Libration Point Orbits and Applications
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INVARIANT MANIFOLDS, THE SPATIAL THREE–BODY PROBLEM AND PETIT GRAND TOUR OF JOVIAN MOONS

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The invariant manifold structures of the collinear libration points particular, the stable and unstable invariant manifold "tubes" associated to for the spatial restricted three-body problem provide the framework for understanding complex dynamical phenomena from a geometric point of view. In libration point periodic orbits are phase space structures that provide a conduit for orbits between the primary bodies in separate three-body systems. These invariant

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manifold tubes can be used to construct new spacecraft trajectories, such as a "Petit Grand Tour" of the moons of Jupiter. Previous work focused on the planar circular restricted three-body problem. The current work extends the results to the spatial case.

1. Introduction

New space missions are increasingly more complex, requiring new and unusual kinds of orbits to meet their scientific goals, orbits which are not easily found by the traditional conic approach. The delicate heteroclinic dynamics employed by the *Genesis Discovery Mission* dramatically illustrates the need for a new paradigm: study of the three-body problem using dynamical systems theory as laid out by Poincaré.[2,3,4]

It appears that the dynamical structures of the three-body problem (such as stable and unstable manifolds, and bounding surfaces), reveal much about the morphology and transport of particles within the solar system, whether they are asteroids, dust grains, or spacecraft. The cross-fertilization between the study of the natural dynamics in the solar system and engineering applications has produced a number of new techniques for constructing spacecraft trajectories with desired behaviors, such as rapid transition between the interior and exterior Hill's regions, resonance hopping, temporary capture, and collision.[5]

The invariant manifold structures associated to the collinear libration points for the restricted three-body problem, which exist for an interesting range of energies, provide a framework for understanding these dynamical phenomena from a geometric point of view. In particular, the stable and unstable invariant manifold tubes associated to L_1 and L_2 orbits are phase space structures that conduct particles to and from the smaller primary body (e.g., Jupiter in the Sun-Jupiter-comet three-body system), and between primary bodies for separate three-body systems (e.g., Saturn and Jupiter in the Sun-Saturn-comet and the Sun-Jupiter-comet three-body systems).[6]

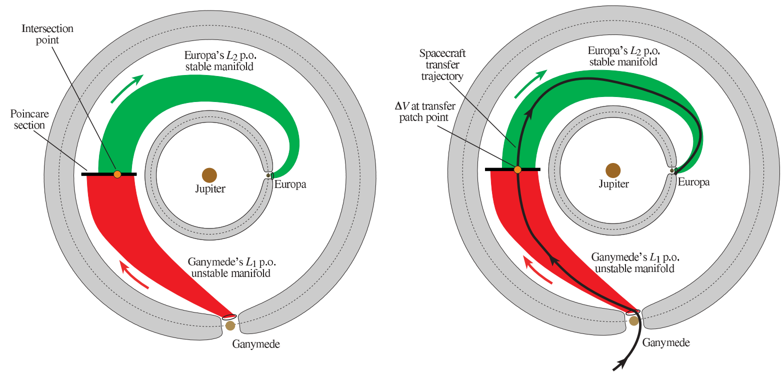
Furthermore, these invariant manifold tubes can be used to produce new techniques for constructing spacecraft trajectories with interesting characteristics. These may include mission concepts such as a low energy transfer from the Earth to the Moon[7] and a "Petit Grand Tour" of the moons of Jupiter.[8] Using the phase space tubes in each 3-body system, we were able to construct a transfer trajectory from the Earth which executes an unpropelled (i.e., ballistic) capture at the Moon. An Earth-to-Moon trajectory of this type, which utilizes the perturbation by the Sun, requires less fuel than the usual Hohmann transfer.[9]

Moreover, by decoupling the Jovian moon n-body system into several three-body systems, we can design an orbit which follows a prescribed itinerary in its visit to Jupiter's many moons. In an earlier study of a transfer from Ganymede to Europa,[8] we found our transfer ΔV to be half the Hohmann transfer value. As an example, we generated a tour of the Jovian moons: starting beyond Ganymede's orbit, the spacecraft is ballistically captured by Ganymede, orbits it once and escapes, and ends in a ballistic capture at Europa. One advantage of this Petit Grand Tour as compared with the Voyager-type flybys is the "leap-frogging" strategy. In this new approach to space mission design, the spacecraft can circle a moon in a loose temporary capture orbit for a desired number of orbits, perform a transfer ΔV and become ballistically captured by another adjacent moon for some number of orbits, etc. Instead of flybys lasting only seconds, a scientific spacecraft can orbit several different moons for any desired duration.

The design of the Petit Grand Tour in the planar case is guided by two main ideas. First, the Jupiter-Ganymede-Europa-spacecraft four-body system is approximated as two coupled planar three-body systems. Then, as shown in Figure 1, the invariant manifold tubes of the two planar three-body systems are used to construct an orbit with the desired behaviors. This initial solution is then refined to obtain a trajectory in a more accurate 4-body model.

The ***coupled* 3*-body model*** considers the two adjacent moons competing for control of the same spacecraft as two nested 3-body systems (e.g., Jupiter-Ganymede-spacecraft and Jupiter-Europa-spacecraft). When close to the orbit of one of the moons, the spacecraft's motion is dominated by the 3-body dynamics of the corresponding planet-moon system. Between the two moons, the spacecraft's motion is mostly planet-centered Keplerian, but is precariously poised between two competing 3-body dynamics. In this region, orbits connecting unstable libration point orbits of the two different 3-body systems may exist, leading to complicated transfer dynamics between the two adjacent moons. We seek intersections between invariant manifold tubes which connect the capture regions around each moon. In the planar case, these tubes separate transit orbits (inside the tube) from non-transit orbits (outside the tube). They are the phase space structures that provide a conduit for orbits between regions within each three-body systems as well as between primary bodies for separate three-body systems.[5]

Extending Results from Planar Model to Spatial Model. Previous work based on the planar circular restricted three-body problem (PCR3BP) revealed the basic structures controlling the dynamics.[5,6,7,8] But current missions (such as Genesis[2]) and future missions will require three-dimensional capabilities, such as control of the latitude and longitude of a spacecraft's escape from and entry into a planetary or moon orbit. For example, the proposed Europa Orbiter mission desires a capture into a high inclination polar orbit around Europa. Three-dimensional capability is also required when decomposing an n-body system into three-body systems that are not co-planar, such as the Earth-Sun-spacecraft and Earth-Moon-spacecraft systems. These demands necessitate the extension of earlier results to the spatial model (CR3BP).



(a) (b)

Fig. 1. **The Coupled 3-Body Model.** (a) Find an intersection between dynamical channel enclosed by Ganymede's L_1 periodic orbit unstable manifold and dynamical channel enclosed by Europa's L_2 periodic orbit stable manifold (shown in schematic). (b) Integrate forward and backward from patch point (with ΔV to take into account velocity discontinuity) to generate desired transfer between the moons (schematic).

In our current work on the spatial three-body problem,[1] we are able to show that the invariant manifold structures of the collinear libration points still act as the separatrices for two types of motion, those inside the invariant manifold "tubes" are transit orbits and those outside the "tubes" are non-transit orbits. We have also designed an algorithm for constructing orbits with any prescribed itinerary and obtained some initial results on the basic itinerary. Furthermore, we have applied the new techniques to the construction of a three dimensional Petit Grand Tour of the Jovian moon system. By approximating the dynamics of the Jupiter-Europa-Ganymede-spacecraft 4-body problem as two 3-body subproblems, we seek intersections between the channels of transit orbits enclosed by the stable and unstable manifold tubes of different moons. In our example, we have designed a low energy transfer trajectory from Ganymede to Europa that ends in a high inclination orbit around Europa. See Figure 2.

Focus of this Paper. In this paper, we will mainly focus on the key ideas that lead to the construction of the Petit Grand Tour. For more details of this work, the reader can consult our full paper published in *Advances in the Astronautical Sciences.*[1]

2. Invariant Manifold as Separatrix

Review of Planar Case. Recall that in the planar Jupiter-Moon-spacecraft 3-body system (PCR3BP), for an energy value just above that of L_2, the Hill's region contains a "neck" about L_1 and L_2 and the spacecraft can make transition through these necks. More precisely, in each equilibrium region around L_1 and L_2, the dynamics of the spacecraft is of the form saddle and center and there exist 4 types of orbits: [10,11]

(1) an unstable periodic orbit (black oval);
(2) four cylinders of asymptotic orbits that wind onto or off this periodic orbit; they form pieces of stable and unstable manifolds;
(3) transit orbits which the spacecraft can use to make a transition from one region to the other; for example, passing from the exterior region (outside moon's orbit) into the moon temporary capture region (bubble surrounding moon) via the neck region;

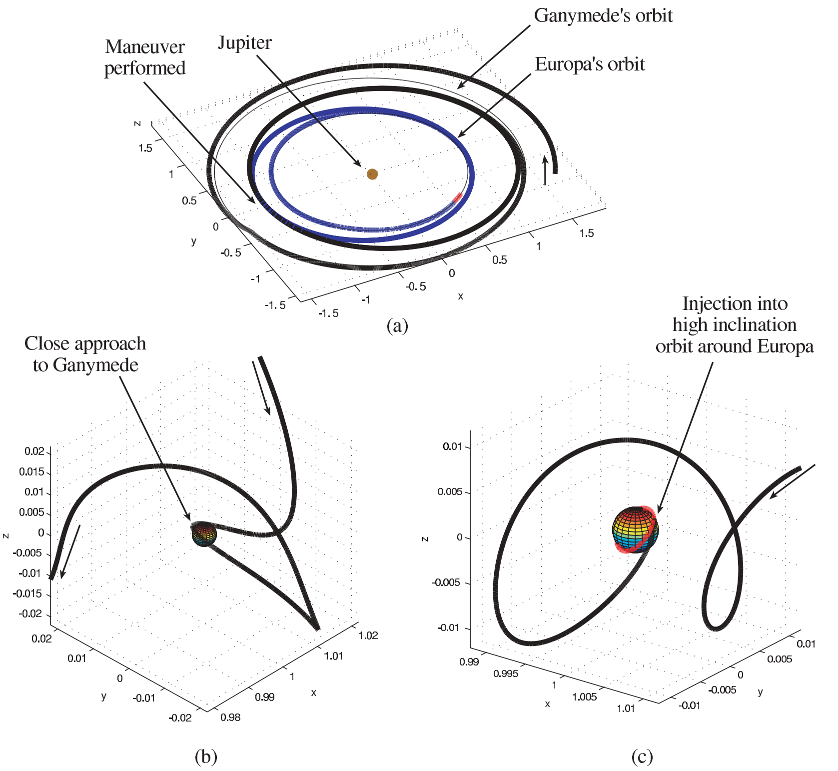


Fig. 2. The three dimensional Petit Grand Tour space mission concept for the Jovian moons. (a) We show a spacecraft trajectory coming into the Jupiter system and transferring from Ganymede to Europa using a single impulsive maneuver, shown in a Jupiter-centered inertial frame. (b) The spacecraft performs one loop around Ganymede, using no propulsion at all, as shown here in the Jupiter-Ganymede rotating frame. (c) The spacecraft arrives in Europa's vicinity at the end of its journey and performs a final propulsion maneuver to get into a high inclination circular orbit around Europa, as shown here in the Jupiter-Europa rotating frame.

(4) nontransit orbit where the spacecraft bounces back to its original region.

Furthermore, these two-dimensional tubes partition the three-dimensional energy manifold and act as separatrices for the flow through the equilibrium region: those inside the tubes are transit orbits and those outside the tubes are non-transit orbits. For example in the Jupiter-moon system, for a spacecraft to transit from outside the moon's orbit to the

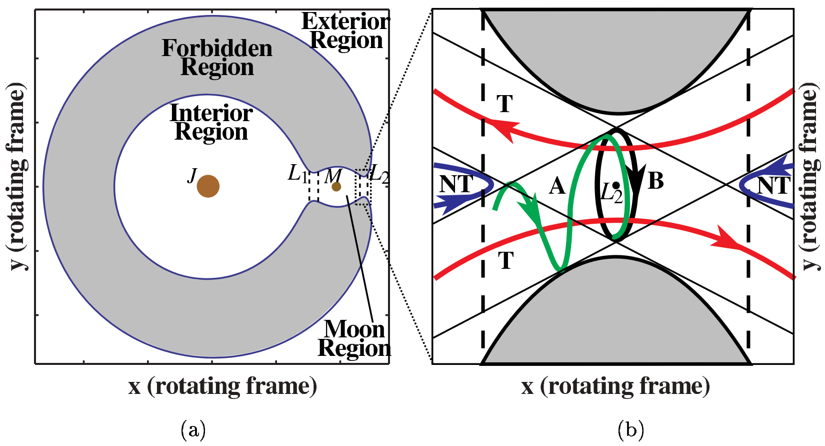


Fig. 3. (a) Hill's region (schematic, the region in white), which contains a "neck" about L_1 and L_2. (b) The flow in the region near L_2, showing a periodic orbit around L_2, an asymptotic orbit winding onto the periodic orbit, two transit orbits and two non-transit orbits. A similar figure holds for the region around L_1.

moon capture region, it is possible *only* through the L_2 periodic orbit stable manifold tube. Hence, stable and unstable manifold tubes control the transport of material to and from the capture region.

Results of the Spatial Case. This planar result generalizes readily to the spatial case.[12] For the dynamics near the equilibrium point, instead of the form saddle and center, we have saddle, center, and center. The last part corresponds to the harmonic motion in the z-direction. Since it is more difficult to draw spatial figures, we will still use the planar case to do the illustration. Again, there are 4 types of orbits, as depcited in Figure 4:

(1) a large number of bounded orbits, both periodic and quasi-periodic, which together form a 3-sphere, i.e., instead of a periodic orbit S^1 in the planar case, you have a S^3 of bounded orbits in the spatial case; it is an example of a *normally hyperbolic invariant manifold* (NHIM)[13] where the stretching and contraction rates under the linearized dynamics transverse to the 3-sphere dominate those tangent to the 3-sphere;
(2) four cylinders of asymptotic orbits that wind onto and off this 3-sphere; the only difference from the planar case is that, instead of two-dimensional invariant manifold tubes, one has four-dimensional in-

variant manifold tube: $S^3 \times \mathbb{R}$;
(3) transit and nontransit orbits.

Now, since the invariant manifold tubes are four-dimensional tubes in a five-dimensional energy manifold, they again act as separatrices for the flow through the equilibrium region: those inside the tubes are transit orbits and they transit from one region to another; those outside the tubes are non-transit orbits and they bounce back to their original region.

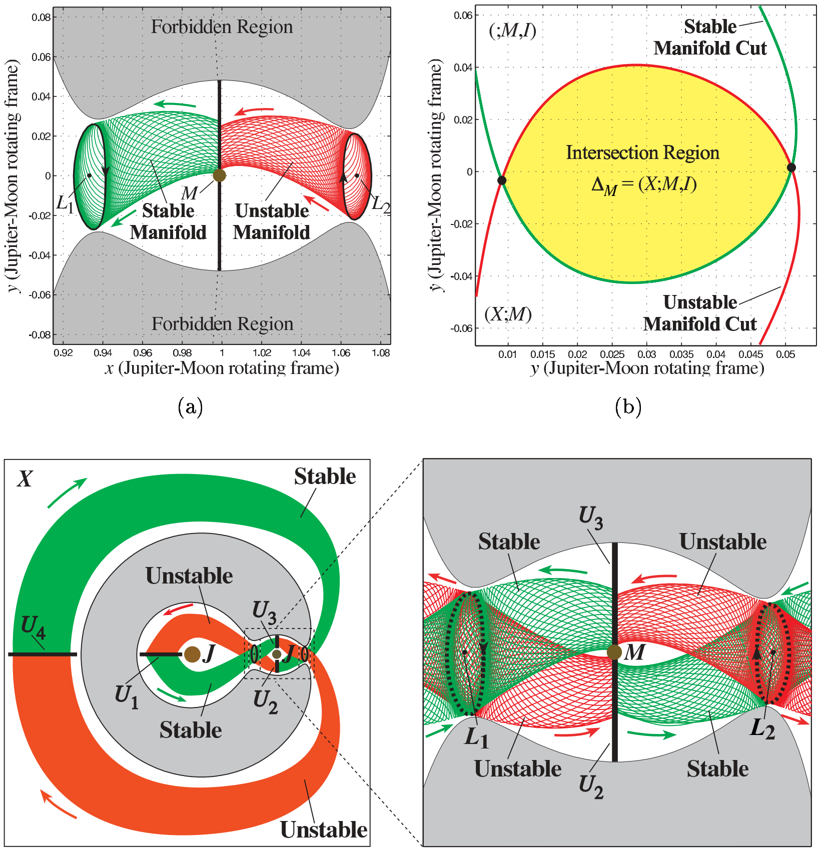
In fact, it can be shown that for a energy value just above that of L_1 (L_2), the nonlinear dynamics in the equilibrium region $\mathcal{R}_1$ ($\mathcal{R}_2$) is qualitatively the same as the linearized picture that we have shown above.[14,15,16,17,18,19] This geometric insight will be used below to guide our numerical explorations in constructing orbits with prescribed itineraries.

3. Constructing Orbits with Desired Itinerary

A key difficulty in the spatial case is to figure out how to link appropriate invariant manifold tubes together to construct orbit that visits the desired regions in a desired order.

Review of Planar Case. In the planar case, it is quite straightforward. Let us take constructing an $(X; M, I)$ orbit as an example. This orbit goes from the exterior region (X) to the interior region (I) passing through the moon region (M). Recall that for the planar case: the invariant manifold tubes separate two types of motion. The orbits inside the tube transit from one region to another; those outside the tubes bounce back to their original region.

Since the upper curve in Figure 4(b) is the Poincaré cut of the stable manifold of the periodic orbit around L_1 in the U_3 plane, a point inside that curve is an orbit that goes from the moon region to the interior region, so this region can be described by the label $(; M, I)$. Similarly, a point inside the lower curve of Figure 4(b) came from the exterior region into the moon region, and so has the label $(X; M)$. A point inside the intersection $\Delta_{\mathcal{M}}$ of *both* curves is an $(X; M, I)$ orbit, so it makes a transition from the exterior region to the interior region, passing through the moon region. Other more



(a) (b)

(c) (d)

Fig. 4. (a) The projection of invariant manifolds $W^{s,\mathcal{M}}_{L_1,\mathrm{p.o.}}$ and $W^{u,\mathcal{M}}_{L_2,\mathrm{p.o.}}$ in the region M of the position space. (b) A close-up of the intersection region between the Poincaré cuts of the invariant manifolds on the U_3 section ($x = 1 - \mu, y > 0$). (c) Location of Lagrange point orbit invariant manifold tubes in position space. Stable manifolds are lightly shaded, unstable manifolds are darkly. The location of the Poincaré sections (U_1, U_2, U_3, and U_4) are also shown. (d) A close-up near the moon.

complicated orbits can be constructed by choosing appropriate Poincare sections and linking invariant manifold tubes in right order.

Extension to Spatial Case. Since the key step in the planar case is to find the intersection region inside the two Poincaré cuts, a key difficulty is to determine how to extend this technique to the spatial case. Take as an example the construction of a transit orbit with the itinerary $(X; M, I)$ that goes from the exterior region to the interior region of the Jupiter-moon system. Recall that in the spatial case, the unstable manifold "tube" of the NHIM around L_2 which separates the transit and non-transit orbits is topologically $S^3 \times \mathbb{R}$. For a transversal cut at $x = 1 - \mu$ (a hyperplane through the moon), the Poincaré cut is a topological 3-sphere S^3 (in $\mathbb{R}^4$). It is not obvious how to find the intersection region inside these two Poincaré cuts (S^3) since both its projections on the $(y, \dot{y})$-plane and the $(z, \dot{z})$-plane are (2-dimensional) disks D^2. (One easy way to visualize this is to look at the equation: $\xi^2 + \dot{\xi}^2 + \eta^2 + \dot{\eta}^2 = r^2 = r_{\xi}^2 + r_{\eta}^2$. that describes a 3-sphere in $\mathbb{R}^4$. Clearly, its projections on the $(\xi, \dot{\xi})$-plane and the $(\eta, \dot{\eta})$-plane are 2-disks as r_ξ and r_η vary from 0 to r and from r to 0 respectively.)

However, in constructing an orbit which transitions from the outside to the inside of a moon's orbit, suppose that we might also want it to have other characteristics above and beyond this gross behavior. We may want to have an orbit which has a particular z-amplitude when it is near the moon. If we set $z = c, \dot{z} = 0$ where c is the desired z-amplitude, the problem of finding the intersection region inside two Poincaré cuts suddenly becomes tractable. Now, the projection of the Poincaré cut of the above unstable manifold tube on the $(y, \dot{y})$-plane will be a closed curve and any point inside this curve is a $(X; M)$ orbit which has transited from the exterior region to the moon region passing through the L_2 equilibrium region. See Figure 5.

Similarly, we can apply the same techniques to the Poincaré cut of the stable manifold tube to the NHIM around L_1 and find all (M, I) orbits inside a closed curve in the $(y, \dot{y})$-plane. Hence, by using z and $\dot{z}$ as the additional parameters, we can apply the similar techniques that we have developed for the planar case in constructing spatial trajectories with desired itineraries. See Figure 5(a).

4. Spatial Petit Grand Tour of Jovian Moons

We now apply the techniques we have developed to the construction of a fully three dimensional Petit Grand Tour of the Jovian moons, extending an earlier planar result.[8] We here outline how one systematically constructs

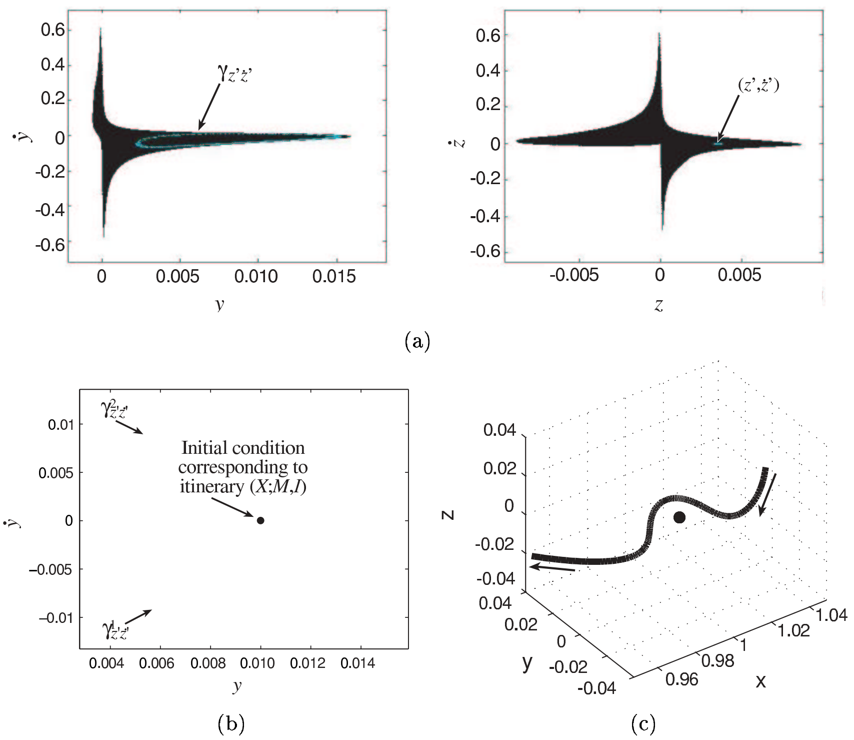


Fig. 5. (a) Shown in black are the $y\dot{y}$ (left) and $z\dot{z}$ (right) projections of the 3-dimensional object $\mathcal{C}_1^{+u2}$, the intersection of $W_+^u(\mathcal{M}_h^2)$ with the Poincaré section $x = 1 - \mu$. The set of points in the $y\dot{y}$ projection which approximate a curve, $\gamma_{z'\dot{z}'}$, all have $(z, \dot{z})$ values within the small box shown in the $z\dot{z}$ projection (which appears as a thin strip), centered on $(z', \dot{z}')$. This example is computed in the Jupiter-Europa system for $C = 3.0028$. (b) The curves ${}^{\mathrm{Gan}}\gamma^1_{z\dot{z}}$ and ${}^{\mathrm{Eur}}\gamma^2_{z\dot{z}}$ are shown, the intersections of ${}^{\mathrm{Gan}}W_+^u(\mathcal{M}^1)$ and ${}^{\mathrm{Eur}}W_+^s(\mathcal{M}^2)$ with the Poincaré section U_1 in the Jupiter-Europa rotating frame, respectively. Note the small region of intersection, $\mathrm{int}(\mathrm{Gan}\gamma^1_{z\dot{z}}) \cap \mathrm{int}(\mathrm{Eur}\gamma^2_{z\dot{z}})$, where the patch point is labeled. (c) The (X, M, I) transit orbit corresponding to the initial condition in (b). The orbit is shown in a 3D view. Europa is shown to scale.

a spacecraft tour which begins beyond Ganymede in orbit around Jupiter, makes a close flyby of Ganymede, and finally reaches a high inclination orbit around Europa, consuming less fuel than is possible from standard two-body methods.

Our approach involves the following three key ideas:

(1) treat the Jupiter-Ganymede-Europa-spacecraft 4-body problem as two

coupled circular restricted 3-body problems, the Jupiter-Ganymede-spacecraft and Jupiter-Europa-spacecraft systems;

(2) use the stable and unstable manifolds of the NHIMs about the Jupiter-Ganymede L_1 and L_2 to find an uncontrolled trajectory from a jovicentric orbit beyond Ganymede to a temporary capture around Ganymede, which subsequently leaves Ganymede's vicinity onto a jovicentric orbit interior to Ganymede's orbit;

(3) use the stable manifold of the NHIM around the Jupiter-Europa L_2 to find an uncontrolled trajectory from a jovicentric orbit between Ganymede and Europa to a temporary capture around Europa. Once the spacecraft is temporarily captured around Europa, a propulsion maneuver can be performed when its trajectory is close to Europa (100 km altitude), taking it into a high inclination orbit about the moon. Furthermore, a propulsion maneuver will be needed when transferring from the Jupiter-Ganymede portion of the trajectory to the Jupiter-Europa portion, since the respective transport tubes exist at different energies.

Ganymede to Europa Transfer Mechanism. The construction begins with the patch point, where we connect the Jupiter-Ganymede and Jupiter-Europa portions, and works forward and backward in time toward each moon's vicinity. The construction is done mainly in the Jupiter-Europa rotating frame using a Poincaré section. After selecting appropriate energies in each 3-body system, respectively, the stable and unstable manifolds of each system's NHIMs are computed. Let ${}^{\mathrm{Gan}}W^u_+(\mathcal{M}^1)$ denote the unstable manifold of Ganymede's L_1 NHIM and ${}^{\mathrm{Eur}}W^s_+(\mathcal{M}^2)$ denote the stable manifold for Europa's. L_2 NHIM. We look at the intersection of ${}^{\mathrm{Gan}}W^u_+(\mathcal{M}^1)$ and ${}^{\mathrm{Eur}}W^s_+(\mathcal{M}^2)$ with a common Poincaré section, the surface U_1 in the Jupiter-Europa rotating frame, defined earlier. See Figure 5(b).

Note that we have the freedom to choose where the Poincaré section is with respect to Ganymede, which determines the relative phases of Europa and Ganymede at the patch point. For simplicity, we select the U_1 surface in the Jupiter-Ganymede rotating frame to coincide with the U_1 surface in the Jupiter-Europa rotating frame at the patch point. Figure 5(b) shows the curves ${}^{\mathrm{Gan}}\gamma^1_{z\dot{z}}$ and ${}^{\mathrm{Eur}}\gamma^2_{z\dot{z}}$ on the $(x, \dot{x})$-plane in the Jupiter-Europa rotating frame for all orbits in the Poincaré section with points $(z, \dot{z})$ within $(0.0160 \pm 0.0008, \pm 0.0008)$. The size of this range is about 1000 km in z

position and 20 m/s in z velocity.

From Figure 5(b), an intersection region on the $x\dot{x}$-projection is seen. We pick a point within this intersection region, but with two differing y velocities; one corresponding to ${}^{\mathrm{Gan}}W_+^u(\mathcal{M}^1)$, the tube of transit orbits coming from Ganymede, and the other corresponding to ${}^{\mathrm{Eur}}W_+^s(\mathcal{M}^2)$, the orbits heading toward Europa. The discrepancy between these two y velocities is the ΔV necessary for a propulsive maneuver to transfer between the two tubes of transit orbits, which exist at different energies.

Four-Body System Approximated by Coupled PCR3BP. In order to determine the transfer ΔV, we compute the transfer trajectory in the full 4-body system, taking into account the gravitational attraction of all three massive bodies on the spacecraft. We use the dynamical channel intersection region in the coupled 3-body model as an initial guess which we adjust finely to obtain a true 4-body bi-circular model trajectory.

Figure 5(c) is the final end-to-end trajectory. A ΔV of 1214 m/s is required at the location marked. We note that a traditional Hohmann (patched 2-body) transfer from Ganymede to Europa requires a ΔV of 2822 m/s. Our value is only 43% of the Hohmann value, which is a substantial savings of on-board fuel. The transfer flight time is about 25 days, well within conceivable mission constraints. This trajectory begins on a jovicentric orbit beyond Ganymede, performs one loop around Ganymede, achieving a close approach of 100 km above the moon's surface. After the transfer between the two moons, a final additional maneuver of 446 m/s is necessary to enter a high inclination (48.6°) circular orbit around Europa at an altitude of 100 km. Thus, the total ΔV for the trajectory is 1660 m/s, still substantially lower than the Hohmann transfer value.

5. Conclusion

In our current work on the spatial three-body problem, we have shown that the invariant manifold structures of the collinear libration points still act as the separatrices for two types of motion, those inside the invariant manifold "tubes" are transit orbits and those outside the "tubes" are non-transit orbits. We have also designed a numerical algorithm for constructing orbits with any prescribed finite itinerary in the spatial three-body planet-moon-spacecraft problem. As our example, we have shown how to construct a spacecraft orbit with the basic itinerary $(X; M, I)$ and it is straightforward to extend these techniques to more complicated itineraries.

Furthermore, we have applied the techniques developed in this paper toward the construction of a three dimensional Petit Grand Tour of the Jovian moon system. Fortunately, the delicate dynamics of the Jupiter-Europa-Ganymede-spacecraft 4-body problem are well approximated by considering it as two 3-body subproblems. One can seek intersections between the channels of transit orbits enclosed by the stable and unstable manifold tubes of the NHIM of different moons using the method of Poincaré sections. With maneuvers sizes (ΔV) much smaller than that necessary for Hohmann transfers, transfers between moons are possible. In addition, the three dimensional details of the encounter of each moon can be controlled. In our example, we designed a trajectory that ends in a high inclination orbit around Europa. In the future, we would like to explore the possibility of injecting into orbits of all inclinations.

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