Feedback control for quantum and classical uncertainty management

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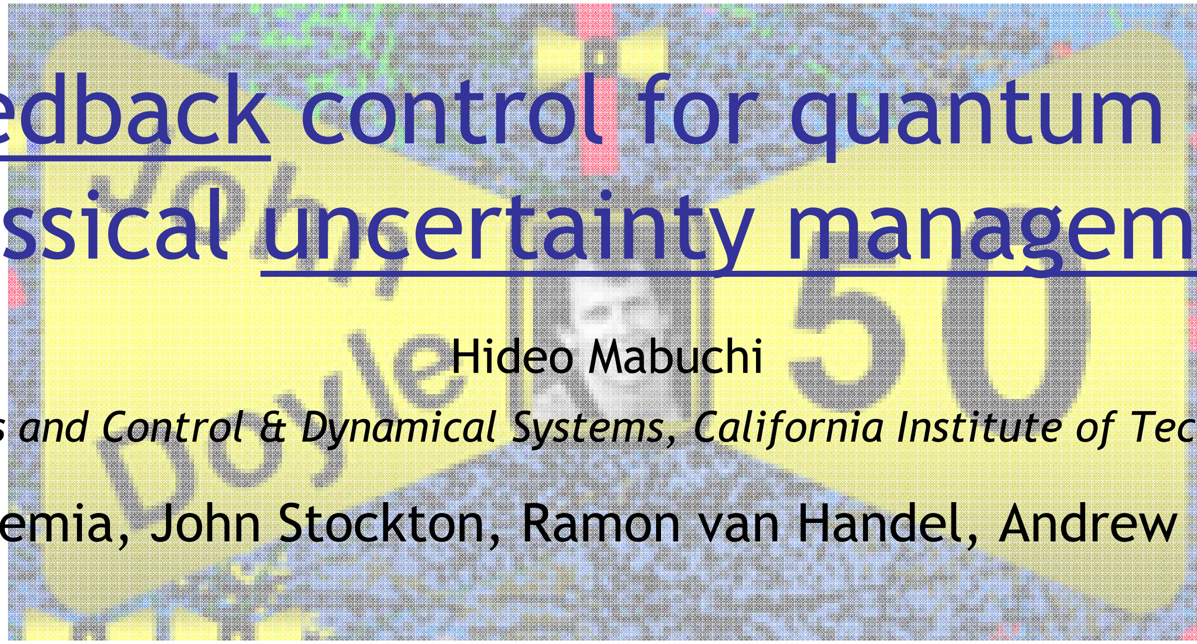
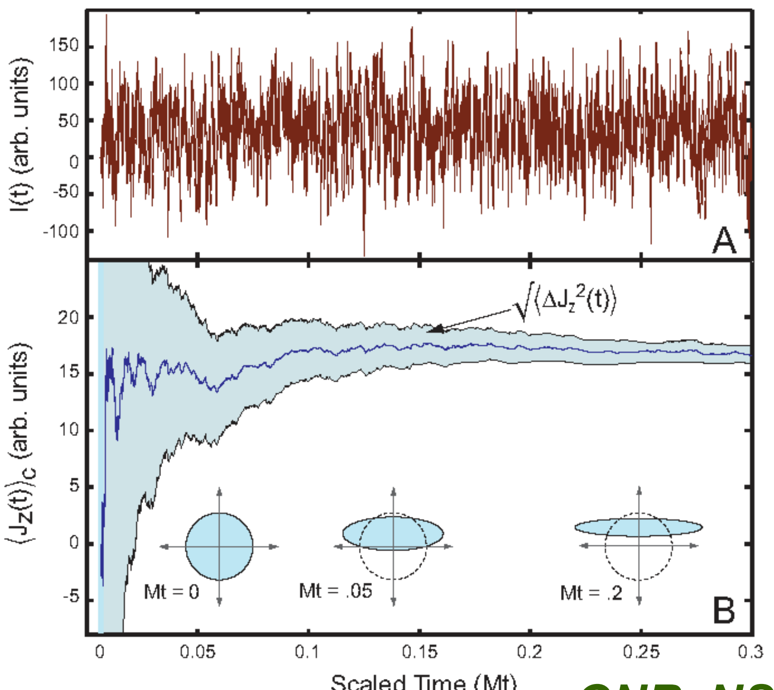
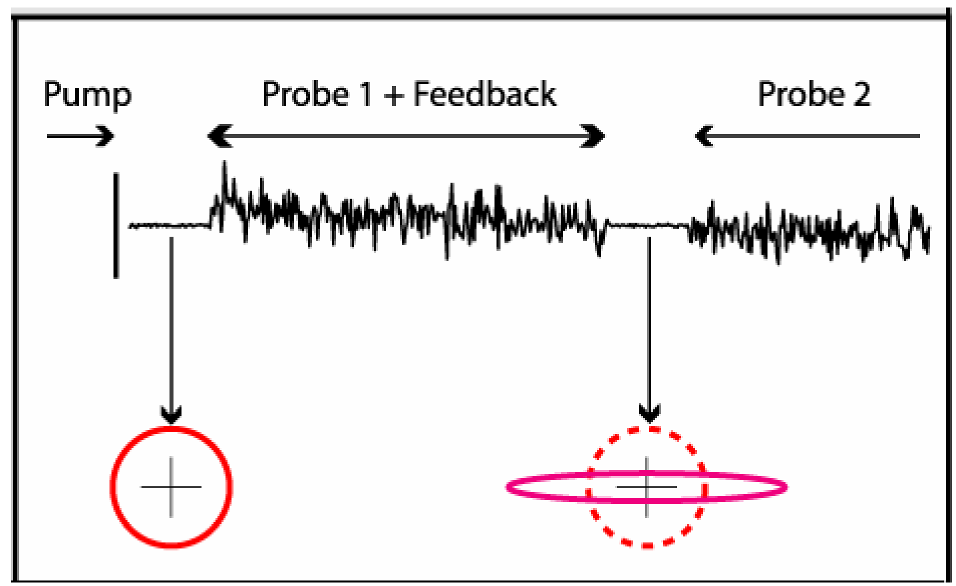
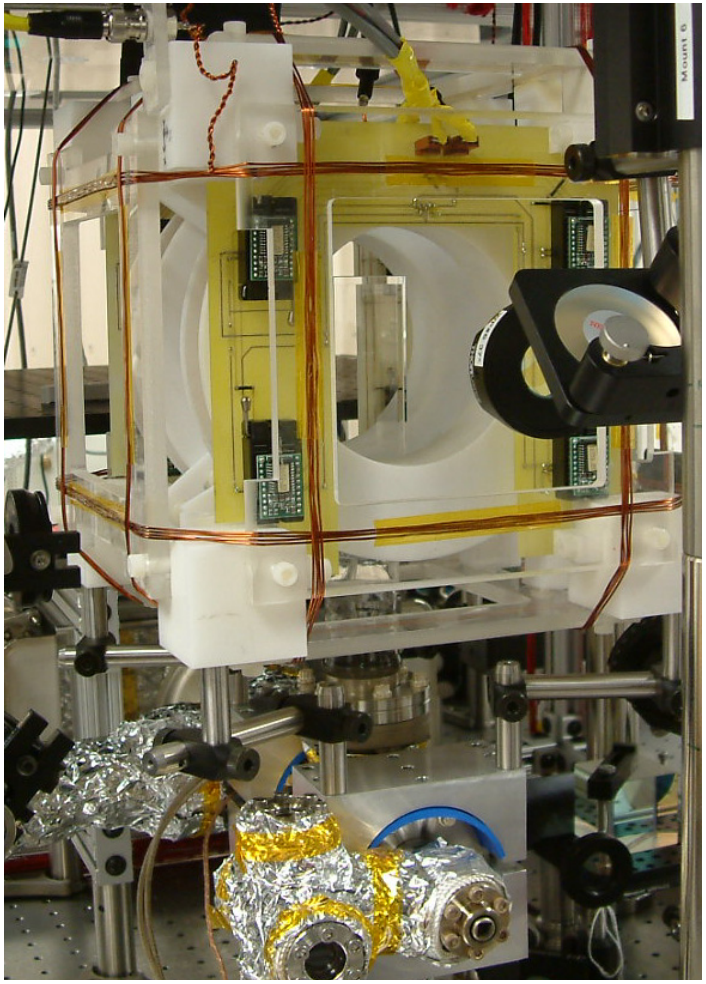


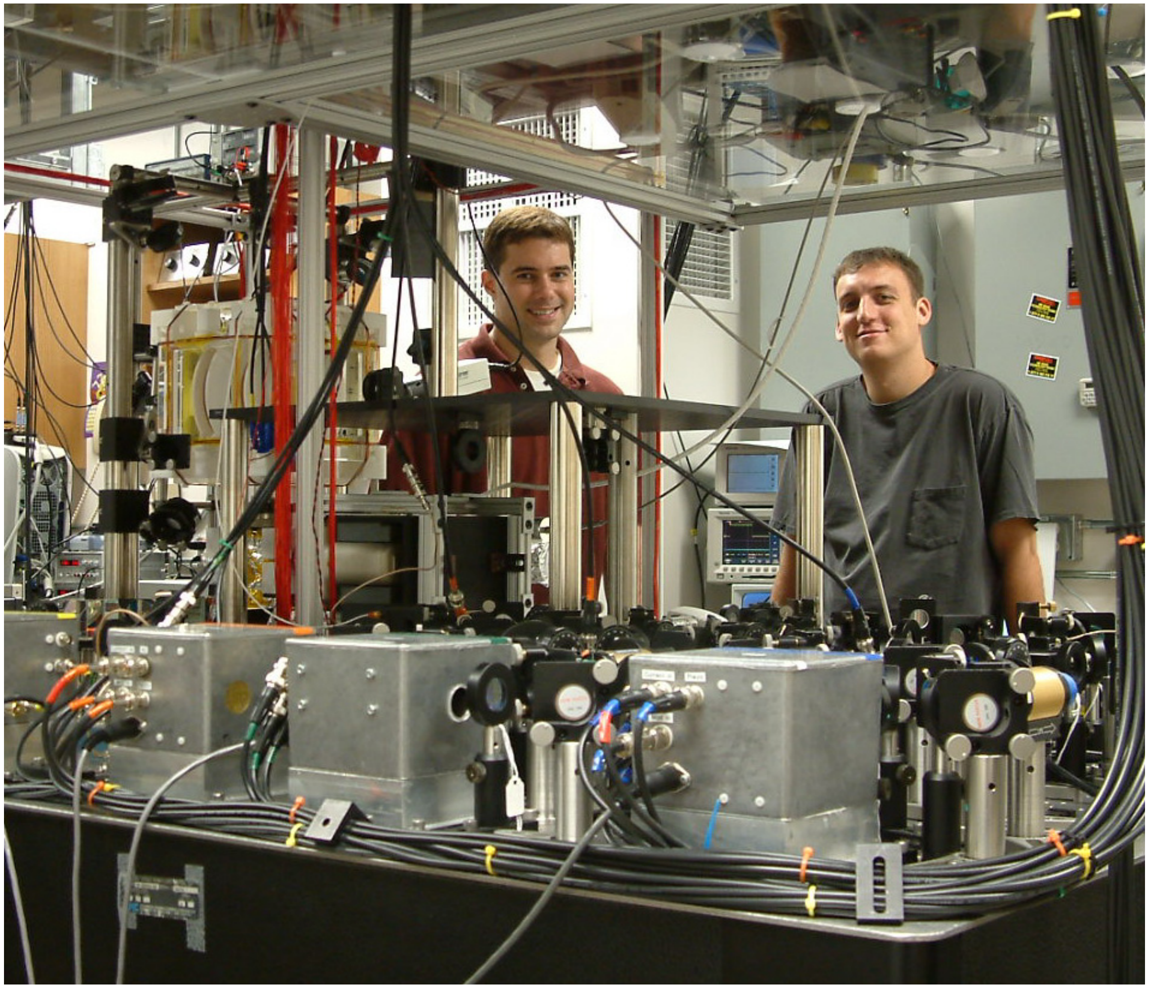




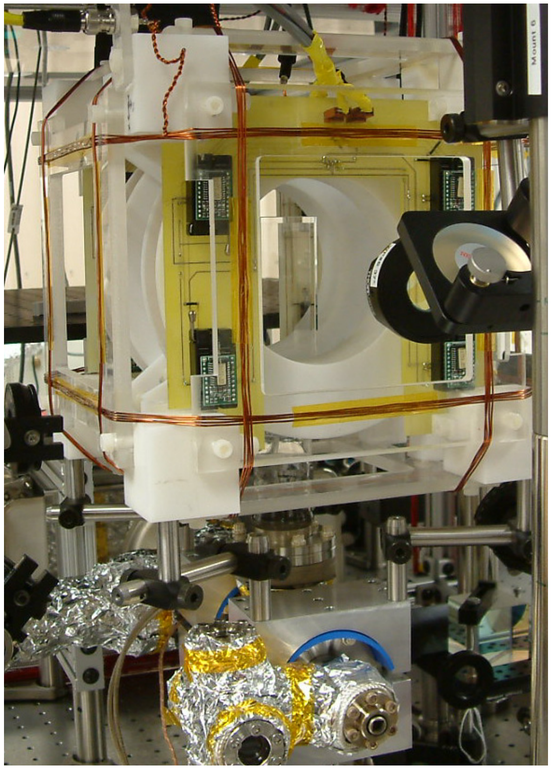
ONR, NSF, NSA, ARO/MURI

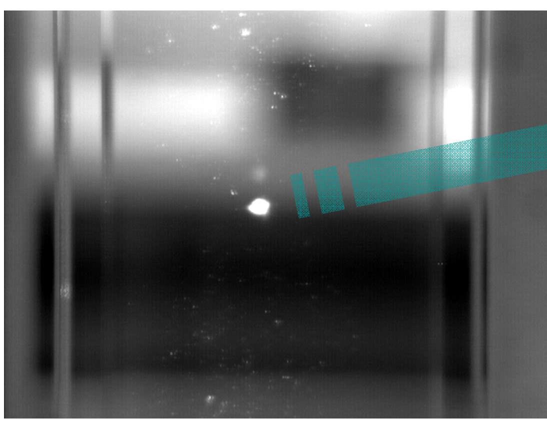
Atomic magnetometry with cold atoms

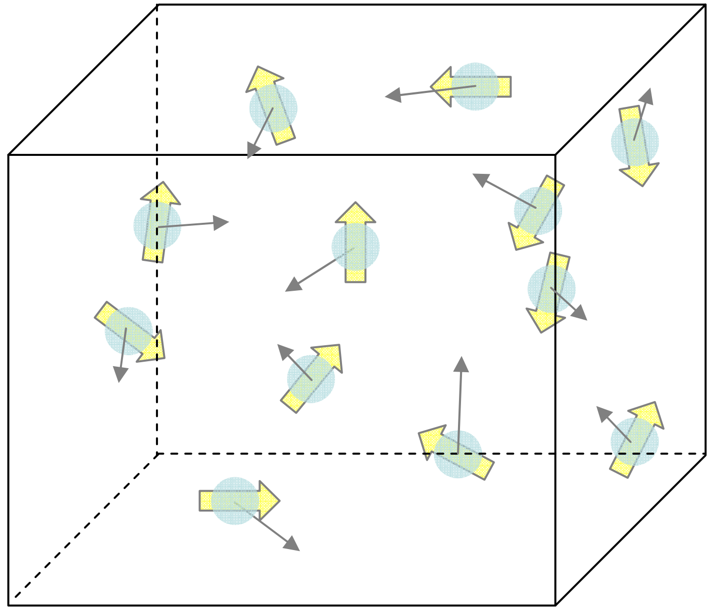




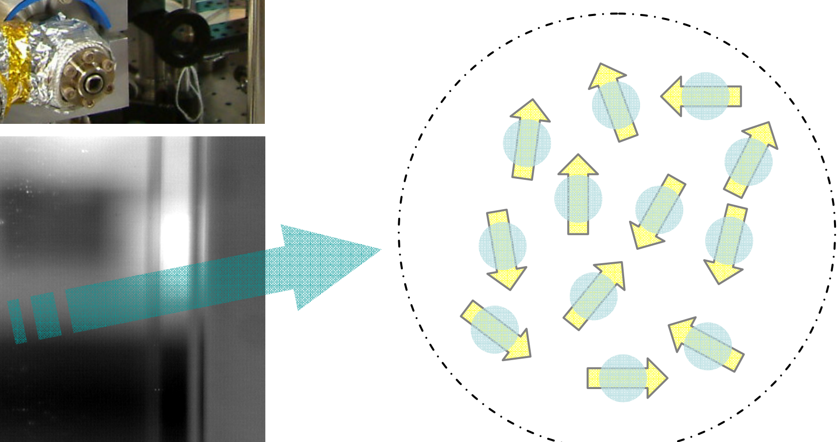
Trapped *cold* atoms; optical pumping



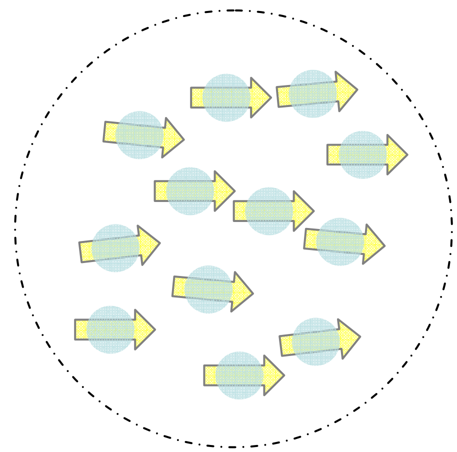




At room temperature, Cesium atoms fly around at speeds in the neighborhood of 300 meters per second



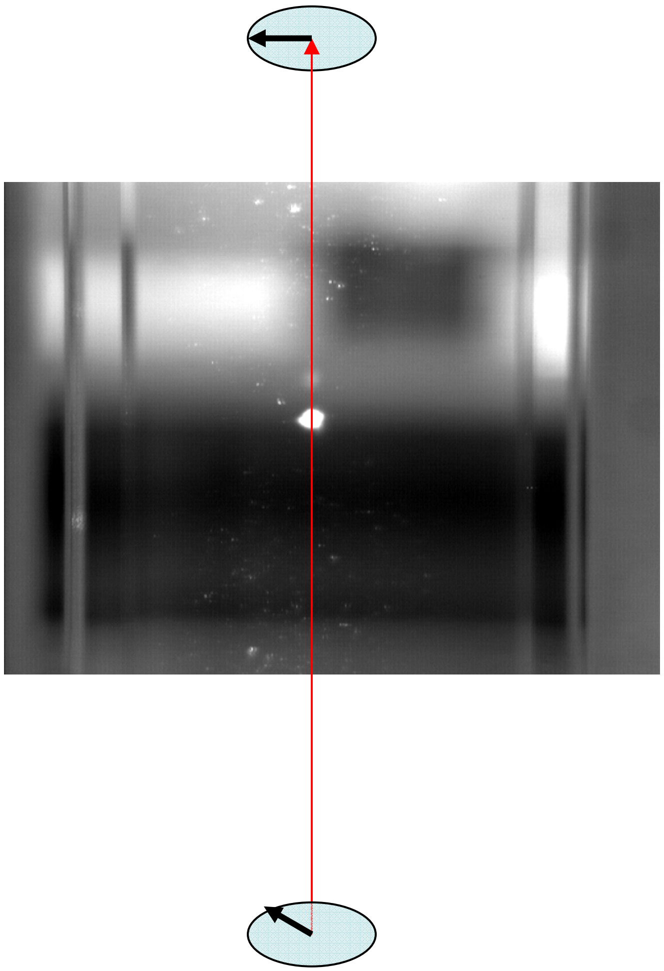
laser cooling creates trapped atoms ~ 1 μK (speeds more like 1 cm per second)

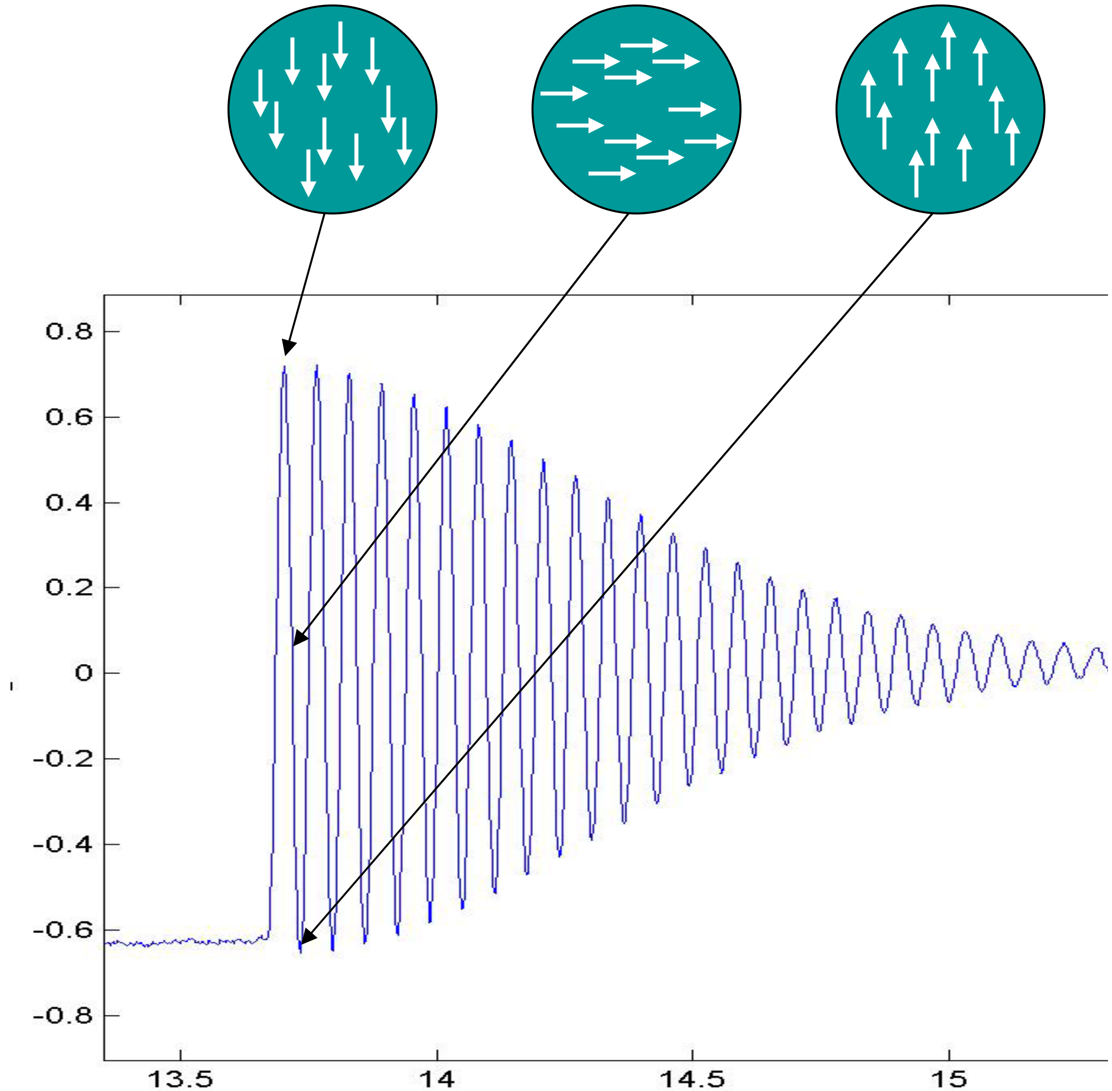


lasers can also be used to align the atomic “spins” (optical pumping)

Optical real-time measurement of collective spin

Bigelow, Polzik, Jessen, …

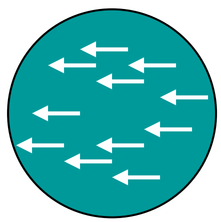




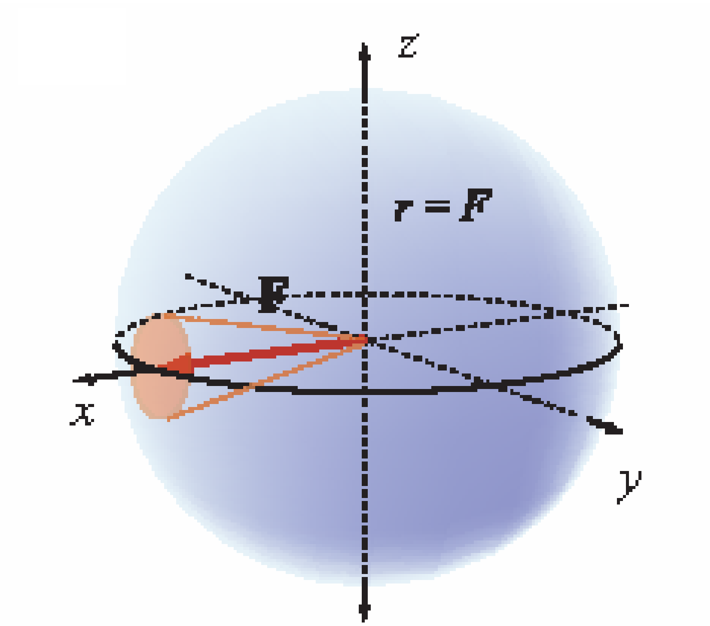
Like NMR free induction decay, but *much higher sensitivity* relative to hΔ F_z^2i

Quantum description of collective spin states

Cold cloud of Cs atoms
(gas phase, non-interacting)



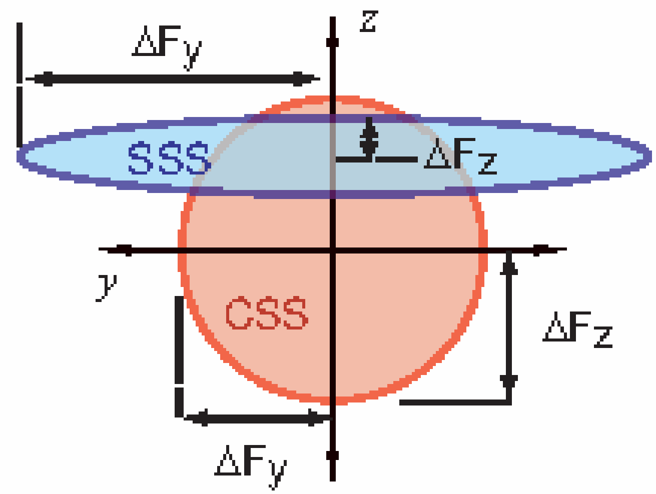
align to common axis



Each atom carries a “spin” and
associated magnetic moment

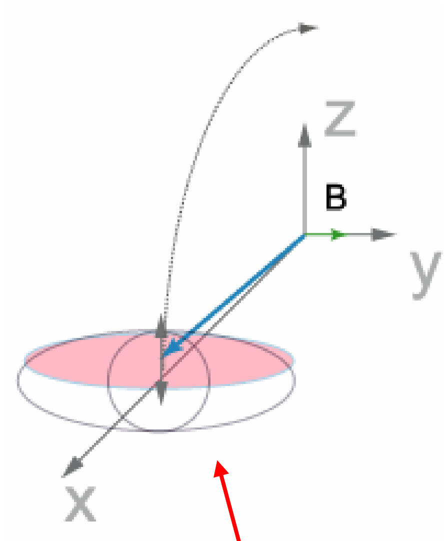
$$|\psi\rangle \in \mathcal{H}, \qquad \hat{\mathbf{F}} = [\hat{F}_x\ \hat{F}_y\ \hat{F}_z]$$

$$\Delta \hat{F}_y \Delta \hat{F}_z \geq \frac{1}{2} |\langle \hat{F}_x \rangle|$$



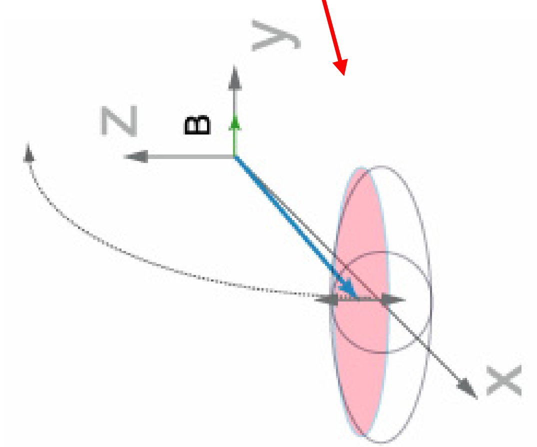
Spin-squeezing and quantum metrology

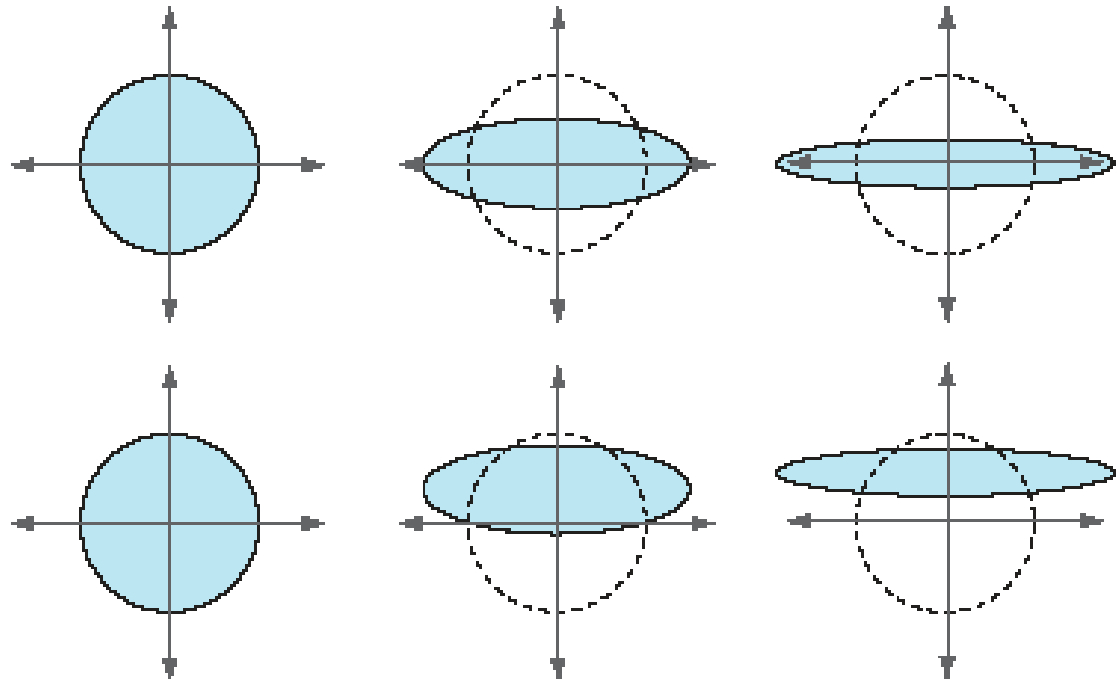
(Kitagawa & Ueda, Wineland *et al.*)



magnetometry-like

clock-like



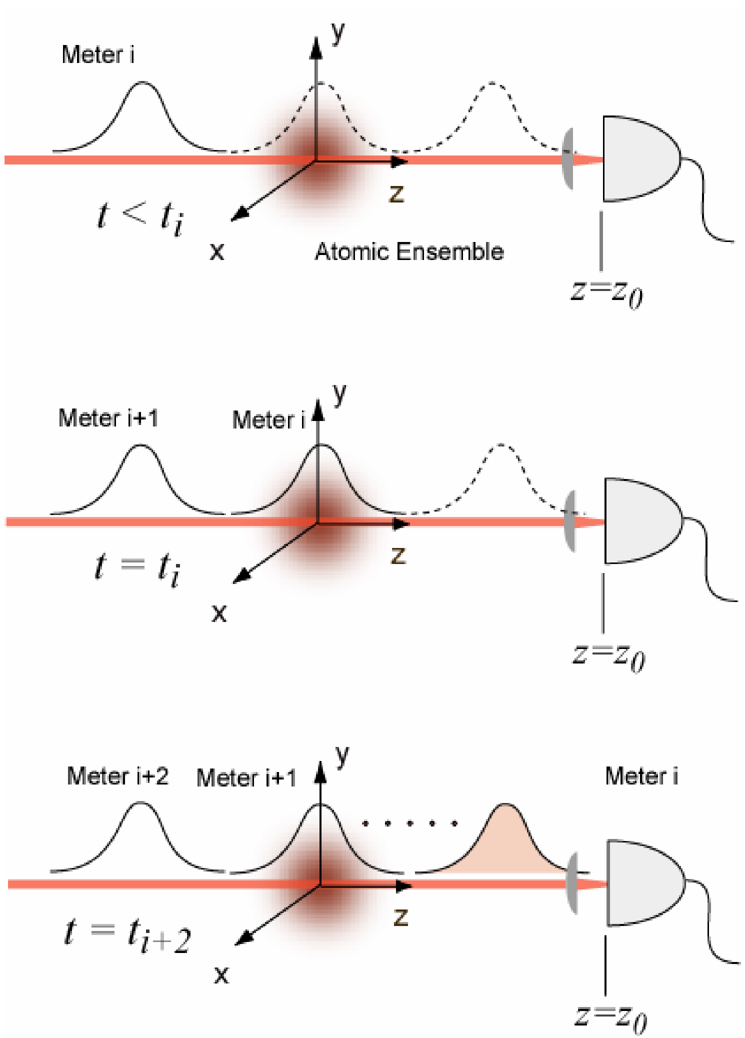


but how to distinguish rotations from projection noise?

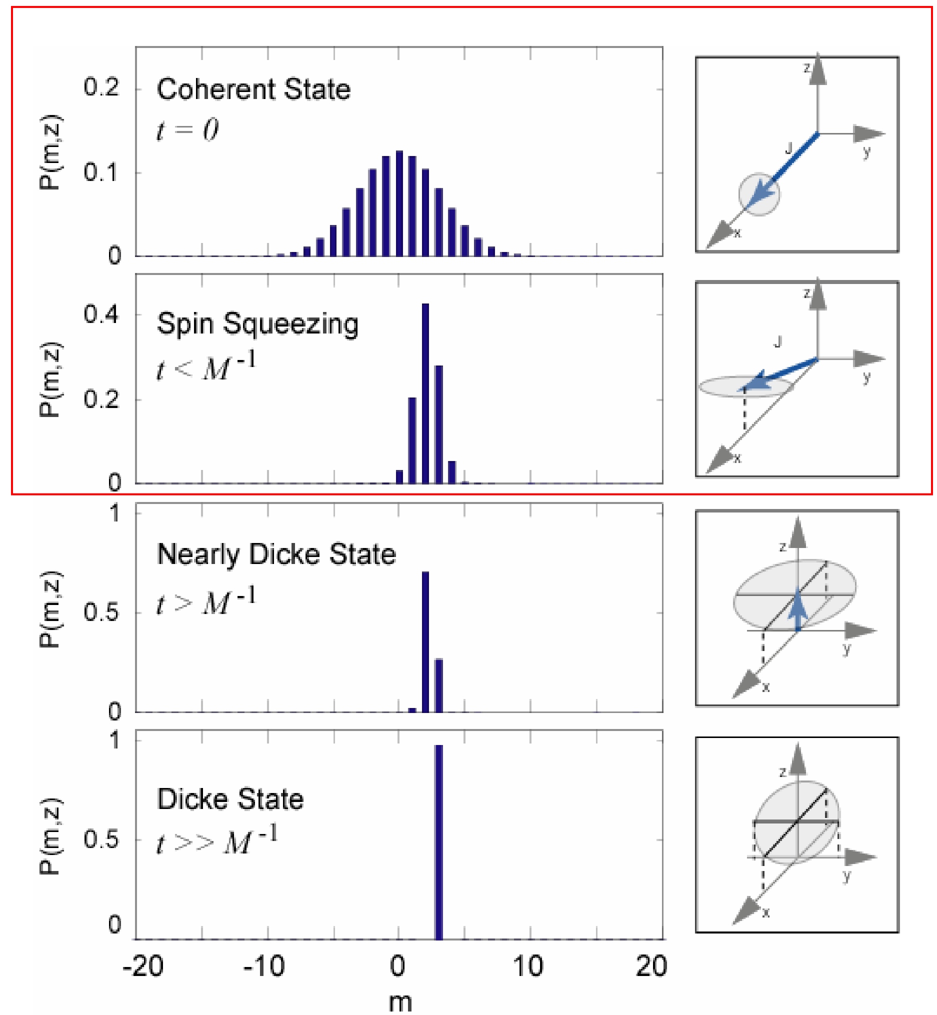
- quantum filtering (+ related)
- real-time feedback...

Continuous Faraday rotation – quantum filtering

Quantum Markov process



Quantum filtering; gradual “collapse”

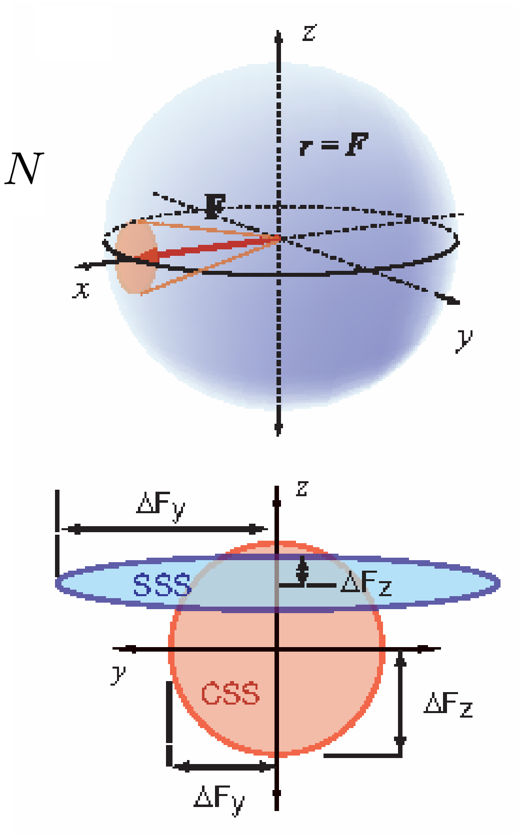


Stochastic Master Equation - conditional spin state

$$\begin{aligned} d\rho_b(t) &= -i[\gamma F_y b, \rho_b(t)]dt + \mathcal{D}[\sqrt{M}F_z]\rho_b(t)dt \\ &\quad + \sqrt{\eta}\mathcal{H}[\sqrt{M}F_z]\left(2\sqrt{M\eta}[y(t)dt - \langle F_z\rangle_b dt]\right)\rho_b(t) \\ \mathcal{D}[c]\rho &\equiv c\rho c^\dagger - (c^\dagger c\rho + \rho c^\dagger c)/2 \\ \mathcal{H}[c]\rho &\equiv c\rho + \rho c^\dagger - \mathrm{Tr}[(c + c^\dagger)\rho]\rho \end{aligned}$$

Hilbert space dimension very large $\sim (2f+1)^N$

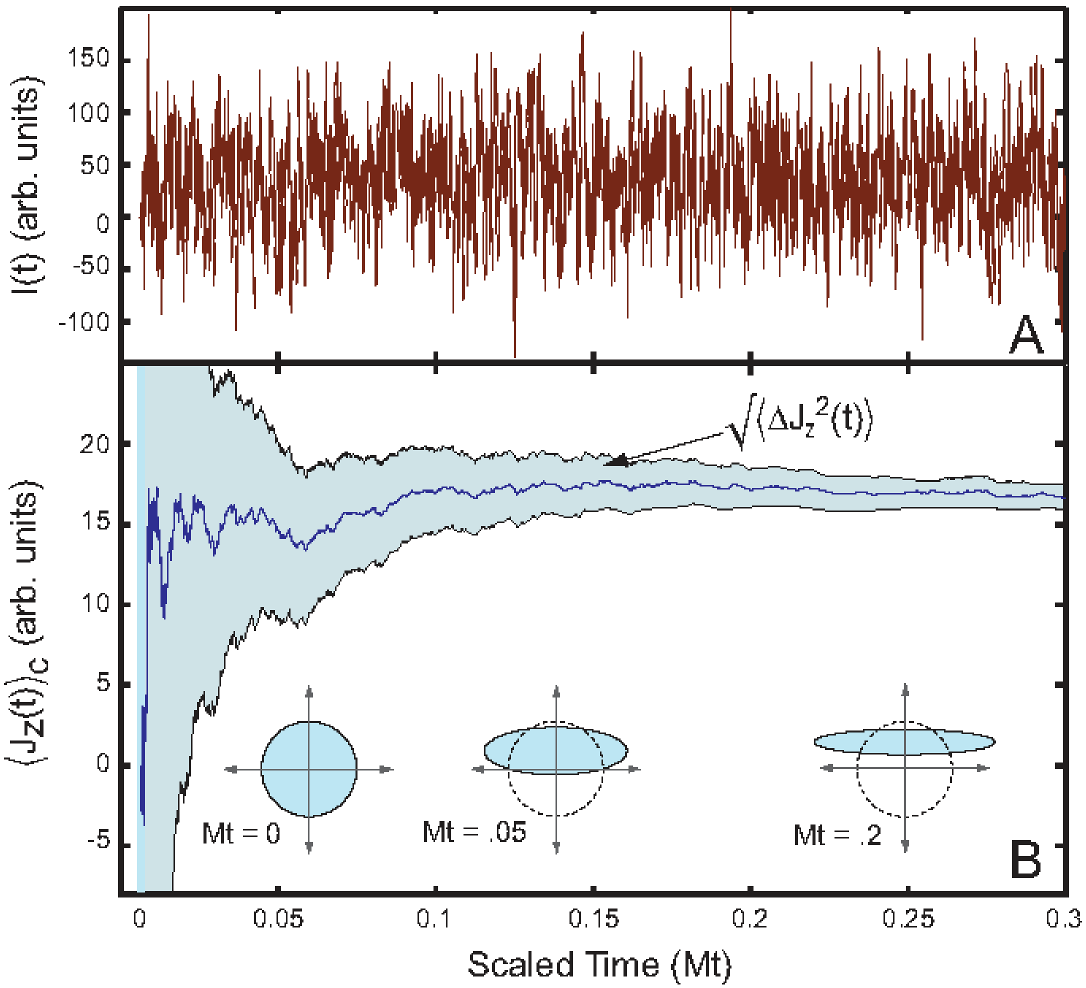
Reduction by symmetry to $\sim 2f \times N$



- restriction to “back-action evading” operators
- flat approximation to local phase space
- moment expansion in one operator of interest
- Gaussian truncation...

Filtering equations for atomic spins

$$d\left\langle \hat{F}_z \right\rangle = \gamma B F e^{-Mt/2} dt + 2\sqrt{M\eta}\left\langle \Delta \hat{F}_z^2 \right\rangle dW_t,$$
$$d\left\langle \Delta \hat{F}_z^2 \right\rangle = -4M\eta \left\langle \Delta \hat{F}_z^2 \right\rangle^2 dt.$$



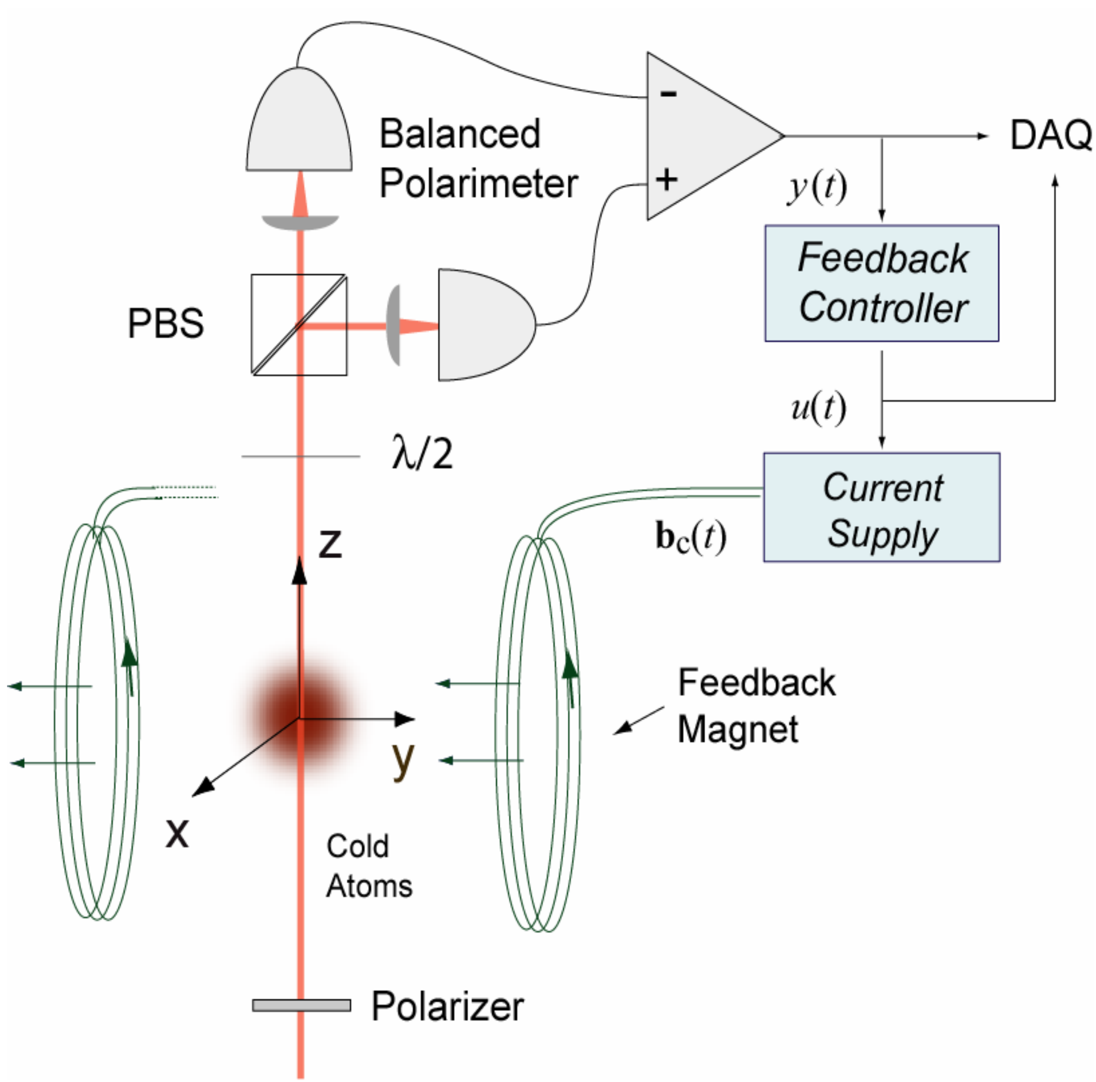
$$d\Xi = \frac{I_c(t)dt}{2\eta\sqrt{M}}$$
$$= z dt + \frac{1}{2\sqrt{M\eta}} dW_t$$

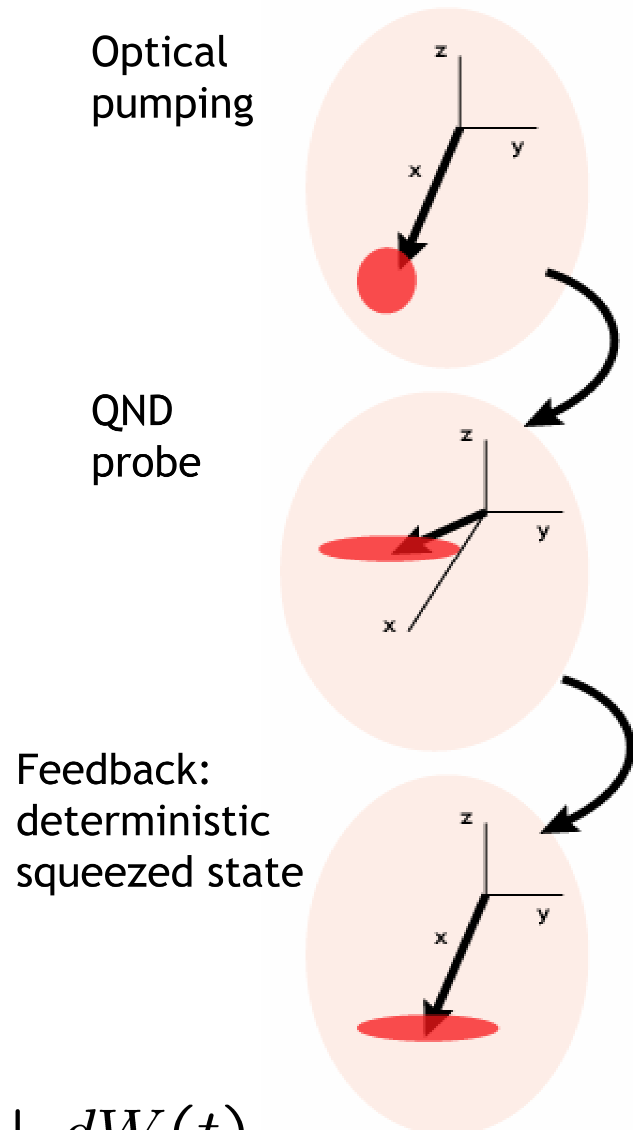
Conditioning of evolution on observed photocurrent gives rise to stochastic localization of F_z on timescale ~ F^{-1}

Model validation...?

Deterministic preparation of spin-squeezed states

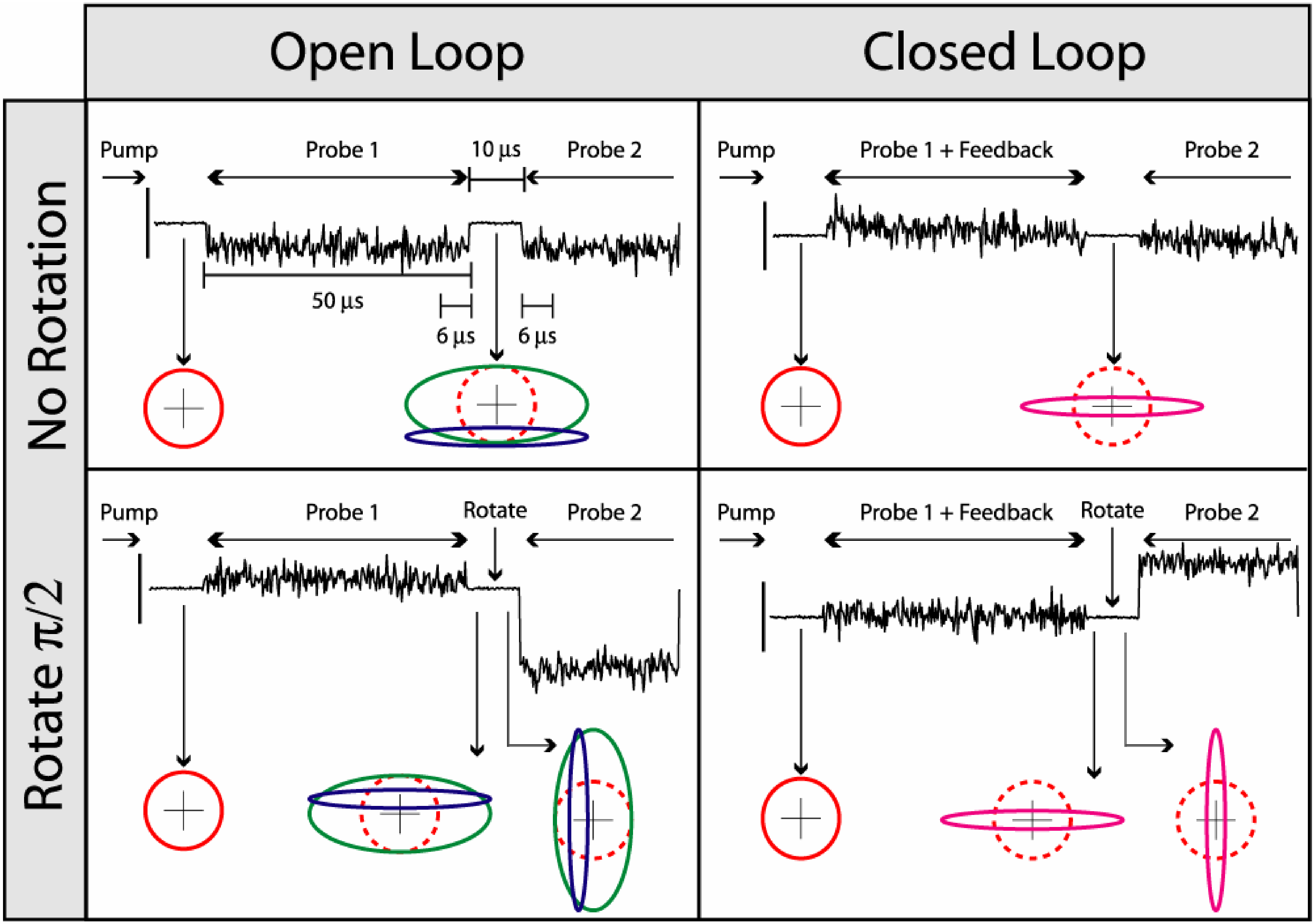
(L.K. Thomsen, S. Mancini, and H.M. Wiseman: PRA **65**, 061801(R) (2002))





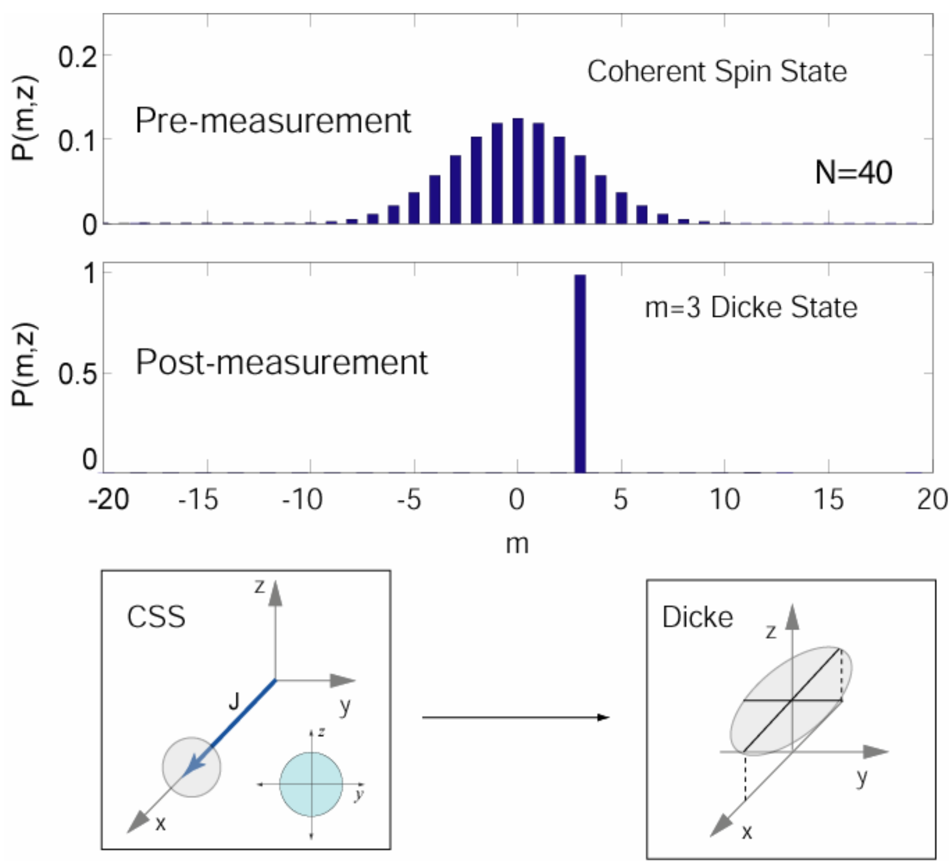
$$I_c(t)dt = 2\sqrt{M}\langle \hat{J}_z(t)\rangle_c dt + dW(t)$$

Trajectory data: random vs. deterministic states

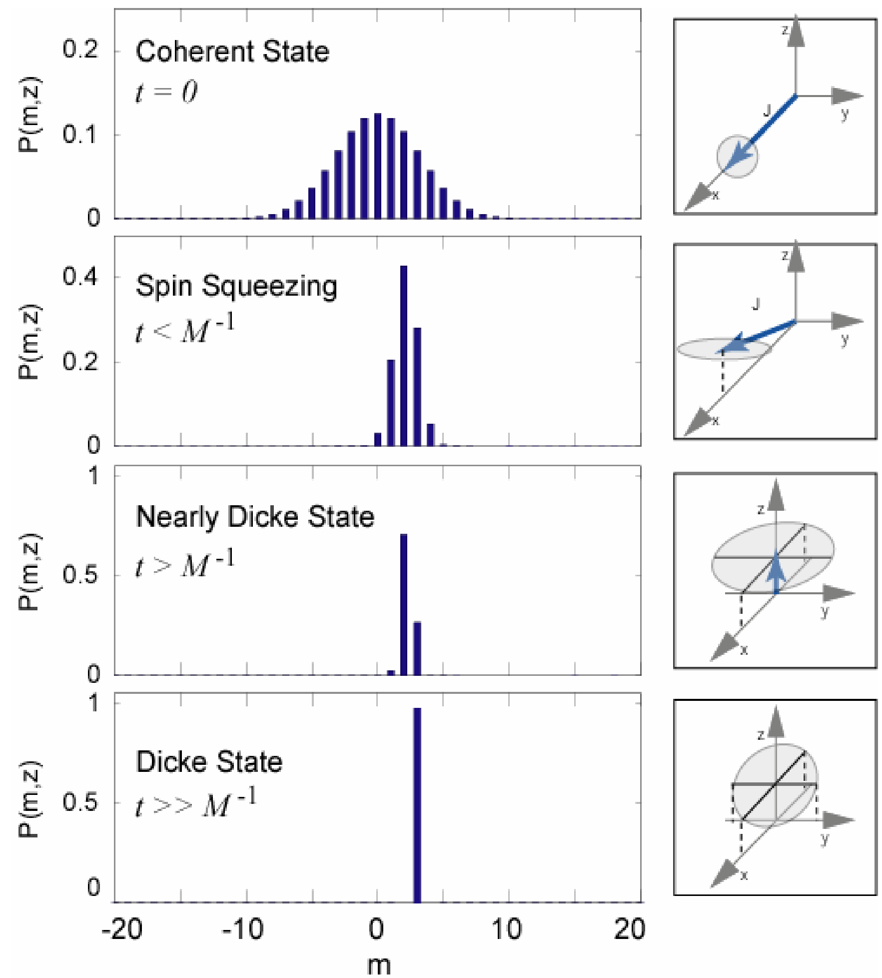


Resource requirements for deterministic state prep

<u>Impulsive</u> QND measurement:
random outcome, no simple means of conversion among Dicke states

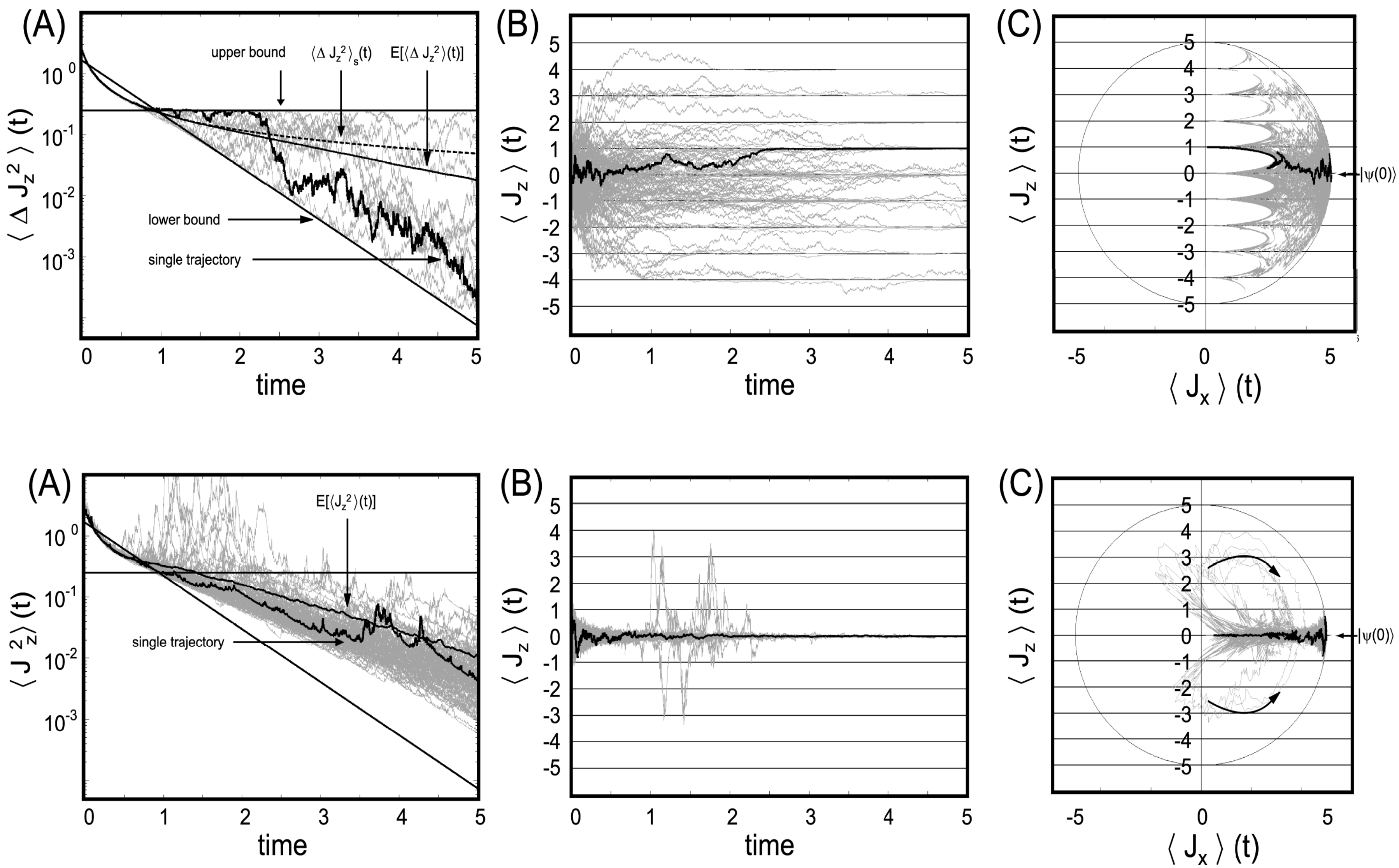


<u>Continuous</u> QND measurement:
convergence to Dicke state can be "guided" by applied magnetic field



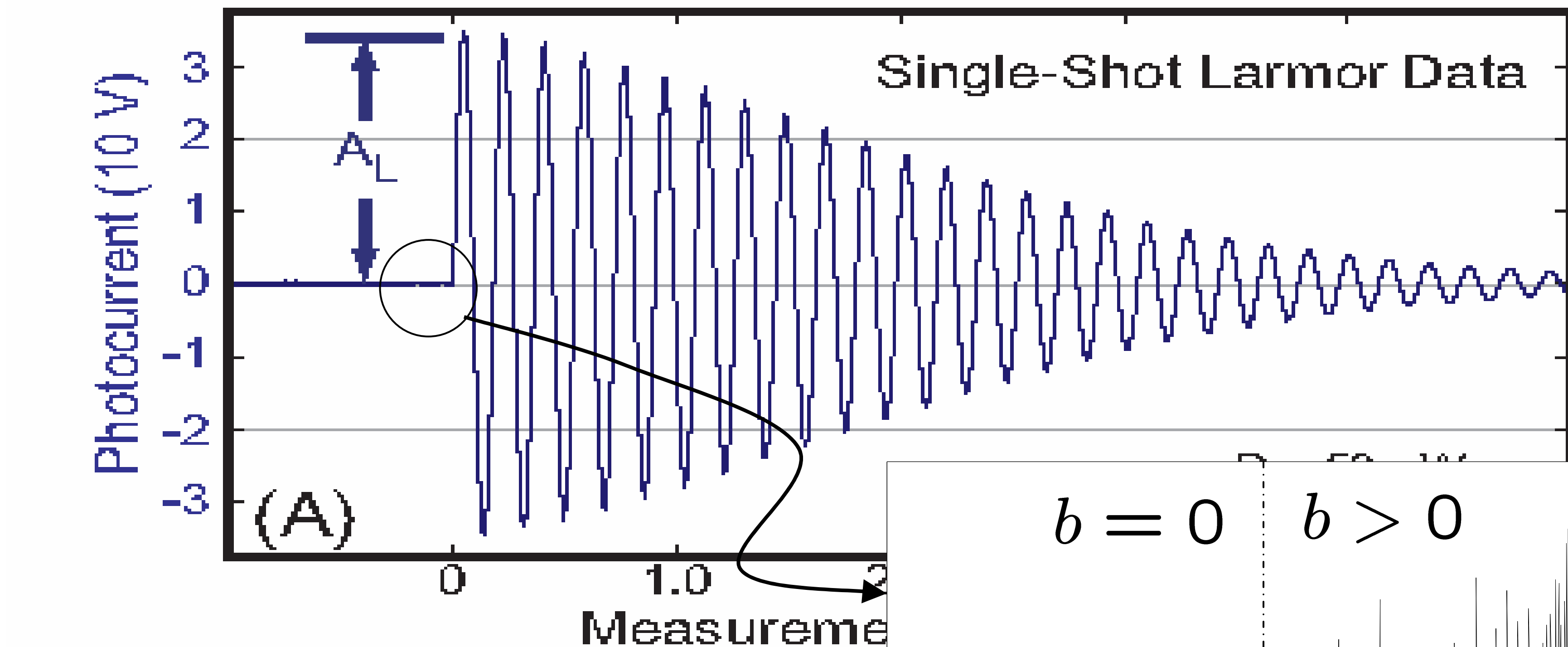
Deterministic Dicke-state preparation via feedback

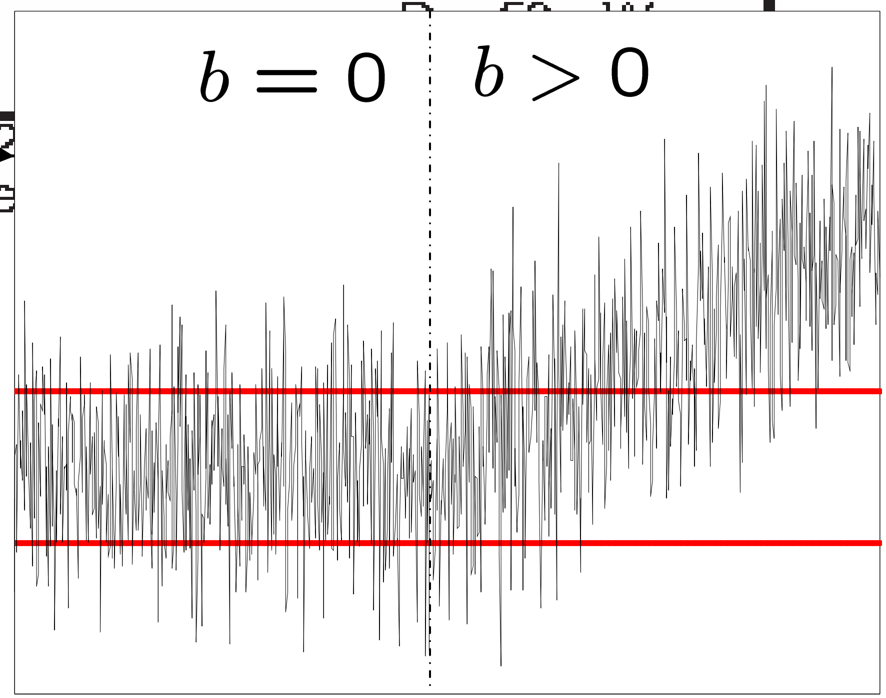
John Stockton



Broadband atomic magnetometry

Phys. Rev. Lett. **91**, 250801 (03)





- Spins integrate *b*
- estimate *b* from slope
- in practice Kalman filter…

- quantum variance hΔF_z^2i?

Open-loop sensitivity to parameter variation

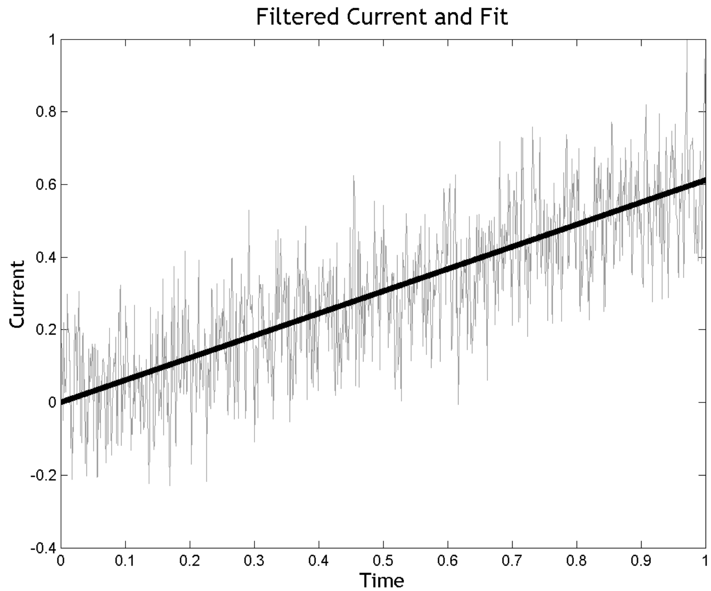
Canonical magnetometry is sensitive to atom number fluctuations

Photocurrent: $I = \gamma F b t + dW/dt$

Estimated Slope: $\hat{m}$

Correct Estimate: $\hat{b}_0 = \hat{m}/\gamma F$

Minimal Error: $E[(\hat{b}_0 - b)^2] \propto 1/F^2$



Incorrect assumed number: $\tilde{F} \neq F$

Incorrect estimate: $\hat{b} = \hat{m}/\gamma\tilde{F} = \hat{b}_0 F/\tilde{F}$

Resulting error: $E[(\hat{b} - b)^2] = E[(\hat{b}_0 - b)^2] + \frac{E[\Delta F^2]}{\tilde{F}^2}(b^2 + E[(\hat{b}_0 - b)^2])$

$E[\Delta F^2]$ represents shot-to-shot fluctuations of actual number

Robustness to uncertainty in atom number

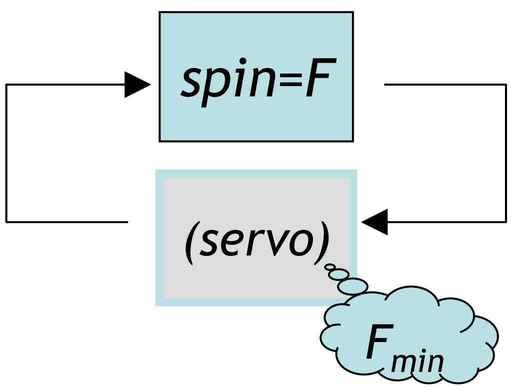
Simple Conservative Strategy (still valid in spin-squeezing regime):

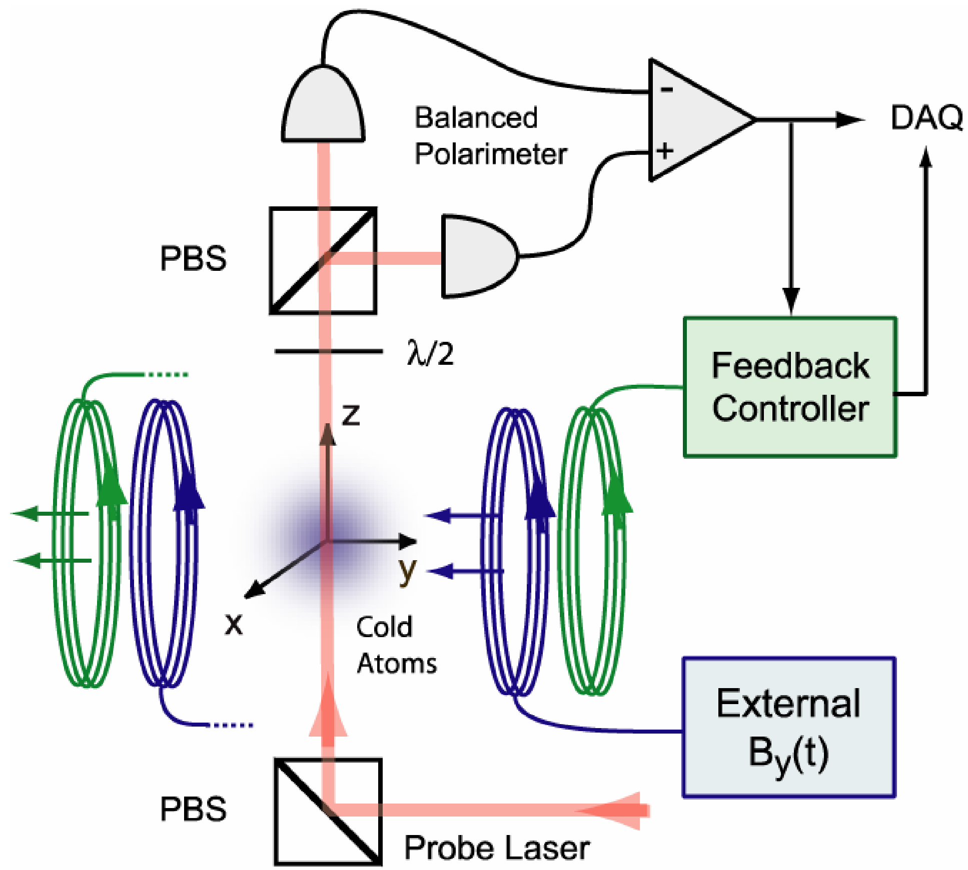
Given F distribution with minimum F_{min}, design the controller for F_{min}.

Design controller with $\tilde{F}$

Black: $\tilde{F} = F$

Blue: $\tilde{F} = F_{min} < F$

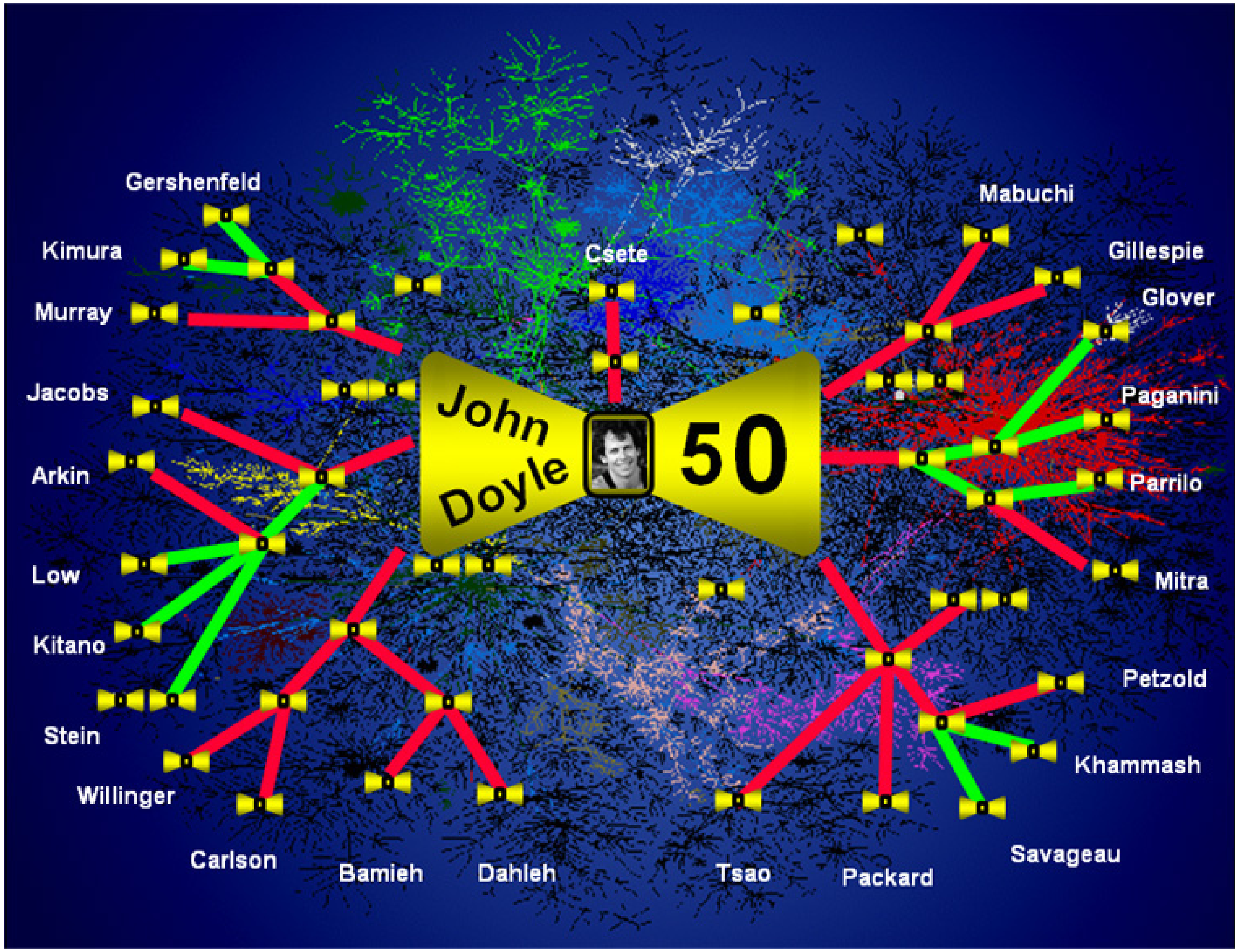




For $F \geq F_{min}$, it can be shown that the tracking error obeys the relation

$$E((\hat{b} - b)^2, F, t*) \leq E((\hat{b} - b)^2, F_{min}, t*) = C/F_{min}^2 \text{ (LQG, robust } \mathcal{H}_\infty)$$

Feedback and robustness in *quantum* systems



- Suppress quantum+classical uncertainties via real-time feedback
- Huge need for rigorous input-output model (filter) reduction
- Need nonlinear robust control (evaluation & design of stability)