Invariant Manifolds, Spatial 3-Body Problem and Space Mission Design

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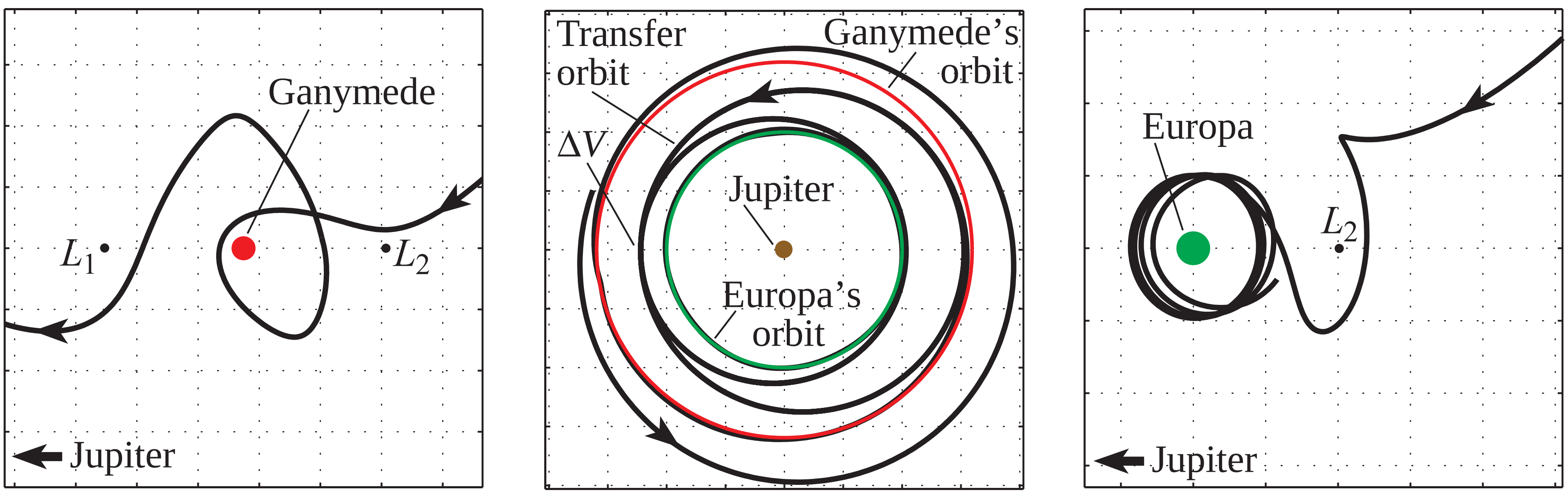
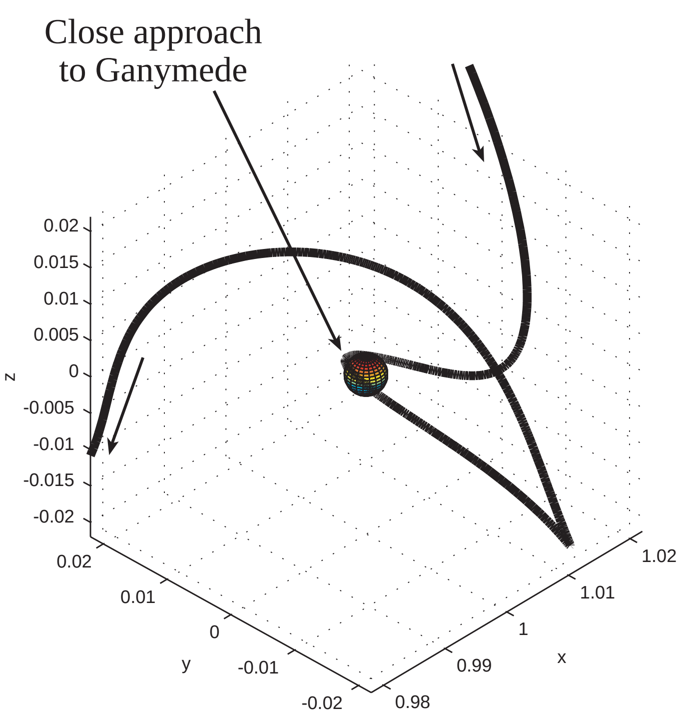
Petit Grand Tour of Jupiter's Moons (Planar Model)

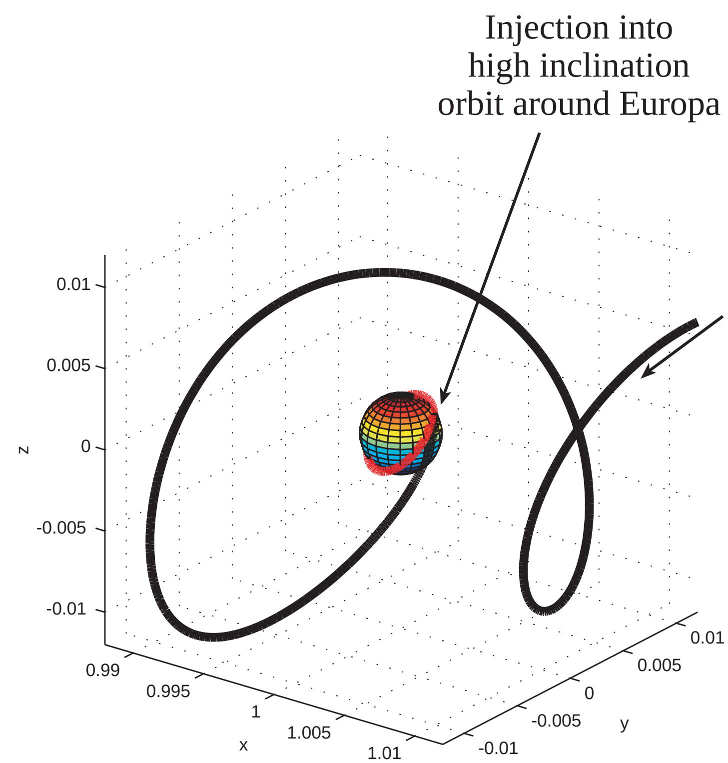
- Used **invariant manifolds** to construct trajectories with interesting characteristics:
  - Petit Grand Tour of Jupiter's moons. 1 orbit around **Ganymede**. 4 orbits around **Europa**.
  - A ΔV nudges the SC from **Jupiter-Ganymede** system to **Jupiter-Europa** system.
- Instead of **flybys**, can orbit several moons for **any duration**.



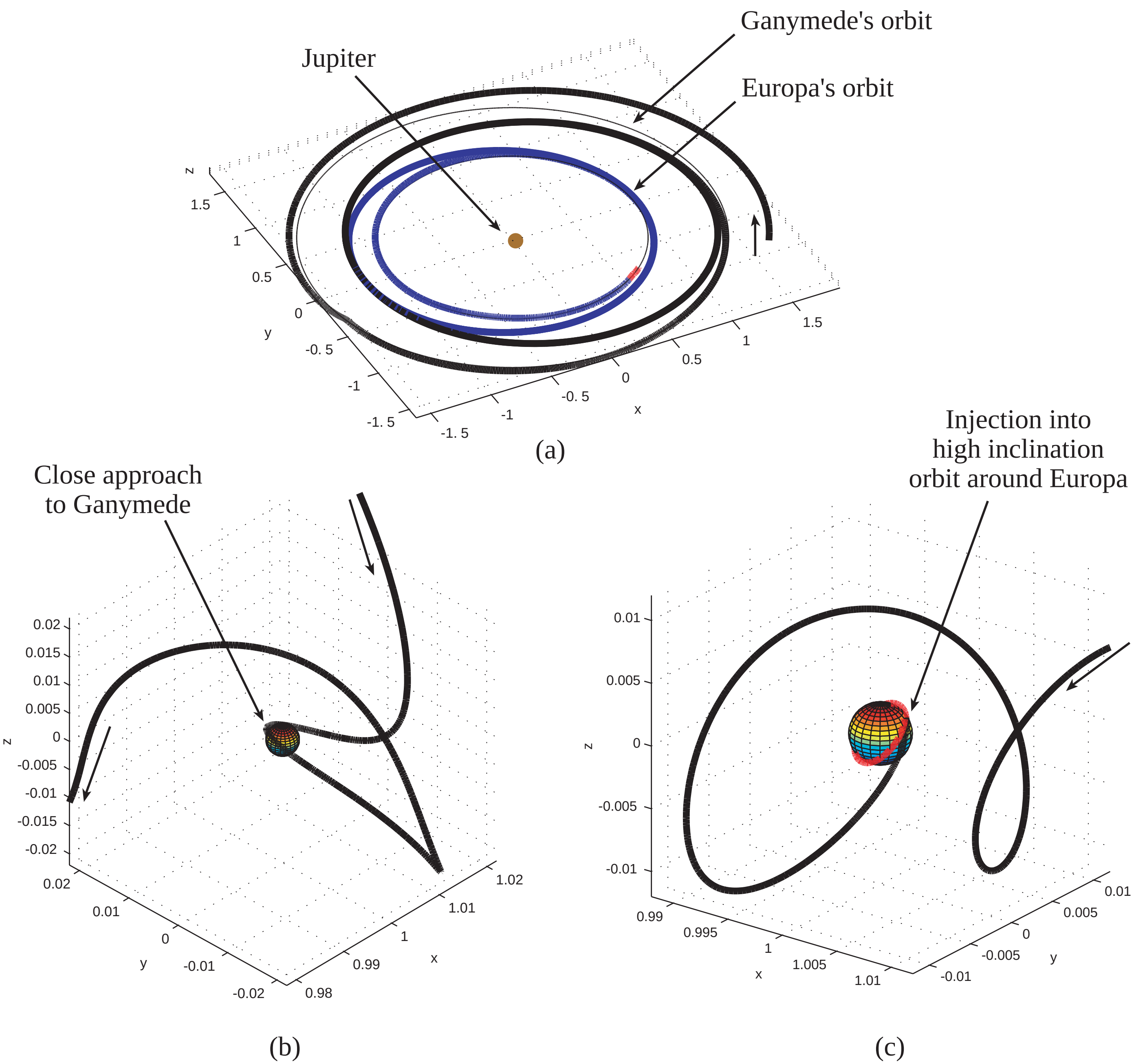
Extend from Planar Model to Spatial Model

- Previous work based on **planar** 3-body problem.
- Future missions will require **3D** capabilities.
  - Europa Orbiter mission needs a capture into a **high inclination orbit** around Europa.
- Current study has extended from **planar** to **spatial model**.



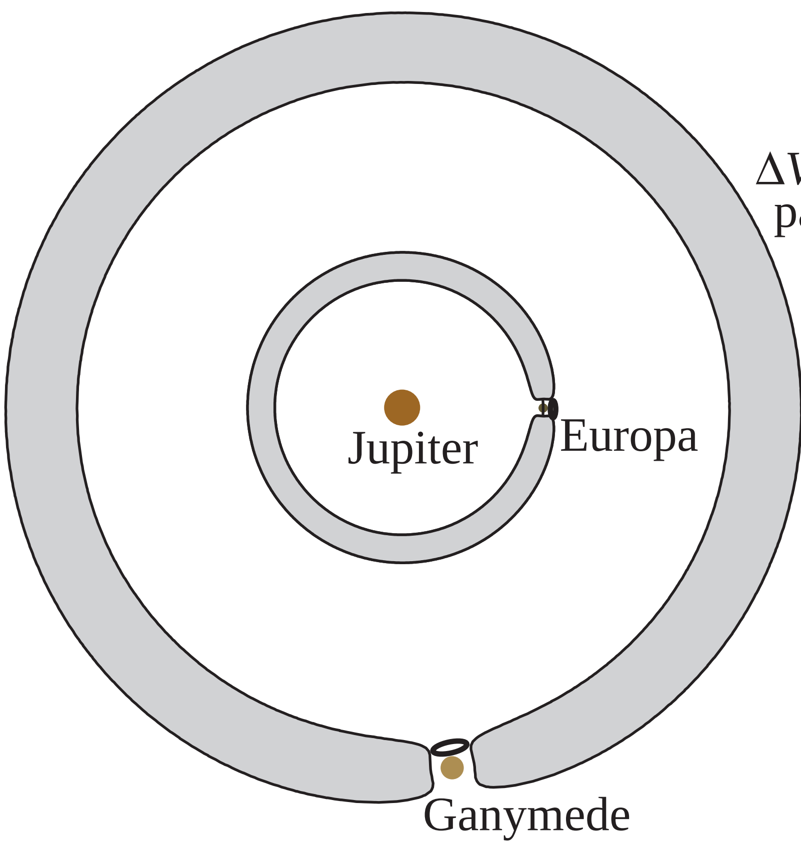


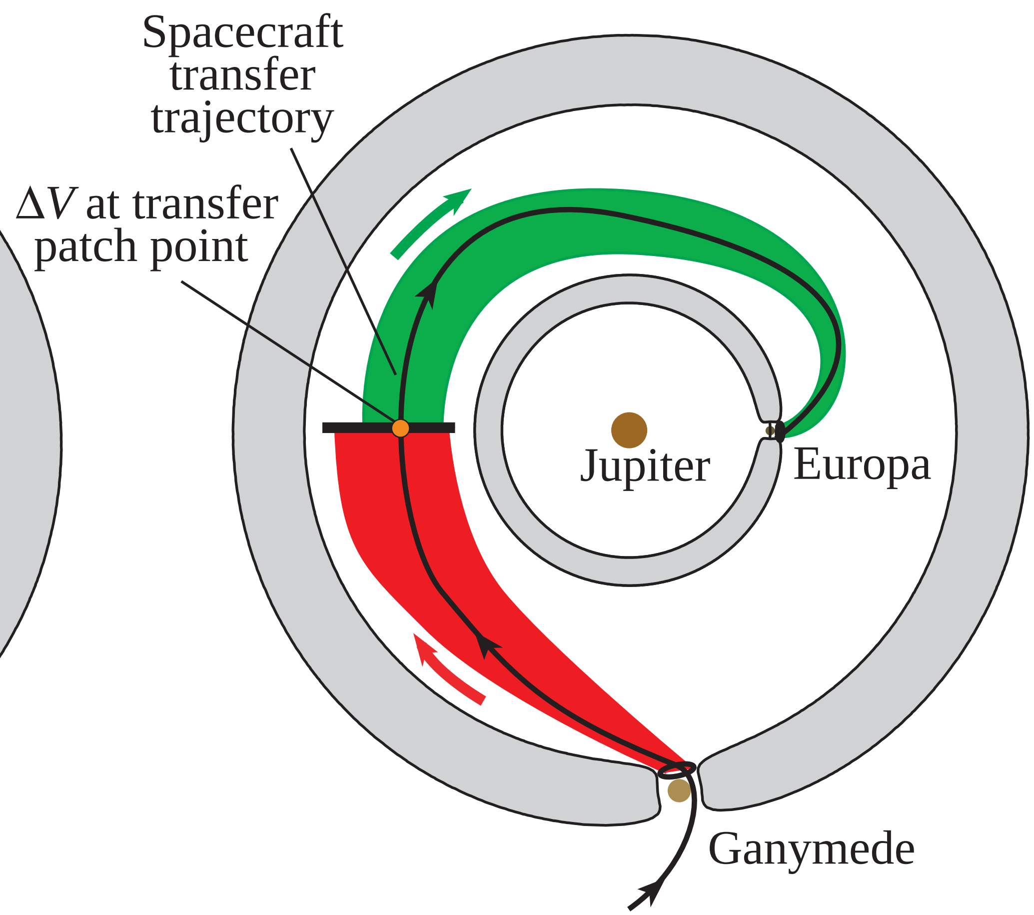
Extend from Planar Model to Spatial Model



Petit Grand Tour of Jupiter's Moons

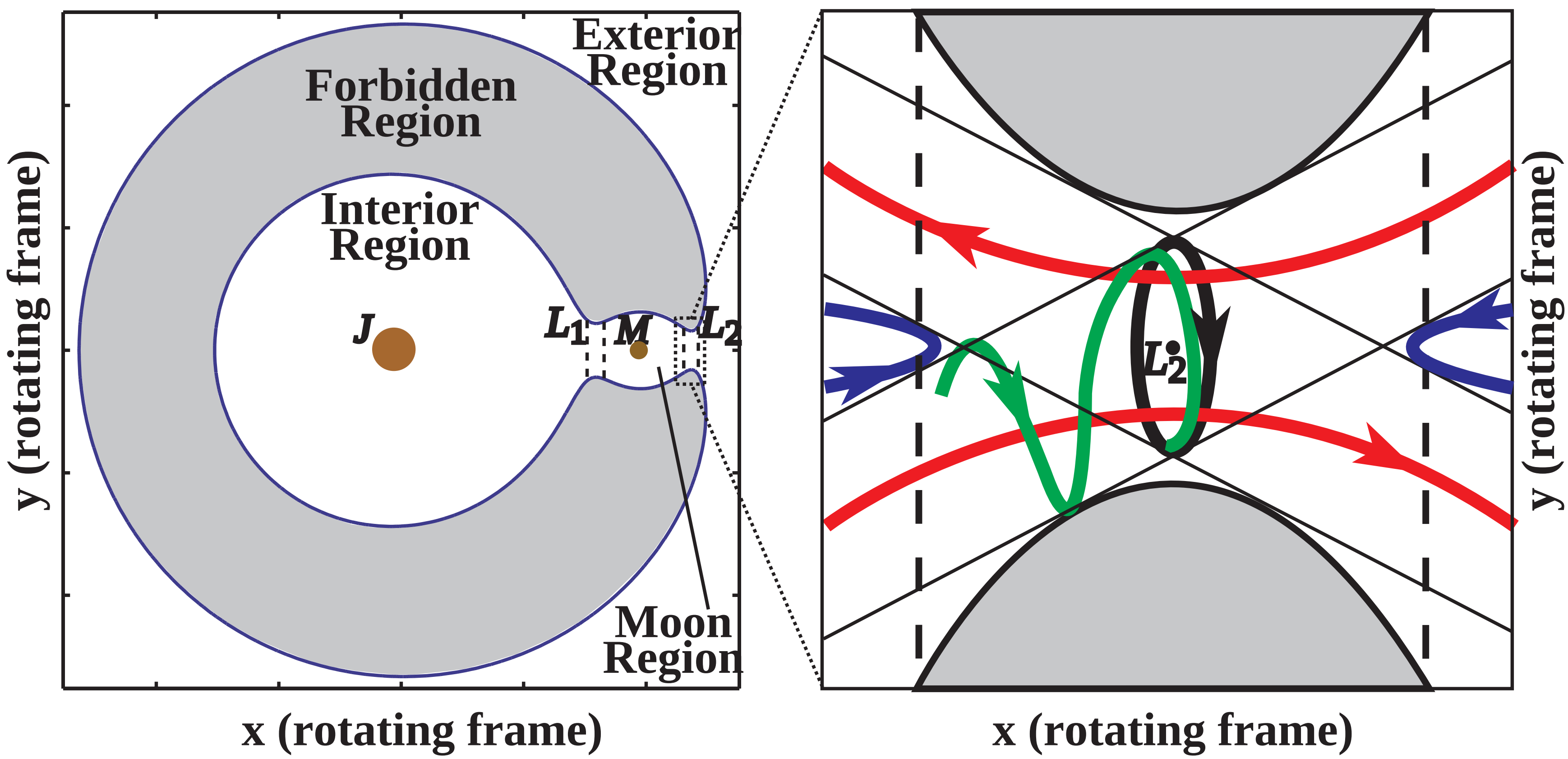
- Jupiter-Ganymede-Europa-SC 4-body system approximated as 2 **coupled 3-body systems**
- **Invariant manifold tubes** of **spatial** 3-body systems are linked in right order to construct orbit with desired itinerary.
- Initial solution refined in **4-body model**.





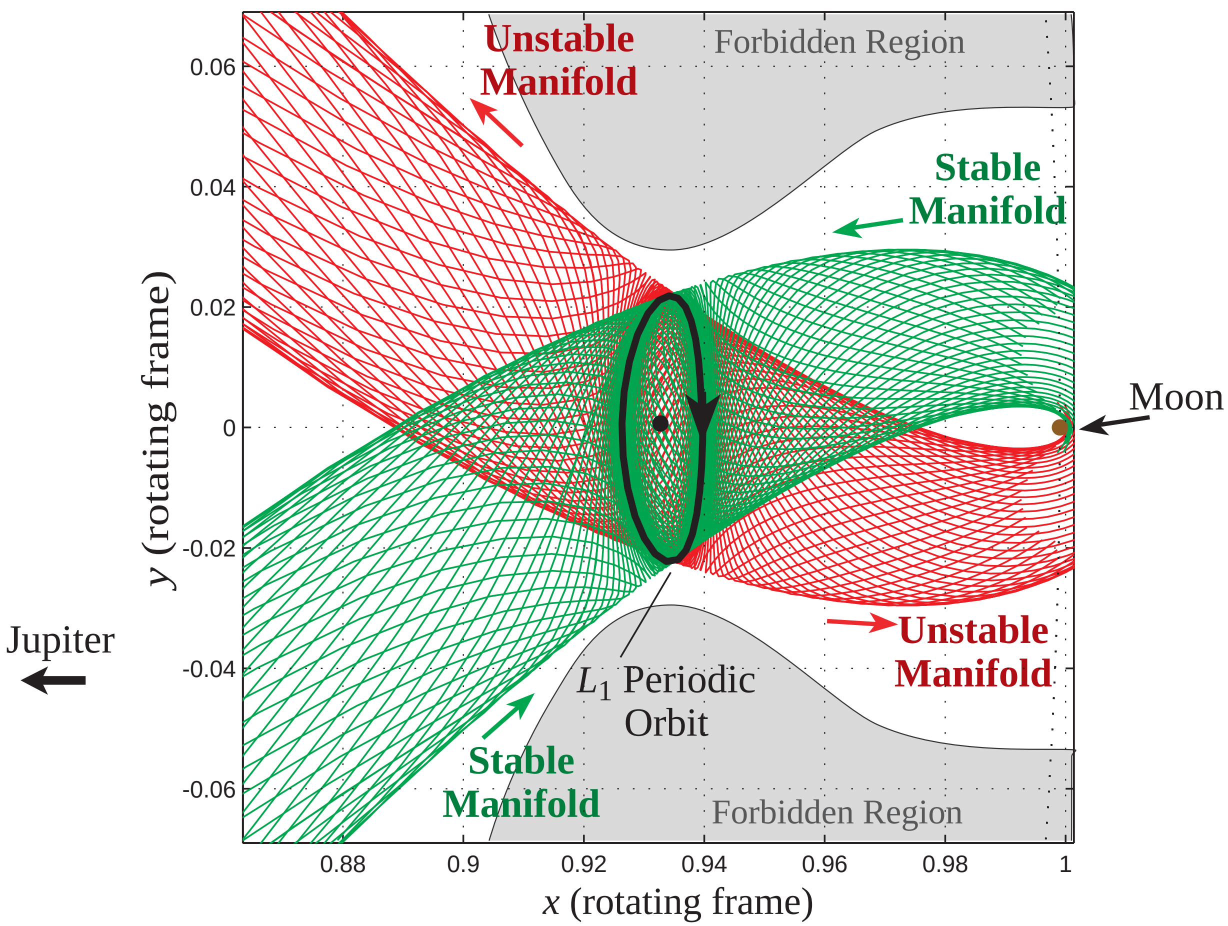
Planar Restricted 3-Body Problem

- **Recall:** for **energy value** just above that of L_2, **Hill's region** contains a **"neck"** about L_1 & L_2.
- Dynamics in each equilibrium region: **saddle x center**.
- 4 types of orbits: **periodic**, **asymptotic**, **transit** & **nontransit**.



Planar: Invariant Manifold as Separatrix

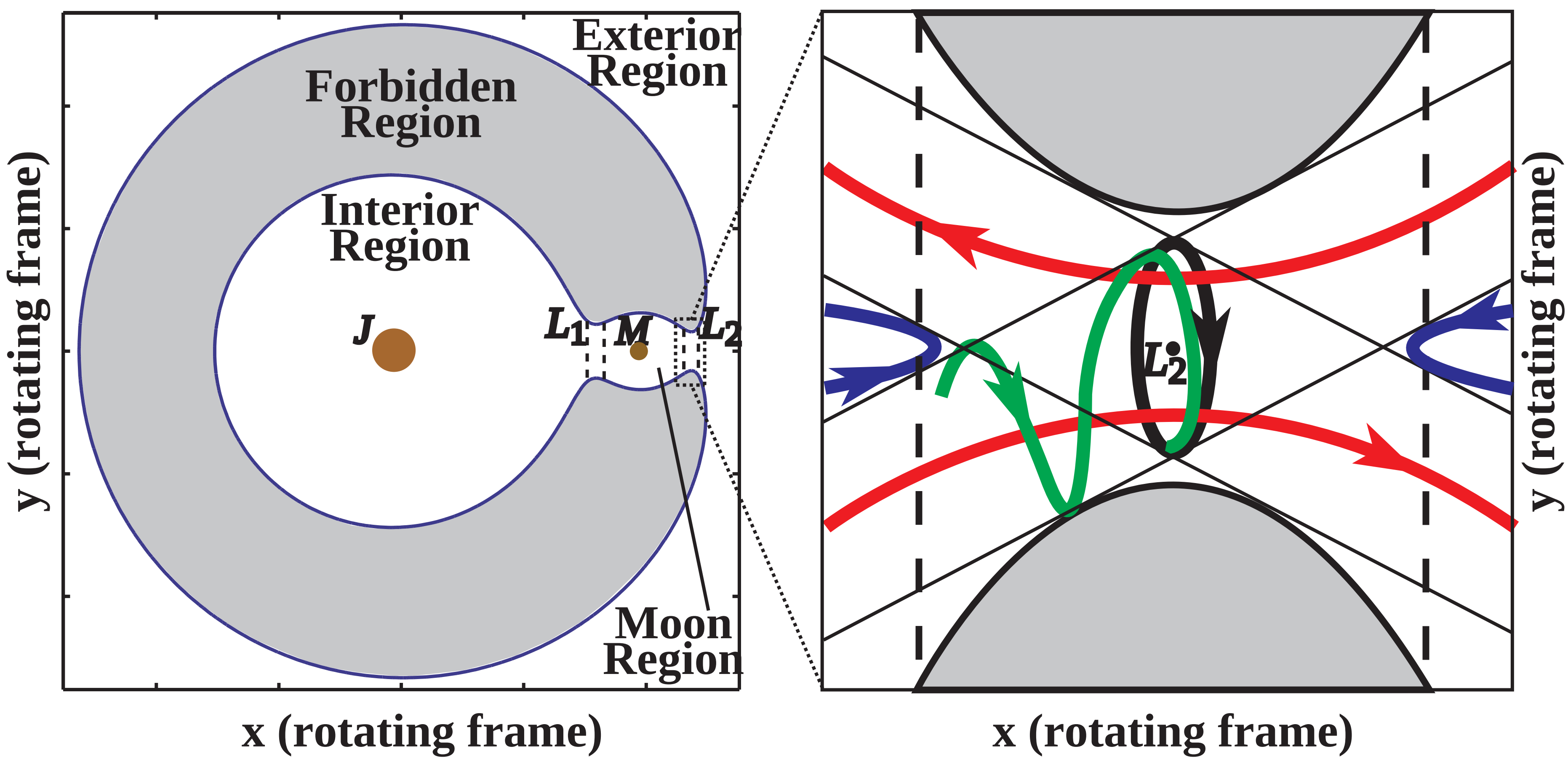
- Asymptotic orbits form **2D invariant manifold tubes** in **3D energy surface**.
- They separate transit and non-transit orbits:
  - **Transit orbits** are those inside the tubes.
  - **Non-transit orbits** are those outside the tubes.



■ Spatial Restricted 3-Body Problem (CR3BP)

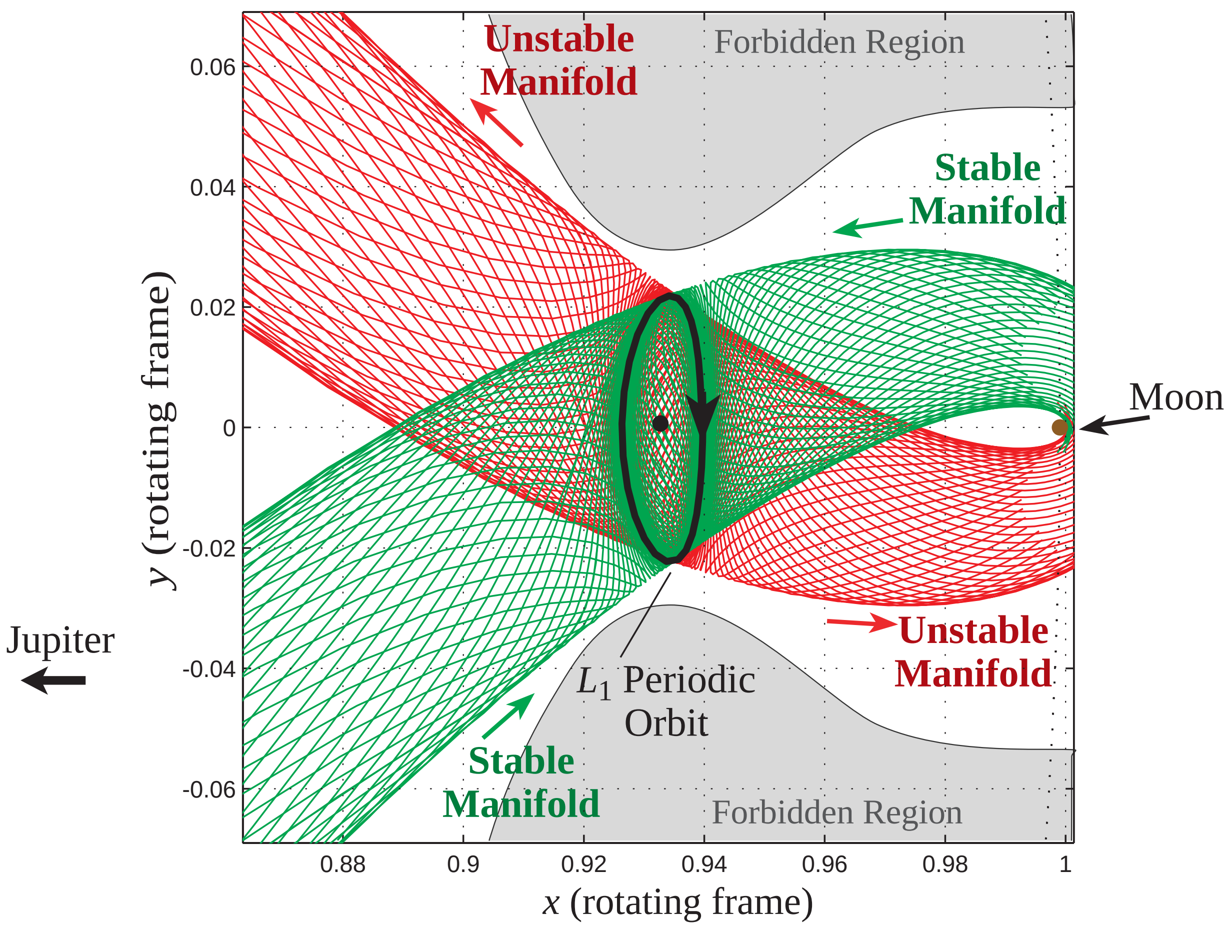
▶ Dynamics near equilibrium point: **saddle x center x center**.

- **bounded orbits** (periodic/quasi-periodic): S^3 (3-sphere)
- **asymptotic orbits** to 3-sphere: $S^3 \times I$ (**"tubes"**)
- **transit** and **nontransit** orbits.



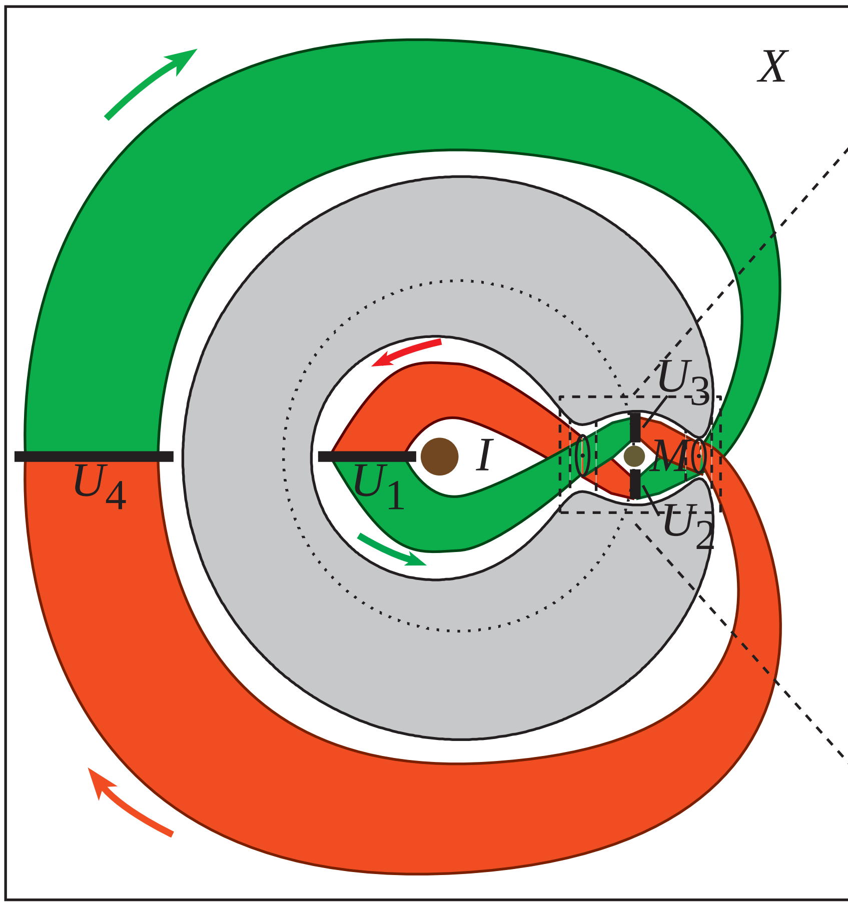
Spatial: Invariant Manifold as Separatrix

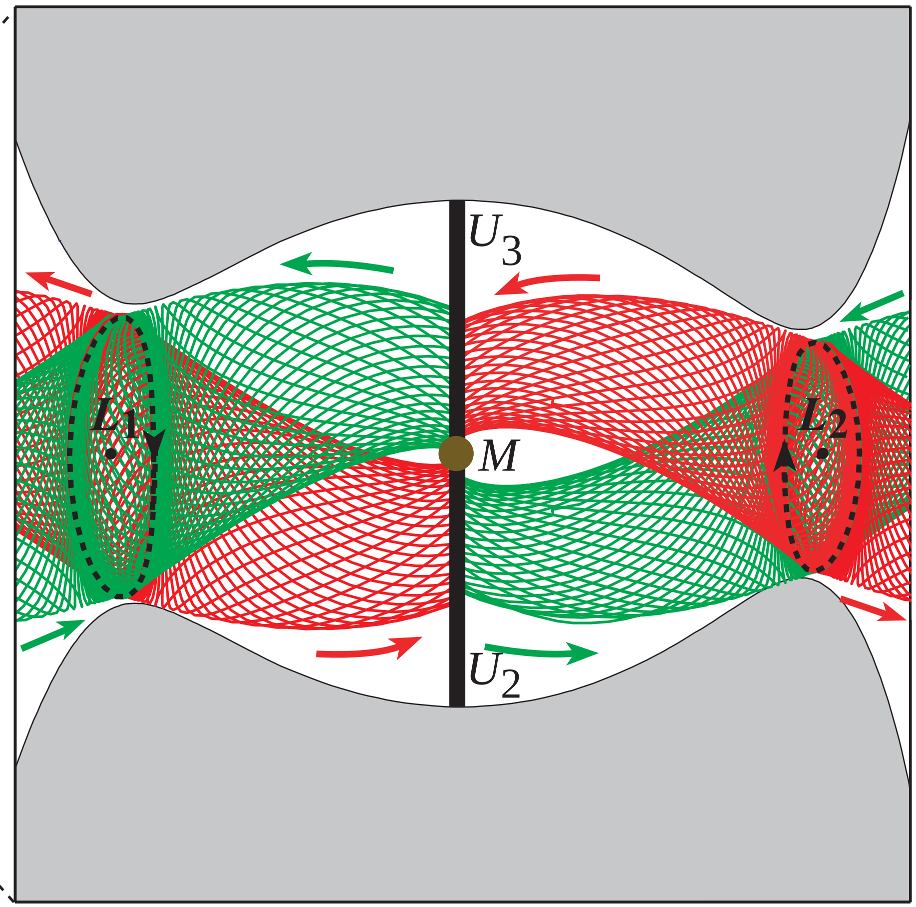
- Asymptotic orbits form **4D invariant manifold "tubes"** $(S^3 \times I)$ in **5D energy surface**.
- They separate transit and non-transit orbits:
  - **Transit orbits** are those inside the "tubes".
  - **Non-transit orbits** are those outside the "tubes".



Construct Orbit with Desired Itinerary

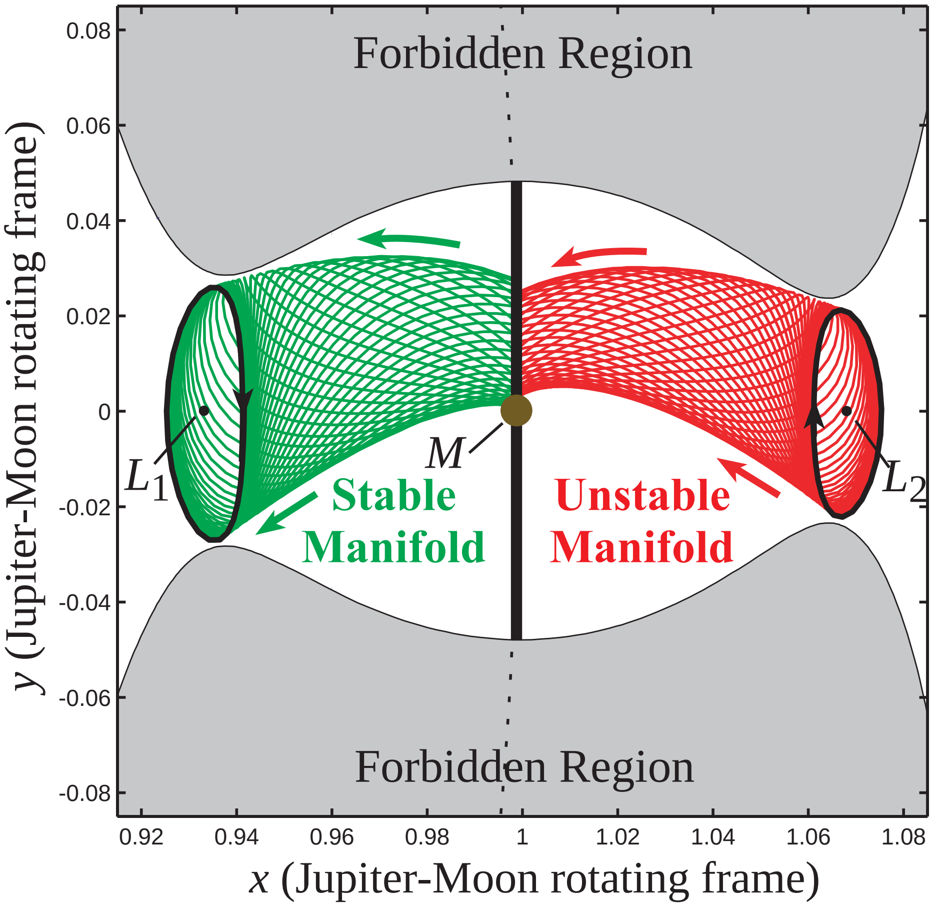
- How to **link** invariant manifold tubes to construct orbit with **desired itinerary**.
- Construction of $(X; M, I)$ orbit.

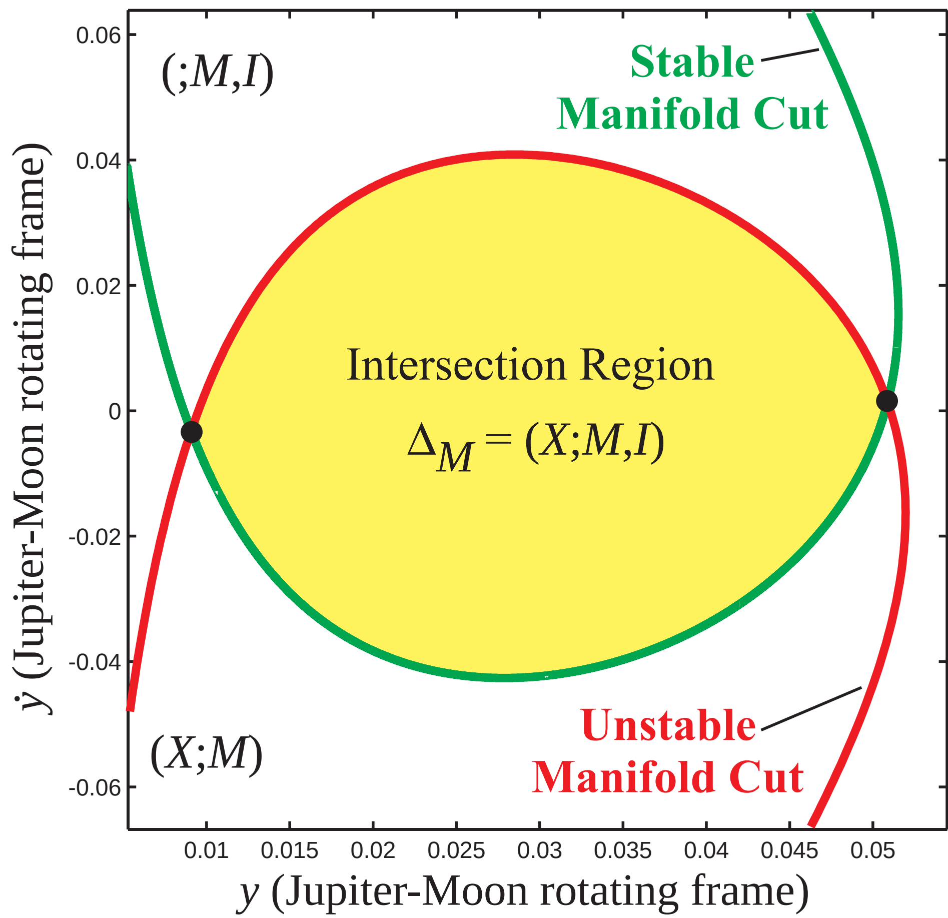




Planer: Construction of $(\mathrm{X}; \mathrm{M}, \mathrm{I})$ Orbits

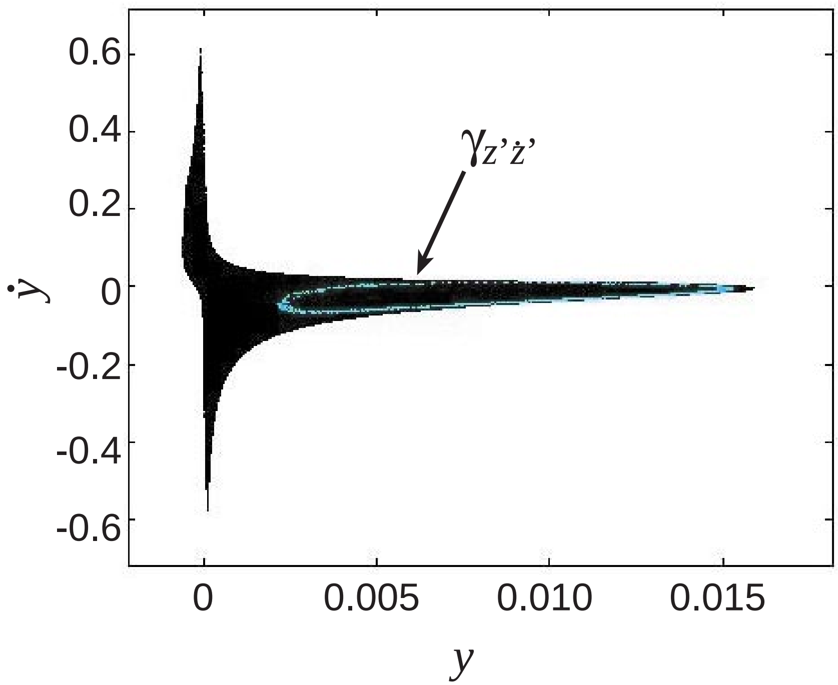
- Invariant mfd. **tubes** ($S \times I$) separate transit/nontransit orbits.
- **Red curve** (S^1) (Poincaré cut of L_2 **unstable manifold**).
  Green curve (S^1) (cut of L_1 **stable manifold**).
- Any point inside the intersection region Δ_M is a $(X; M, I)$ orbit.

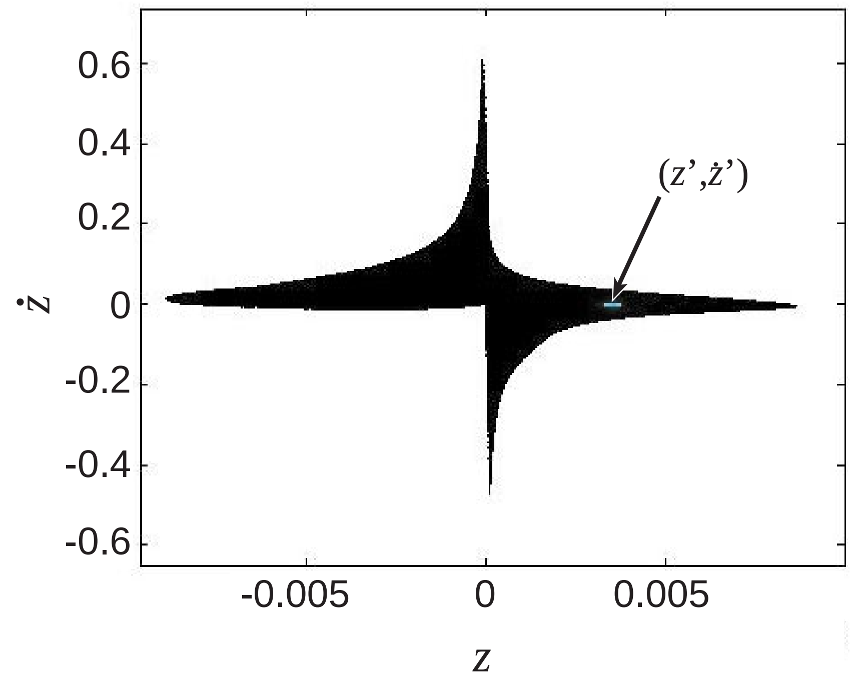




Spatial: Construction of $(X; M, I)$ orbits

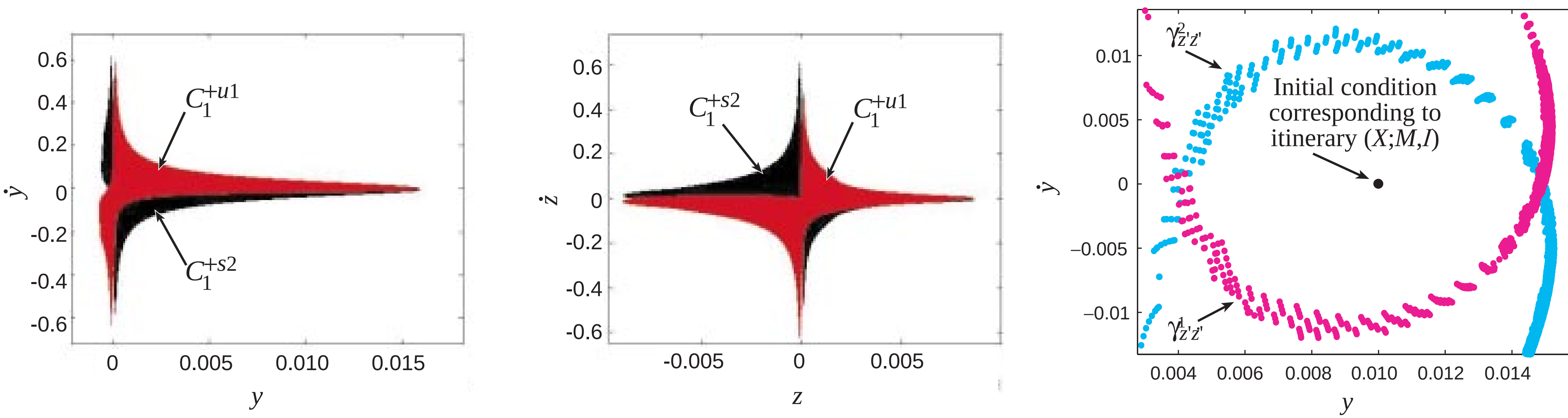
- Invariant manifold **tubes**: $(S^3 \times I)$.
- Poincaré **cut** is a topological **3-sphere** S^3 in $\mathbb{R}^4$.
  - S^3 looks like **disk x disk**: $\xi^2 + \dot{\xi}^2 + \eta^2 + \dot{\eta}^2 = r^2 = r_\xi^2 + r_\eta^2$
- If $z = c, \dot{z} = 0$, its **projection** on $(y, \dot{y})$ **plane** is a **curve**.
- Any point inside this **curve** is a $(X; M)$ orbit.



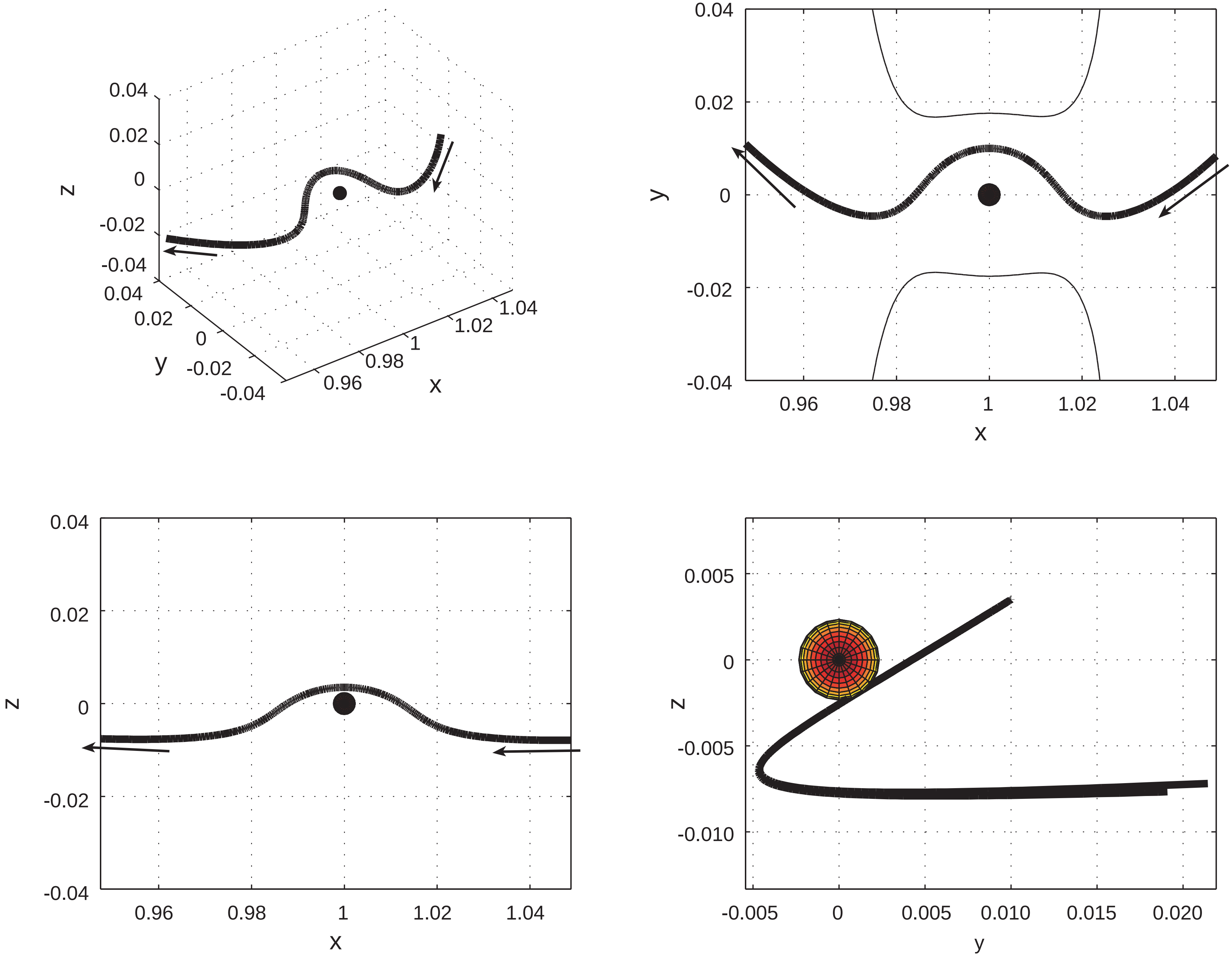


Spatial: Construction of $(\mathrm{X};\mathrm{M},\mathrm{I})$ orbits

- Similarly, while **cut** of stable manifold **"tube"** is a S^3, its **projection** on $(y,\dot{y})$ **plane** is a **curve** for $z=c, \dot{z}=0$.
- Any point inside this **curve** is a (M,I) orbit.
- Hence, any point inside the **intersection region** Δ_M is a $(X;M,I)$ orbit.

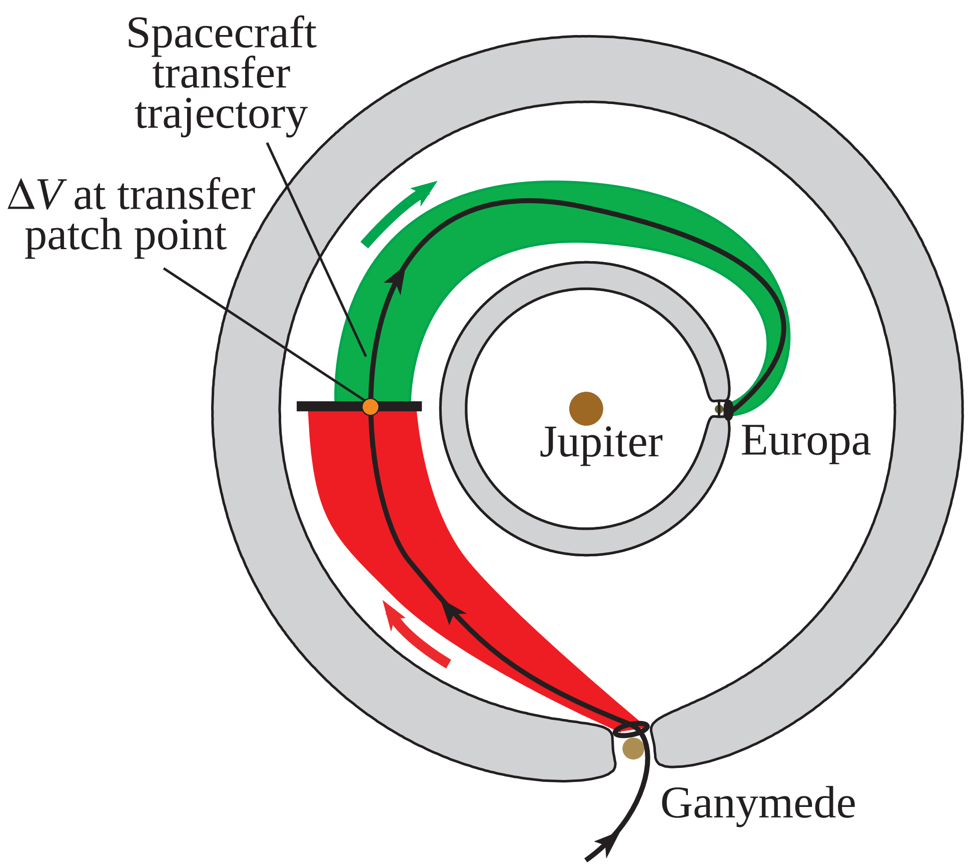


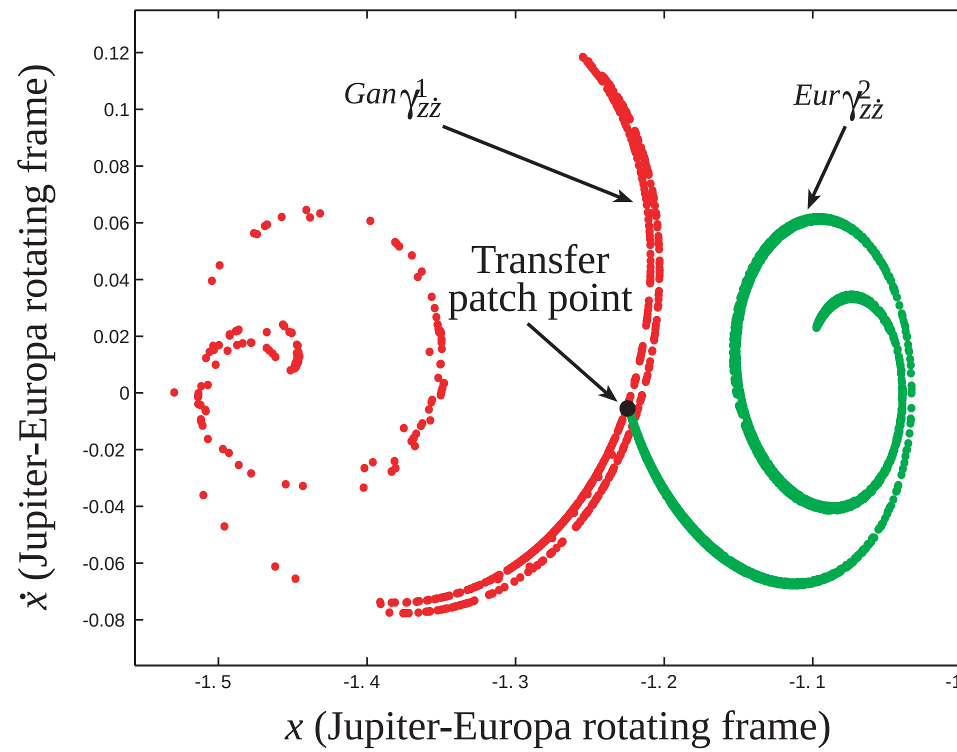
Spatial: Construction of $(X; M, I)$ orbits



Petit Grand Tour of Jupiter's Moon

- Petit Grand Tour can be constructed similarly
  - Approximate 4-body system as 2 nested **3-body systems**.
  - Choose appropriate **Poinaré section**.
  - Link invariant **manifold tubes** in right order.
  - Refine initial solution in **4-body model**.





Conclusion and Future Work

- In our study of **spatial** CR3BP:
  - Invariant manifolds still act as **separtrices**.
  - Construct orbit with **prescribed itinerary**.
  - Construct **Petit Grand Tour** of Jupiter's moons that ends in a **high inclination** orbit around Europa.
- To construct useful trajectories in **Sun-Earth-Moon** system.