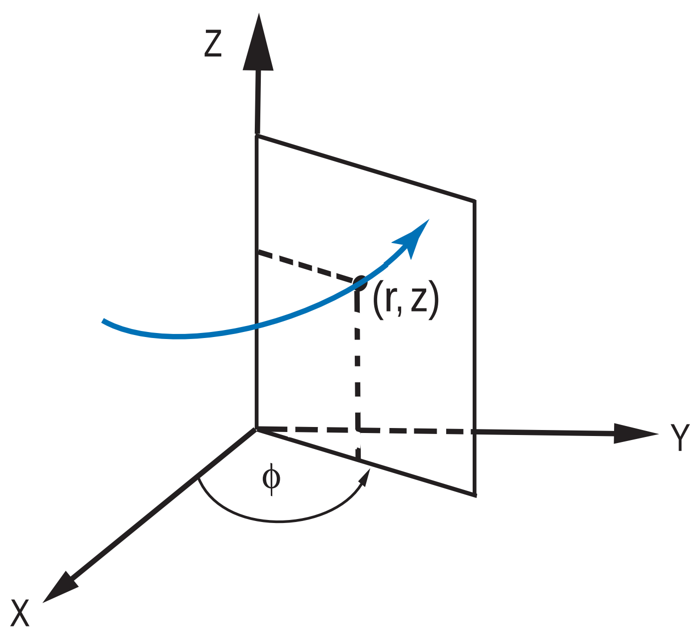
- **Symmetry reduction** allows to study first
  - reduced dynamics in **meridian plane** of satellite
  - before dealing with dynamics in ϕ direction.

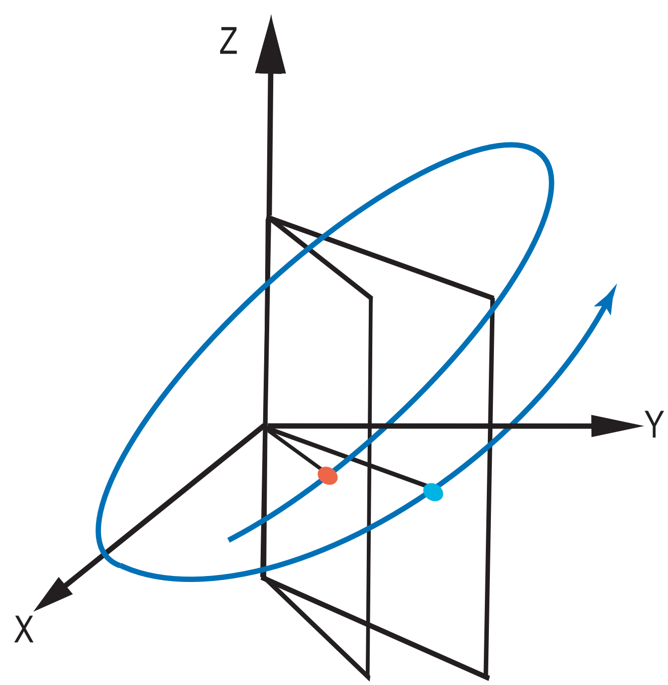




Poincaré Section

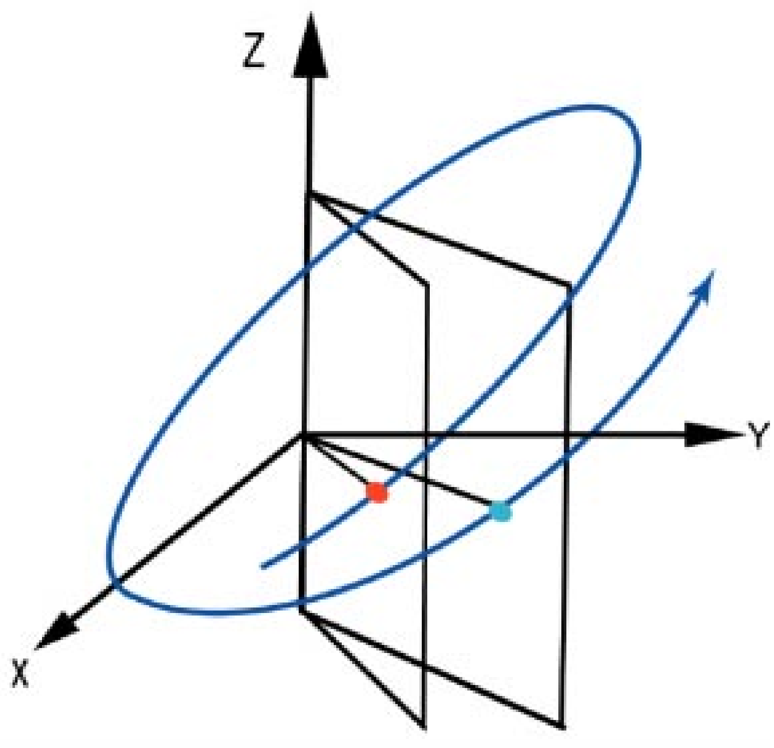
- Consider **4D** reduced phase space $(r, z, \dot{r}, \dot{z})$.
  - Since **energy** E is conserved,
    for fixed E, reduced dynamics is **3D**: $\dot{z} = \dot{z}(r, \dot{r}, z, E)$.
  - For $z = 0$, obtain a **2D** Poincaré section: $(r, \dot{r})$.
- **Poincaré section** made by plotting a point $(r, \dot{r})$ whenever SC crosses **equator** from south to north.

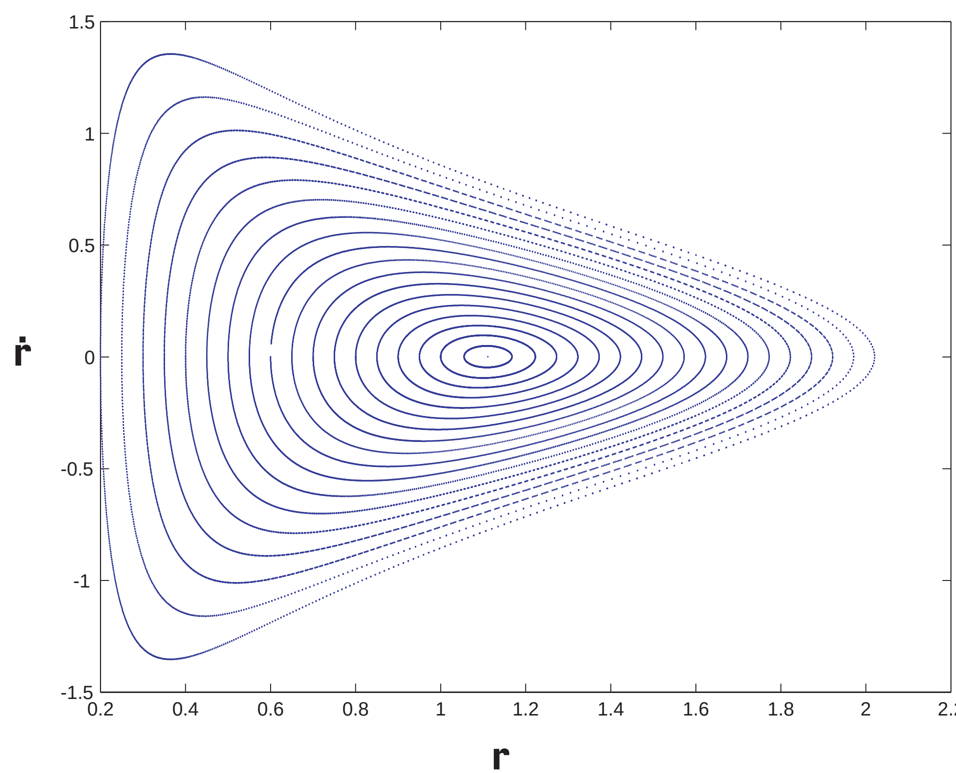




Poincaré Section

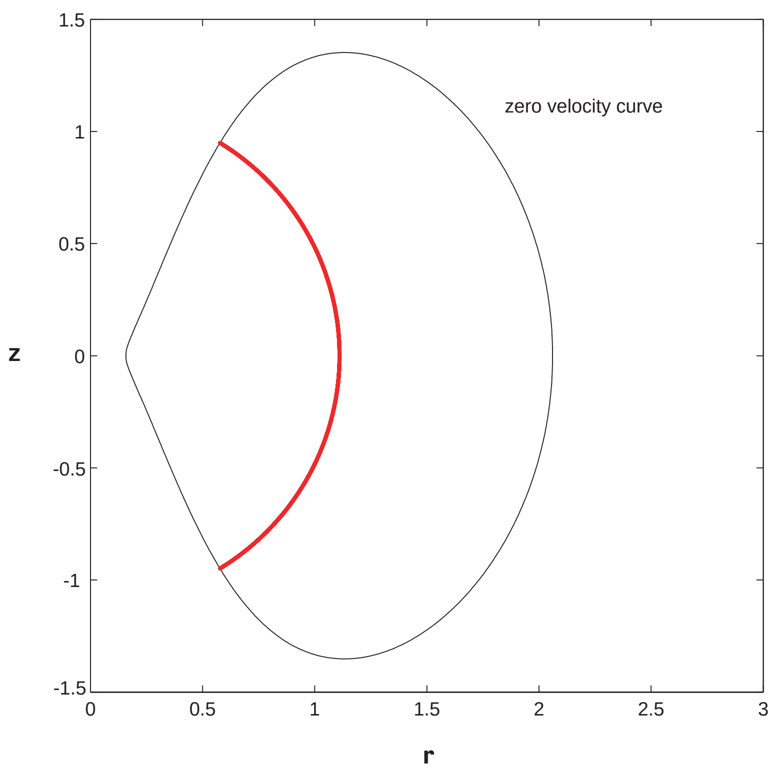
- Each $(r, \dot{r})$ gives **initial conditions** $(r, z, \phi, \dot{r}, \dot{z}, \dot{\phi})$ for an orbit of **full system**.
  - $z = 0$. $\dot{z} = \dot{z}(r, \dot{r}, 0, E)$ and $\dot{\phi} = H_z/r^2$ can be computed from E and H_z.
  - ϕ (symmetry) can be chosen arbitrary: $\phi = 0$ at $t = 0$.
  - Hence, $(r, 0, \phi, \dot{r}, E, H_z)$ provides all the informations.

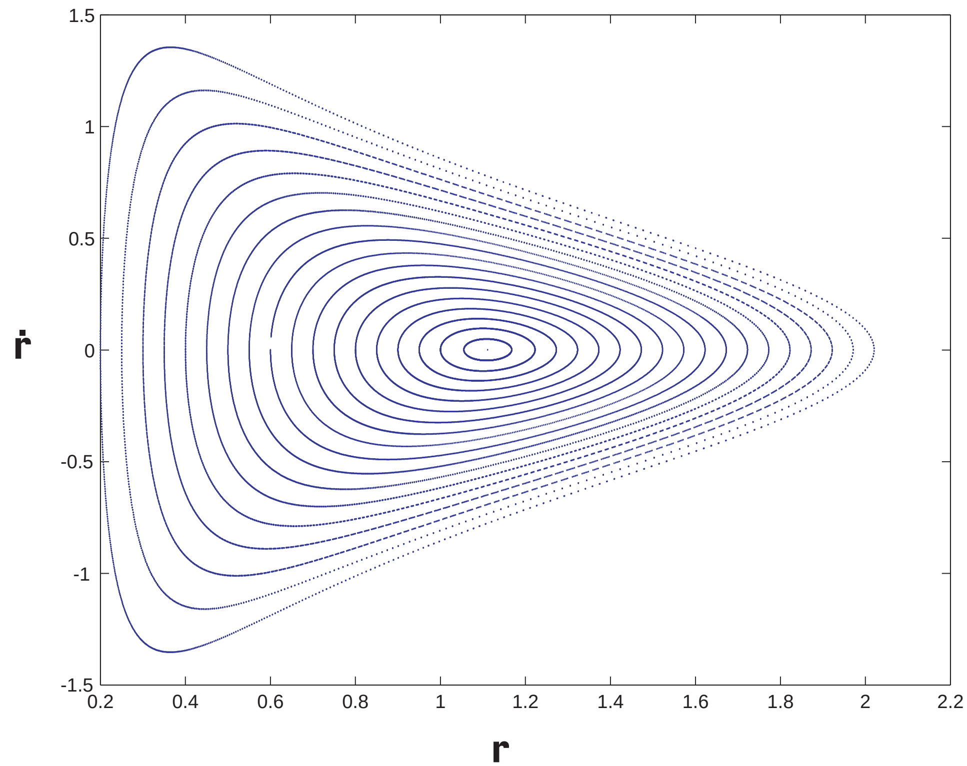




Almost-Circular Orbit.

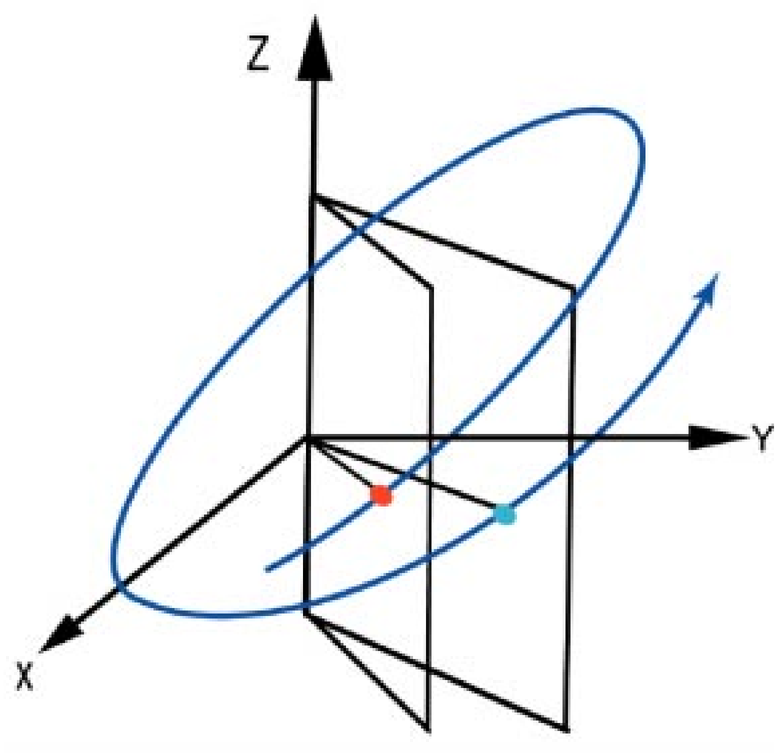
- Stable fixed point gives **almost-circular** orbit:
  - **periodic** in reduced system $(r, z, \dot{r}, \dot{z})$,
  - modulo procession in ϕ,
    trajectory repeats itself (**almost periodic**),
  - mean eccentricity nearly **zero**.

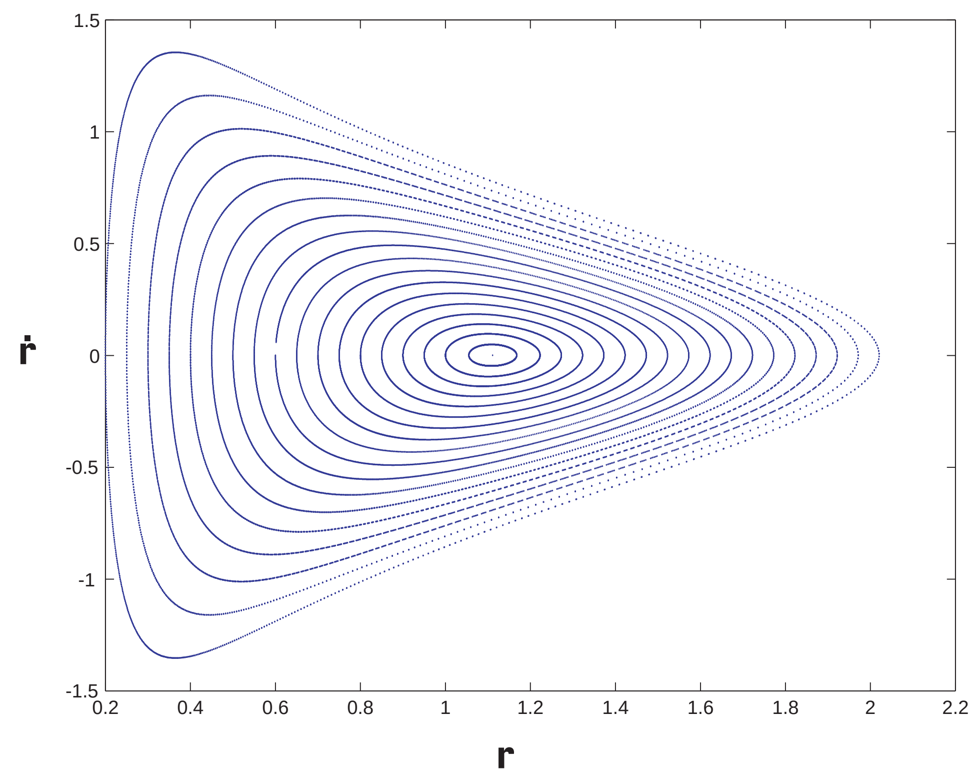




Triangular Cluster near Almost-Circular Orbit

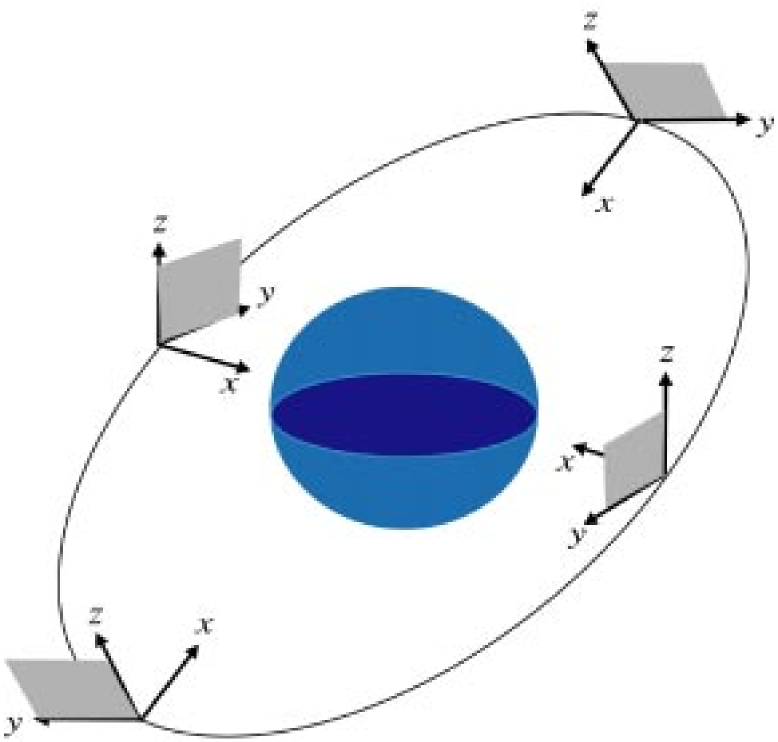
- Can construct **cluster** that stay toghether for long time
  - using **fixed point** and **nearby points**,
  - making **slight changes** in ϕ or in t,
- Find initial conditions for a **triangular cluster**
  - about **710 km** above Earth, with **inclination** near 58^o
  - each side in the order of 100 **meters**.

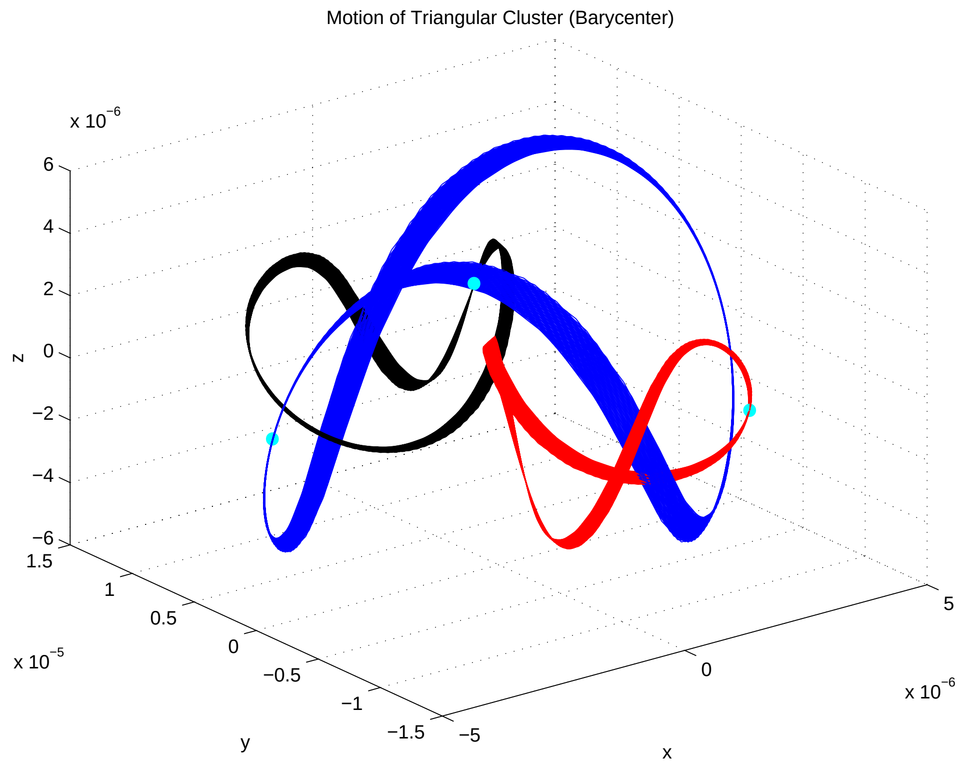




Triangular Cluster near Almost-Circular Orbit

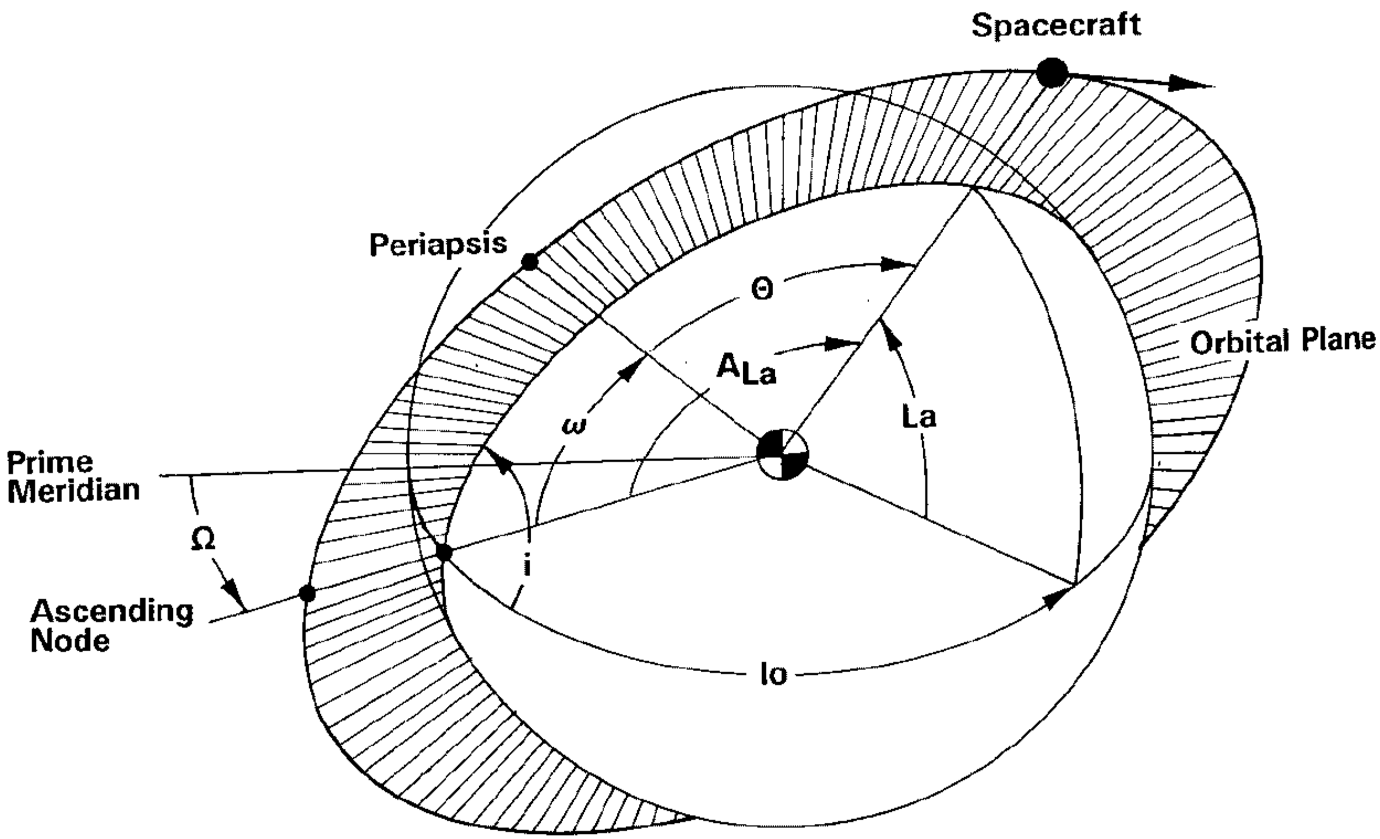
- Plot of **relative motions** of 3 satellites in moving frame.
- Even after 5000 revs, cluster remain in **same box**. Dispersion in a year measured in meters.





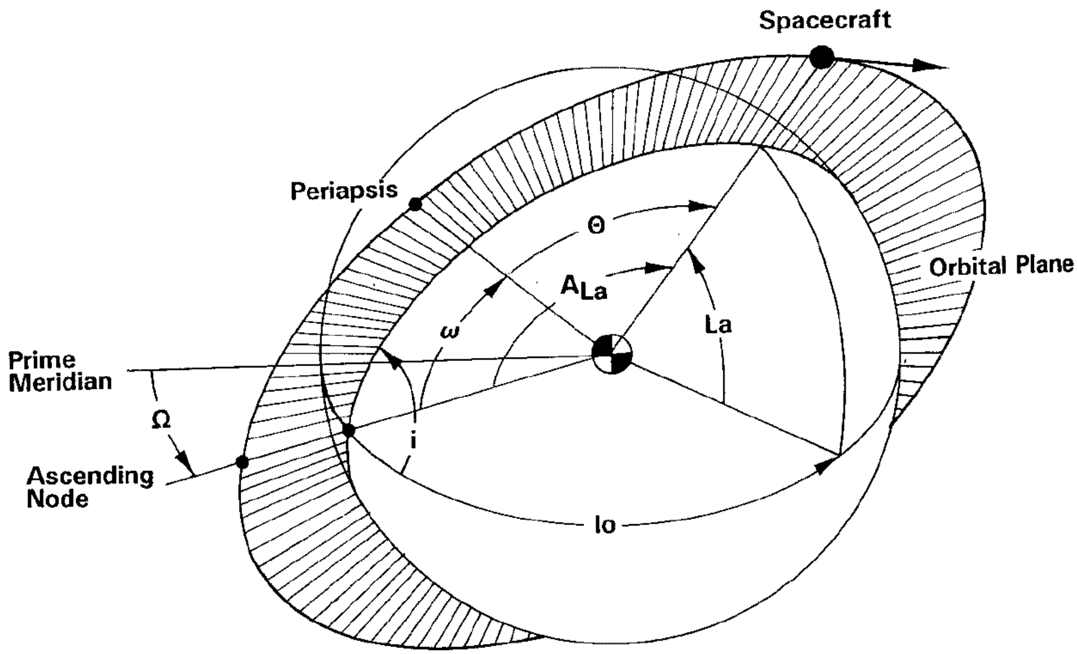
Why the Method Works So Well

- Recall budge of Earth (J_2 term) causes 3 **secular drifts**:
  - **regression of nodes** Ω (orbital plane)
  - **rotation of apsides** ω (semi-major axis)
  - **mean anomaly** θ (mean distance since last perigee passage)
- For tracking SC, should use **mean distance** to node $\omega + \theta$.



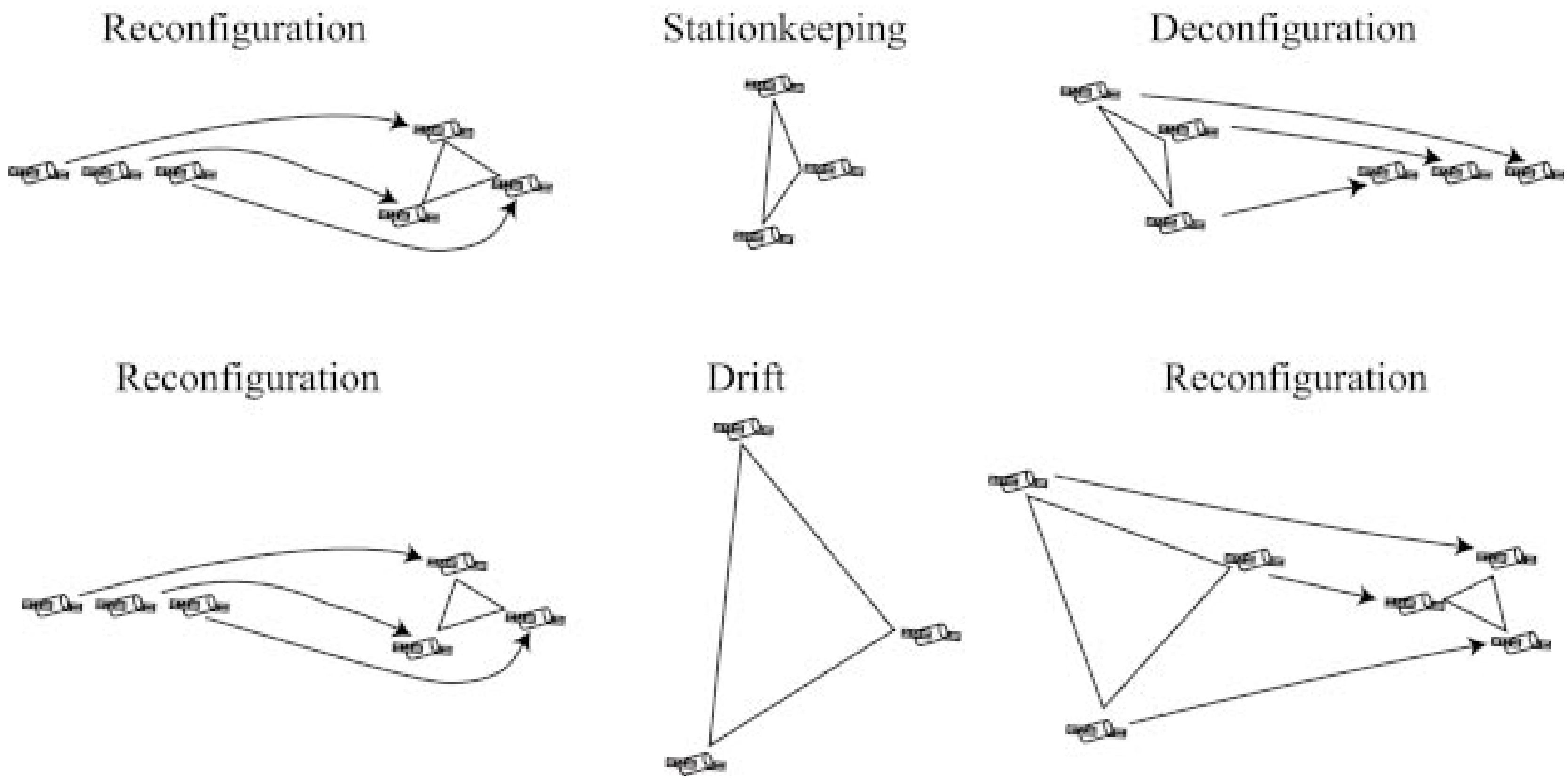
Why the Method Works So Well

- 3 main conditions for the cluster
  - **same energy** E
  - **same polar component** of angular momentum H_z
  - close to **almost-circular orbit** (nearly zero eccentricity)
- Using **Brouwer's theory**, 3 conditions imply microsats
  - same **drift rate** in node Ω
  - same **drift rate** in mean distance to node $\omega + \theta$



Conclusion and Future Work

- Use **dynamical systems** ideas to find orbits
  - **cluster remains close for many years, with very little dispersion, even with no controls.**
- Will couple with **optimal control** for **fuel-efficient** way in
  - formation **maintenance** and formation **reconfiguration.**



References and Other Informations

- **Koon, W., J. Marsden, J. Masdemont, R. Murray,** **J2 Dynamics and Formation Flight,** *Proceedings of AIAA Guidance, Navigation, and Control Conference, Montreal, Canada, August, 2001.* **AIAA 2001-4090.**
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