From control to networks, and into microeconomics?

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Optimal control: an authoritarian view

Theory of control design applies mainly to problems with:
- Single performance objective.
- Single decision maker (centralized information).

Complexity in analysis or design comes from the description of the plant $G$: nonlinearity, or the amount of structure in an uncertainty description $G \in G$.

- Q: Is this picture relevant to large-scale systems?
- A: Rarely. As dimension grows, we run into
  - Multiple objectives
  - Decentralized information
- These requirements lead to complexity, even for a simple $G$. 
Control with no central authority: the Internet.

- Multiple decision makers, decentralized information. Nobody knows the system, let alone others’ signals.
- Multiple, and not very explicit objectives.
- Anarchy? Not quite, some standards, IETF, etc. But limited quantitative understanding of its behavior.
Bringing some order: Kelly’s formulation.

S source-destination pairs (long flows) share L links.

Routing matrix $R$ (fixed for now)

$$R_{li} = \begin{cases} 1 & \text{if source } i \text{ uses link } l \\ 0 & \text{otherwise} \end{cases}$$

Source $i$ has rate $x_i$, receives utility $U_i(x_i)$.

Link $l$ has rate $y_l = \sum R_{li}x_i$, and capacity $c_l$.

$$y = Rx$$

Social welfare optimization: $\max_x \sum_i U_i(x_i)$ subject to $Rx \leq c$.

A market way to decentralize the optimization:

- Link $l$ generates congestion measure or price $p_l$
- Source $i$ sees price $q_i = \sum R_{li}p_l$, optimizes own profit: $\max_{x_i} U_i(x_i) - q_i x_i$

Issue: generating prices that align individual and social welfare; they should be the Lagrange multipliers of the dual of the convex problem.

Finding them dynamically: $\dot{p}_l = \gamma \left[ y_l - c_l \right]^{+}_{p_l}$ (Low-Lapsley '99)
Beyond quasi-statics \(\rightarrow\) Control viewpoint.

Time-scales determined by the plant dynamics: network delays.

- **Challenges for design of source and link laws:**
  - Highly decentralized information.
  - Unknown network: topology, parameters.
  - Accommodating both equilibrium point and dynamic behavior.

- **Solutions feature:**
  - Handcrafted designs, use equilibrium information to bound loop gains.
  - Mathematical proofs of local stability in any network, using multivariable control.
  - Time-scale separation between desired equilibrium and dynamic behaviors.

\[
y_i(t) = \sum_i R_{li} x_i(t - \tau_{i,l}^f)
\]

Delayed model in Laplace transform (for fixed delays):

\[
[R_f(s)]_{l,i} = \begin{cases} 
e^{-\tau_{i,l}^f s} & \text{if } i \text{ uses } l \\ 0 & \text{otherwise} \end{cases}
\]

\[
RTT: \tau_i = \tau_{i,l}^f + \tau_{i,l}^b
\]

\[
q(t) = \sum_i R_{li} p_i(t - \tau_{i,l}^b)
\]

\[
x: \text{source rates.}
\]

\[
y: \text{aggregate link rates}
\]

\[
p: \text{link prices}
\]

\[
q: \text{aggr. source prices.}
\]

\[
Sources
\]

\[
Routing-Delay
\]

\[
Links
\]
Example: A “primal-dual” congestion control
[joint work with Z. Wang, S. Low and J. Doyle, Infocom ’03]

\[ \dot{x}_i = - \frac{\alpha_i M_i \tau_i}{x_i \dot{q}_i + \beta_i (U_i'(x_i) - q_i) x_i} \]

\[ \dot{p}_l = \frac{1}{c_{0l}} \left[ y_l - c_{0l} \right]^+ \]

\( \alpha_i, \beta_i \) parameters,
\( \tau_i \): round-trip time
\( M_i \): bound on # of bottlenecks

- **Design strategy:**
  - Start from local dynamics, compensate for RTT at the source, and use network equilibrium to control the overall gain in a distributed manner.
  - Incorporate equilibrium features as a slower dynamics
- **Given a bound on RTT, one can provably satisfy:**
  - Almost full network utilization, empty equilibrium queues.
  - Resource allocation according to utility functions.
  - Local dynamic stability for any network.
  - Global stability: for simple networks, conditions similar to local ones.
Still, we have a single objective.

We have avoided the inherently multi-objective nature of the problem by imposing operation in the maximizer of \( \sum_i U_i(x_i) \). Justification?

More precisely, consider the "utility possibility set" \( U \) achievable by sources, given the capacity constraints \( Rx \leq c \). It is clearly desirable to be Pareto optimal, but which one to pick?

An argument from microeconomics:
if utilities are expressed in units of a tradeable "numeraire" good (money), then this is the only market equilibrium: from any other point, a side payment between users makes it advantageous to switch to this point.

This, however, does not appear too realistic for the networking context, especially at fast time-scales.
The viewpoint of welfare economics.
A policy maker can arbitrate between multiple utilities by maximizing a social welfare function \( W(u_1, \ldots, u_S) \), increasing and concave. Examples:

"Utilitarian"
\[
W = \sum u_i
\]

"Rawlsian"
\[
W = \min u_i
\]

"Constant elasticity"
\[
W = \left( \sum u_i^{1-\rho} \right)^{\frac{1}{1-\rho}}
\]

In the limit as \( \rho \to 1 \), the constant elasticity welfare functions gives
\[
W = \sum \log(u_1).
\]
The resulting Pareto optimal point gives the so-called "Nash Bargaining Solution", which satisfies the following axioms:
(i) Symmetry,  (ii) Invariance to change of scale in utilities (iii) Independence of irrelevant alternatives: reducing \( U \) by non-optimal points does not change choice.
Two interpretations for “proportionally fair” rate allocation.

\[
\max_x \sum_i \log(x_i) \quad \text{subject to} \quad Rx \leq c.
\]

1. (Kelly): Utility is \( \log(x_i) \), use utilitarian welfare function.
2. (Yaiche-Mazumdar): Utility is \( x_i \), use Nash bargaining solution.

Asymmetric generalization: \( \max_x \sum_i w_i \log(x_i) \)

1. (Kelly) Utility is \( w_i \log(x_i) \), \( w_i \) is "willingness to pay".

2. Utility is \( x_i \), use social welfare function \( \sum_i w_i \log(u_i) \), Nash with asymmetric bargaining power.
The supply side of this economy

A dual problem would be to fix a demand, and allocate network resources so as to minimize a certain cost.

- At long time-scales, this could include bandwidth provisioning.
- At short time scales, the main degree of freedom is *routing*.

Traffic engineering problem: given a traffic matrix of flows between source-destination nodes, minimize $\sum \phi_i(y_i)$ subject to meeting demands. Cost $\phi_i(y_i)$ could represent, e.g. a delay at the link. For single-path (IP) routing, the problem is hard.
Allowing multiple routes per source: multicommodity flow problem: let $z_i^l$ be the rate of source $i$ going through link $l$, solve

$$\min \sum_l \phi_l(y_l) \text{ subject to}$$

$$y_l = \sum_l z_i^l,$$

$$\sum_{l \text{ in}} z_i^l = \sum_{l \text{ out}} z_i^l \text{ at interior node},$$

$$\sum_{l \text{ in}} z_i^l = x_i \text{ at source node.}$$

- If $\phi_l(y_l)$ are convex, optimization can be solved by a central planner.
- Using duality, we can obtain weights for which the routes are which give routes via shortest path.
- Decentralized versions involve sources, or edge routers keeping track of routes.
Combining demand and supply?

A "partial equilibrium" formulation from microeconomics would be to maximize the "Marshallian surplus" \[ \sum U_i(x_i) - \sum \phi_i(y_i) \] subject to "market-clearing" (supply=demand). For a simple divisible good, solution is "competitive equilibrium" with price \[ p = U_i'(x_i) = \phi_i'(y_i). \]

- Applying this to networks:
  - Combining Traffic Engineering and congestion control; dynamics, time-scales (see J.Wang et al '03)
  - Justification? Note that “network cost” is really QoS for sources.
  - If demand is an aggregate of elastic flows with random duration, what is a good model?
Other results from the economic viewpoint.

- Demand problem in which sources are not price-takers. i.e., they anticipate their influence on prices. Problem becomes a game, overall utility deteriorates by < 25% (Johari-Tsitsiklis ’04, Sanghavi-Hajek-’04).

- Traffic engineering problem with “selfish routing”. Again, this is a game, and there are results on deterioration of the overall cost (Roughgarden & Tardos ‘02).

- The network as a profit-seeking entity: monopoly and oligopoly theory, (Hajek & Gopal ’04).

- Bidding formulations to differentiated services: Vickrey-Clarke-Groves method (e.g., Shu & Varaiya ‘03).

- Open questions:
  - Decentralization, and simplicity of implementation?
  - Dynamics.
Another layer: Wireless power control

Noise $N$

$\gamma_i = \frac{W}{B} \cdot \frac{h_i p_i}{N + \sum_{j \neq i} h_j p_j}$

CDMA: SIR for mobile $i$:

Assume:

- Elastic traffic sources, utility is proportional to rate $x_i$.
- For simplicity, assume the rate depends on SIR through the Gaussian Shannon capacity formula. So pick $U_i = k_i B \log(1 + \gamma_i)$.

Non-convex utility possibility set.

Maximizing total utility would give extreme unfairness!
Other ways of arbitration.

Extending Nash bargaining

Extended Kazai-Smorodnisky solution

A convex approximation (Johansson et al.)

Maximizer of $\sum \log(u_i)$ over the convex hull.

Choose Pareto optimum which is proportional.

Choose Pareto optimum where each source gets a fixed proportion of their maximum utility.

Assuming $\gamma_i \gg 1$ so that $U_i \approx \log(\gamma_i)$, a logarithmic change of variables convexifies the problem. Optimize over this set.
Decentralized information? A non-cooperative version.

Each user plays the best response assuming others' powers are fixed. If we simply use the utility $U_i(p_i, p_{-i}) = \log(1 + \gamma_i)$, then every user will blast at maximum power.

Idea: add a cost term proportional to power. Users then maximize $U_i(p_i, p_{-i}) = \log(1 + \gamma_i) - a_i p_i \quad \text{over} \quad p_i \in [0, p_{\text{max}}]$.

This is convex, the solution is solution $p^+ = F(p) = \text{sat}[Mp + b]$, affine map saturated to $[0, p_{\text{max}}]$.

Properties:

- Nash equilibrium exists, unique if the number of users $n < \frac{B}{W}$. In that case, iteration $p[k+1] = F(p[k])$ converges to it.
- If $a_i = kh_i$, (so cost is proportional to received power from source $i$) then equilibrium will equalize SIRs.
- Nash equilibrium need not be Pareto optimal.
Most of microeconomic theory applies to characterizations, properties of equilibria. In regard to resource allocation, we have much to draw from this field.

On the other hand, less is known about dynamics outside equilibrium. One classical model is the “Tatonnement” dynamics for price,

\[ \frac{dp}{dt} = \gamma \left[ \text{agg.demand} - \text{supply} \right] \]

Exactly what was used before in link laws.

However economists question its descriptive value.

Nevertheless, for “designed economies” as in networks, dynamics can have very precise meaning. Control insight could be valuable.

Another example: dynamics of fictitious play in games with learning (Shamma and Arslan ’03).
Conclusions

• When migrating control theory to large scale networks, we immediately face challenges of decentralized info, and often multiple objectives.
• Feedback design becomes more difficult to automate, so we use handcrafted designs and mathematical analysis (“design-for provability”).
• Elements of economic theory come in naturally in network resource allocation problems: we’re still on a learning curve.
• It is likely, still, that a control viewpoint can go beyond what is known about dynamics, especially for “designed markets” where all models are by definition valid.