

An ICP variant using a point-to-line metric

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ICP: Iterative Closest/Corresponding Point

- Used in many contexts – vision being one of the largest.
- But robotics scan-matching is special:
 - few data points, large uncertainties
 - performance:
 - convergence is slow
 - correspondence search is expensive
 - occlusions and dynamic environments
 - need to characterize uncertainty for SLAM

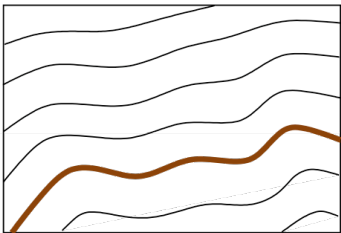
The problem with ICP

- We cannot characterize the **global** behavior.
- ... but we can say something about the **local** behavior, using a mix of statistics and computational geometry
 - ICRA’07: “On achievable accuracy for range-finder localization”
 - ICRA’07: “An accurate closed-form estimate of ICP’s covariance”
- ... if we concede some *white lies*:
 - let’s assume it converges “near” the true solution
 - let’s ignore trimming and other “impurities”

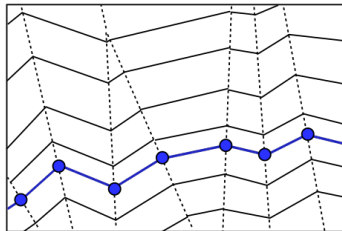
PL-ICP: ICP with point-to-line metric

- What happens if we use a point-to-line metric?
- Good things:
 - quadratic convergence instead of linear
 - convergence in a finite number of steps
 - much faster in practice
- Bad things:
 - less robust for large rotations

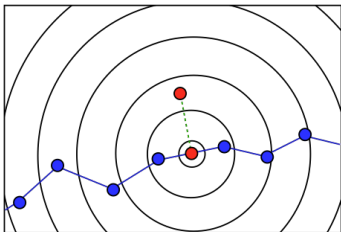
Metrics zoo



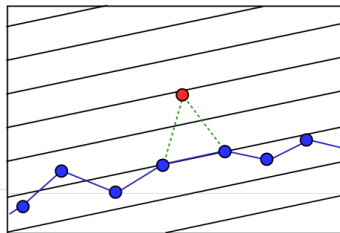
“True” distance to surface (unknown)



Distance to polyline



Point-to-point (vanilla ICP)



Point-to-line (PL-ICP)

Metrics zoo

- Vanilla ICP and PL-ICP both **use the same global function:**

$$\min_q \sum_i \|p_i \oplus q - \Pi\{\mathcal{S}^{\text{ref}}, p_i \oplus q\}\|^2$$



same **statistical** properties

- but a **different incremental** one:



different **numerical** properties

ICP: point-to-point

$$\min_{q_{k+1}} \sum_i \|p_i \oplus q_{k+1} - \Pi\{\mathcal{S}^{\text{ref}}, p_i \oplus q_k\}\|^2$$

PL-ICP: point-to-line

$$\min_{q_{k+1}} \sum_i (n_i^T [p_i \oplus q_{k+1} - \Pi\{\mathcal{S}^{\text{ref}}, p_i \oplus q_k\}])^2$$

q robot pose (world frame)

p_i points in the first scan

$\Pi\{\mathcal{S}^{\text{ref}}, \cdot\}$ projection on the reference surface

\oplus rototranslation: $p_i \oplus q_k = \mathbf{R}(\theta_k) p_i + t_k$

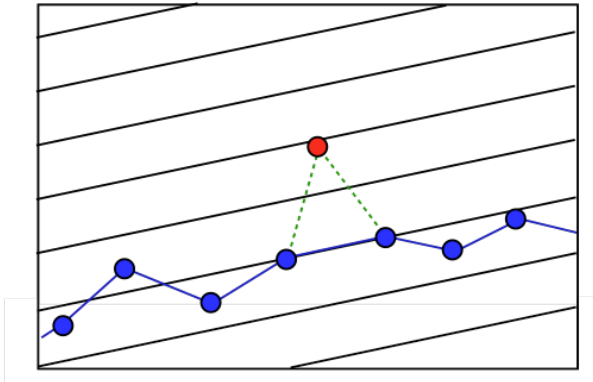
n_i normal to surface

Convergence theorems

- The following results obtained by Pottman *et al.*, in the general 3D case.
- **Point-to-point** metric:
 - linear convergence.
 - convergence rate depends on approaching direction
- **Point-to-line** metric:
 - equivalent to Gauss-Newton iteration.
 - quadratic convergence for a zero-residual problem
- Note: these results are not robust to tricks like trimming (valid in practice, however)
- (No closed-form exists for 3D case)

Convergence in a finite number of steps

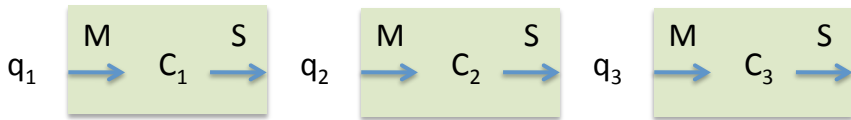
- Key observation: the error function depends only on point-to-segment correspondences.



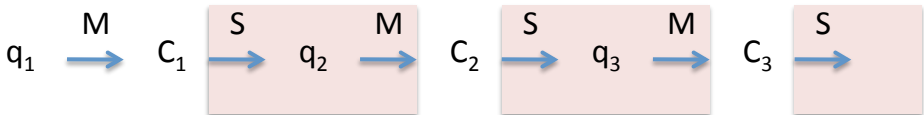
- There is a finite number of such correspondences.

Proof

- ICP is the iterative application of two operations:
 - **Match**: given current pose q_k , find correspondences C_k
 - **Solve**: given C_k , solve for next pose q_{k+1} .
- We always see ICP as iteration among poses:



...but for PL-ICP it's more convenient to group S+M:



- Because there are a finite number of C s, after a **finite number of steps**, we reach either a fixed point or a loop.

Experimental results

- Same experiments as Minguez *et al* (T-RO 2007).
- Precision:

	Method	MbICP	IDC	ICP	PLICP	GPM	GPM \circ PLICP
	Precision (m,rad)	(%)	(%)	(%)	(%)	(%)	(%)
Experiment 3 (0.15m, 0.15m, 8.6°)	< 0.001	80.84	82.95	56.62	99.51	0.91	99.95
	(0.001, 0.005)	19.15	16.96	43.37	0.03	18.64	0.01
	(0.005, 0.01)	0.00	0.00	0.00	0.05	24.58	0.02
	(0.01, 0.05)	0.00	0.05	0.00	0.33	50.16	0.02
	> 0.05	0.00	0.03	0.002	0.08	5.71	0
Experiment 6 (0.20m, 0.20m, 45°)	< 0.001	80.38	74.94	52.18	73.46	0.61	99.79
	(0.001, 0.005)	18.86	16.53	42.01	0.23	14.05	0.01
	(0.005, 0.01)	0.00	0.37	0.00	0.35	21.79	0.02
	(0.01, 0.05)	0.00	0.81	0.01	1.14	54.25	0.07
	> 0.05	0.75	7.32	5.78	24.81	9.3	0.11

- Speed:

	<i>avg. iterations</i>	<i>avg. execution time</i>
MbICP	31.2	0.076 s (13.1 Hz)
ICP	34.7	0.083 s (12.0 Hz)
IDC	30.4	0.240 s (4.1 Hz)
PLICP	7.2	0.0018 s (539 Hz)

Note: MbICP, ICP, IDC stopped by threshold, they have **infinite** iterations.

Summary

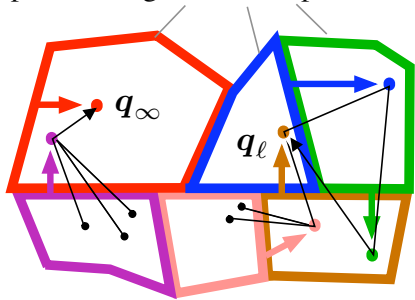
- Good properties of PL-ICP:
 - quadratic convergence instead of linear
 - convergence in a finite number of steps (if polyline)
 - great improvement in practice
- Drawbacks: less robust for large rotations
- Software and logs available at my website.

TODO

- Trivial extensions to algorithm hopefully improve robustness:
 - Use Gaussian prior from odometry.
 - Mix point-to-point and point-to-line.

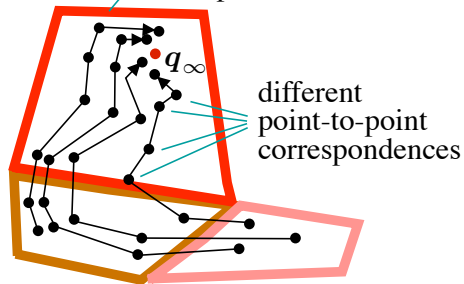
PL-ICP

sets of poses having same point-to-segment correspondences



Vanilla ICP

same point-to-segment correspondences



- No epsilons for termination in PL-ICP.
 - Check for fixed point or loop using hash of correspondences.

	Method	MbICP	IDC	ICP	PLICP	GPM	GPM \circ PLICP
	Precision (m,rad)	(%)	(%)	(%)	(%)	(%)	(%)
Experiment 1 (0.05m, 0.05m, 2°)	< 0.001	81.27	83.31	57.78	99.85	1.86	99.98
	(0.001, 0.005)	18.72	16.68	42.22	0.01	36.34	0
	(0.005, 0.01)	0.00	0.00	0.00	0.01	38.42	0.01
	(0.01, 0.05)	0.00	0.01	0.00	0.13	23.37	0.01
	> 0.05	0.00	0.00	0.00	0.00	0.01	0
Experiment 2 (0.10m, 0.10m, 4°)	< 0.001	80.97	83.12	56.62	99.71	1.3	99.98
	(0.001, 0.005)	19.02	16.84	42.48	0.02	24.56	0.00
	(0.005, 0.01)	0.00	0.00	0.00	0.03	29.12	0.01
	(0.01, 0.05)	0.00	0.03	0.00	0.22	42.85	0.01
	> 0.05	0.00	0.00	0.00	0.02	2.17	0.00
Experiment 3 (0.15m, 0.15m, 8.6°)	< 0.001	80.84	82.95	56.62	99.51	0.91	99.95
	(0.001, 0.005)	19.15	16.96	43.37	0.03	18.64	0.01
	(0.005, 0.01)	0.00	0.00	0.00	0.05	24.58	0.02
	(0.01, 0.05)	0.00	0.05	0.00	0.33	50.16	0.02
	> 0.05	0.00	0.03	0.002	0.08	5.71	0
Experiment 4 (0.20m, 0.20m, 17.2°)	< 0.001	81.28	81.96	56.30	98.43	0.61	99.79
	(0.001, 0.005)	18.71	16.79	43.58	0.088	14.05	0.01
	(0.005, 0.01)	0.00	0.00	0.00	0.13	21.79	0.02
	(0.01, 0.05)	0.00	0.80	0.00	0.44	54.25	0.07
	> 0.05	0.00	0.44	0.10	0.92	9.3	0.11
Experiment 5 (0.20m, 0.20m, 32°)	< 0.001	80.92	79.54	54.00	84.48	0.61	99.79
	(0.001, 0.005)	18.79	16.36	43.13	0.20	14.05	0.01
	(0.005, 0.01)	0.0	0.04	0.00	0.28	21.79	0.02
	(0.01, 0.05)	0.0	0.81	0.00	0.93	54.25	0.07
	> 0.05	0.28	3.05	2.85	14.11	9.3	0.11
Experiment 6 (0.20m, 0.20m, 45°)	< 0.001	80.38	74.94	52.18	73.46	0.61	99.79
	(0.001, 0.005)	18.86	16.53	42.01	0.23	14.05	0.01
	(0.005, 0.01)	0.00	0.37	0.00	0.35	21.79	0.02
	(0.01, 0.05)	0.00	0.81	0.01	1.14	54.25	0.07
	> 0.05	0.75	7.32	5.78	24.81	9.3	0.11