Optimal uncertainty quantification for renewable energy utilization

Shuo Han, Ufuk Topcu, Molei Tao, Houman Owhadi, and Richard M. Murray, California Institute of Technology

Uncertainty as a major challenge
- Key feature of renewables: highly uncertain
- Can compromise stability of power grids
- Need stochastic models

Theory: From infinite to finite
- Consider a similar, but finite dimensional problem
\[ \max_{\{\theta, p\}} \sum_{k=1}^{n} p_k f(\theta) \]

This is equivalent to assuming the probability distribution is discrete (i.e., consisting of Dirac masses)

Surprise: This optimization yields the same optimal value as the infinite dimensional one!
- A simple example in one dimension:
\[ \max_{\theta \in \mathcal{D}} \Pr(\theta \geq \gamma) \]

Application: Energy storage placement
- Storage: enabling technology to overcome uncertainties in renewable generation
- Abstraction of various implementations: chemical batteries, mechanical flywheels, ...
- Need to evaluate the effectiveness of a given placement strategy

What cost to evaluate?
- Assume:
  - renewables are free
  - transmission is free within the power flow limit
  - the only cost comes from compensation of unsatisfied demand
- Cost is given by the optimal value of
\[ \min_{\tau(t), \alpha(t)} \sum_{t=1}^{T} \sum_{i} \left[ d_i(t) + r_i(t) + \sum_{j} p_{ij}(t) \right]^+ \]

subject to
- \( 0 \leq r_i(t) \leq E_i \), \( \forall t \)
- \( |p_{ij}(t)| \leq Q_{ij} \), \( \forall i, t \)

Applying OUQ
- The cost looks very complicated at first glance, and does not look like piecewise concave

Short answer: Can convert this cost function into piecewise concave after a few technical steps.
- Trick 1: use Lagrange duality (min \( \rightarrow \) max)
- Trick 2: this problem is an LP \( \rightarrow \) enough to check all the vertices of the constraint polytope
- This gives exponentially large \( K \)
  - Tractable for small scales
  - Good approximation exists for large scales

Result: Single-bus case
- Diminishing return: adding more storage will eventually help little (more quickly for buses with less uncertainty)
- Time correlation is important

Result: IEEE 14-bus network
- Standard test case
- Assume only source of uncertainty is from generators: 1-3, 6, & 8
- Assume no spatial correlation
- Time correlation obtained from real wind data

Summary
- OUQ: information-driven worst-case analysis over consistent probability distributions
- Efficient numerical solution for certain cases
- Application on the energy storage placement problem gives some insights

More information
Web: [http://www.cds.caltech.edu/~hanshuo](http://www.cds.caltech.edu/~hanshuo)

Electronic copy of this poster

Acknowledgment
This work is funded by NSF award #0931746.