

ROBUSTNESS ANALYSIS OF A WIND POWER PLANT

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Abstract Stability and performance properties of a wind power plant are analyzed. In particular, the nonlinear effects of time-varying wind speed are considered. The stability analysis makes use of the Nyquist theorem, the Circle Criterion, and a generalization for periodic perturbations. The robust performance analysis is done by optimization of linear matrix inequalities.

Keywords Uncertain System, Robust Stability, Periodic Uncertainty, Wind Power, and Linear System

1. Introduction

A wind power plant offers several challenges for control engineers. It should smoothly produce electrical power under large variations in the working environment. The plant should therefore be robust against wind variations and other types of perturbations. One example of such a perturbation is the lee of the tower. Each time a blade passes behind the tower, some aerodynamic torque is lost.

In analysis of control systems the uncertainty of the model is important since the presence of feedback can amplify the sensitivity to perturbations. Controllers that are designed to optimize certain design criteria, may at the same time increase the sensitivity to unmodelled dynamics.

The theory of robust control offers opportunities to address these issues. The purpose of this paper is to demonstrate this on a realistic application. The example describes a wind power plant with uncertain parameters and significant non-linearities. It is analyzed in a simple framework combining techniques from absolute stability theory and numerical optimization of linear matrix inequalities.

The analyzed plant and controller design has been successfully operating for several years. The theory of this paper is confirmed on a sophisticated and complex (simulation) model of the real system [2]. This nonlinear model is much more advanced than the linearized model used for controller design.

2. The Wind Power Plant Model

In the control design of the power plant the two most important parts are, not surprisingly, the turbine and the generator. The synchronous generator rotates with

synchronous speed imposed by the electrical net. The net is large and the influence from the wind power plant can be neglected. The turbine is connected to the generator through a "soft shaft", which enables the turbine to oscillate relatively to the generator.

The Turbine The driving aerodynamical torque is modeled by static and two-dimensional airfoil theory to the cross sections of the blades. The equation of thrust is linearized to get a simpler equation with partial derivatives with respect to the different expression. For the aerodynamical torque T driving the turbine we get

$$\Delta T = J(K_\beta \Delta \beta + K_U \Delta U_0 + K_\psi \Delta \psi + K_\gamma \Delta \gamma) \quad (1)$$

where ψ is the turbine velocity, U_0 is the wind speed and β is the pitch angle of the blade. The pitch angle β is very important since this is a variable we can control.

The torque $T(\beta, U, \psi, \gamma)$ is very dependent on the wind U and its powers up to the third degree (U^3 , U^2 and U). It also depends linearly on β and on the second power of ψ (ψ^2) (see Section 2.2).

The basic dynamics of importance for pitch angle control is given by the oscillations of the turbine against the electrical grid as depicted in Figure 1. This figure should be interpreted as if the electrical network is rotating with constant synchronous speed (ψ_0). The "soft shaft" illustrated by the damper and spring transfers the kinetic energy of the turbine to the generator. In reality this "soft shaft" is the torsion (γ) of the drive train in the gear, and can be viewed as a dynamic mode linking the turbine to the electrical system. For the imposed motion γ of the turbine the equation of motion is

$$\begin{aligned} \Delta T &= J\Delta\ddot{\gamma} + D_s\Delta\dot{\gamma} + K_s\Delta\gamma \\ \Delta\psi &= \Delta\dot{\gamma} \end{aligned} \quad (2)$$

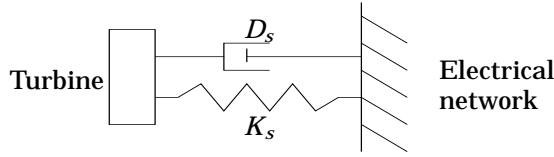


Figure 1. The basic dynamics

where J represents the turbine inertia, and K_s and D_s are spring and damping coefficients.

This simple model contains the first oscillating mode, where the turbine oscillates against the electrical part. The second mode which is the oscillations of the rotor of the generator against the grid can be neglected. The natural frequency of this mode is 25 times higher than the frequency of the first mode and it cannot be excited by the wind variations or damped by the pitch angle control.

The Synchronous Generator generates electrical power, P_E , which is given by $\dot{\psi} T_E$ where T_E is the electrical torque. From Figure 1 we get

$$\Delta P_E = \dot{\psi}_0 T_E = \dot{\psi}_0 (D_s \Delta \dot{\gamma} + K_s \Delta \gamma) \quad (3)$$

The Blade Servo is also important since this will be our only control variable. The dynamics of the servo with the blades is described by

$$T_{bs} \dot{\beta} = \beta_r - \beta \quad (4)$$

where β_r is the control-input to the servo, and T_{bs} is a time constant (0.4s). It is also worth noticing that the servo cannot turn the blade faster than 6 °/s.

A Linearized State Space Model

The linearized state-space representation around an operating point is developed from the dynamic equations.

$$\begin{aligned} \dot{x} &= Ax + B\beta_r + B_U U \\ y &= Cx \end{aligned}$$

$$\begin{aligned} x &= \begin{bmatrix} \Delta \beta \\ \Delta \dot{\gamma} \\ \Delta \gamma \end{bmatrix} & A &= \begin{bmatrix} -\frac{1}{T_{bs}} & 0 & 0 \\ K_\beta & K_{\dot{\psi}} & K_\psi \\ 0 & 1 & 0 \end{bmatrix} \\ B &= \begin{bmatrix} \frac{1}{T_{bs}} \\ 0 \\ 0 \end{bmatrix} & B_U &= \begin{bmatrix} 0 \\ K_U \\ 0 \end{bmatrix} \\ y &= \begin{bmatrix} P_E \\ \dot{\psi} \end{bmatrix} & C &= \begin{bmatrix} 0 & \frac{\dot{\psi}_0 D_s}{10^6} & \frac{\dot{\psi}_0 K_s}{10^6} \\ 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

The torque $T(\beta, U, \dot{\psi}, \psi)$ is dependent on all the variables β , U , $\dot{\psi}$ and ψ . This model is used to design a control system for the plant.

LQG Design

When designing the control of the wind power plant (as well as constructing the whole plant) one of the main goals is to keep the maintenance cost as low as possible. An LQG design has therefore been made, that penalizes β in the cost function to restrict the movement of the servo — thus increase the lifetime of the plant.

When designing the LQR control law for this model, the dynamics for the wind was assumed to be:

$$\Delta \dot{U} = \frac{\Delta U}{T_w} + \sigma_w \sqrt{\frac{2}{T_w}} w \quad (5)$$

where w is white noise with zero mean, the time constant $T_w = 20$ s and the standard derivation σ_w is 5-20% of U (the average wind speed over the disc).

Wind Nonlinearity

The design was based on a linearized model at the operating point of $U=18$ m/s. The power plant is running for winds in the range of 7-27 m/s, which stretches the reliability of the model very far, since K_β , $K_{\dot{\psi}}$ and K_U are dependent on the wind speed. Obviously, the parameter K_U is of no importance when we consider the stability of the model, since it is not in the loop.

The parameters K_β and $K_{\dot{\psi}}$ on the other hand, have a great impact on stability and will be studied in detail. The parameter K_β depends non-linearly on U , but is independent of β . In the state space model K_β is multiplied by β , so there is a linear relationship to the uncertain parameter. Moreover, K_β can be seen as a time varying uncertain parameter. The gain range used is $K_\beta \in (0, 4)$ which is the interval corresponding to the wind variation found in [2]. When analyzing the uncertain parameter it is sometimes convenient to split the nominal part from the uncertain part. The parameter can then be represented as:

$$K_\beta = \bar{K}_\beta + \phi(t) \quad (6)$$

where the nominal value $\bar{K}_\beta = 2$ and $\phi(t) \in [-2, 2]$ is a time varying function.

A study of $K_{\dot{\psi}}$ has shown that it behaves very similar to the uncertain K_β with respect to the wind speed. This makes it possible to lump them together as one scalar uncertainty and write $A(t)$ as:

$$\begin{aligned} A &= \bar{A} + b\phi(t)c \\ &= \begin{bmatrix} -\frac{1}{T_{bs}} & 0 & 0 \\ \bar{K}_\beta & \bar{K}_{\dot{\psi}} & K_\psi \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \phi(t) [1 \quad -0.33 \quad 0] \end{aligned}$$

With

$$M(s) = c(sI - \bar{A})^{-1}b$$

this makes it possible to use classical graphical frequency-domain criteria to analyze the system as the SISO loop shown in Figure 3 .

Periodic Uncertainties — Tower Blockage

When dealing with periodic systems, there is a great chance that the uncertainty also is periodic. When analyzing periodic uncertainty, the periodicity gives new and important information. Thus, more advanced frequency domain stability criteria can be utilized. These criteria take advantage of the periodicity as well as the sector limits of the uncertainties.

When the turbine spins around its own axis with a constant angular-velocity ($P = \dot{\psi}_0 = 2.618 \text{ rad/s}$), the blades align with the tower with a frequency of $2P$. This power plant operates in downwind and the tower, therefore, blocks the wind and steals aerodynamic torque from the blade. Moreover, the parameter concerning K_β varies significantly from its assumed constant value in the design-model of the system. The blade is in the lee for about 10° every time a blade swings behind the shaft. Thus, the aerodynamic torque-drop can be modeled by decreasing K_β to the half for a short while twice every rotation of the wind turbine.

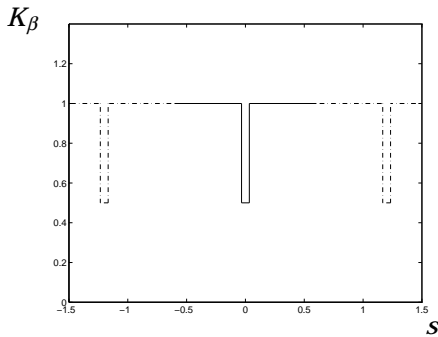


Figure 2. Perturbation due to Tower Blockage.

Nyquist Criterion

To analyze the stability robustness of the system, the nominal model is analyzed together with an uncertain feedback loop as in Figure 3.

A natural starting point for the analysis is the Nyquist criterion, which shows stability for constant values of ϕ . The inverse Nyquist curve was plotted for this system (and scaled so that the x-axis correspondS

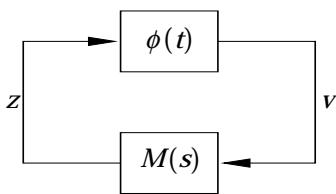


Figure 3. Stability analysis with uncertainty

directly to K_β). For this system the time invariant system was found to be stable for $K_\beta \in [0, \infty)$.

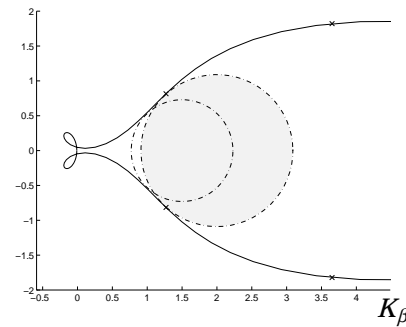


Figure 4. Inverse Circle Criterion used for stability analysis of uncertainty

This means that the system is stable for constants values of ϕ in the whole range of wind speeds considered.

Circle Criterion

For time-varying uncertainty, the circle criterion may be used.

This more conservative evaluation gives circles for which stability can be shown for time varying $\phi(t)$ (see Figure 4). If we choose a circle centered in the operating value of $K_\beta (= 2)$ the interval can be found to be $K_\beta \in [0.91, 3.09]$. Thus, the system is stable for all time varying $\phi(t)$ in this interval.

Periodic Perturbation

The perturbation due to the tower blockage (Figure 2) corresponds to the circles shown in Figure 5. The right edge of the circles describes the value of K_β due to the wind, and the left edge is the droop in K_β due to the tower blockage. Notice that the Circle Criterion leaves us inconclusive for small values of K_β — the circle with nominal condition $K_\beta = 0.5$ crosses the Nyquist curve.

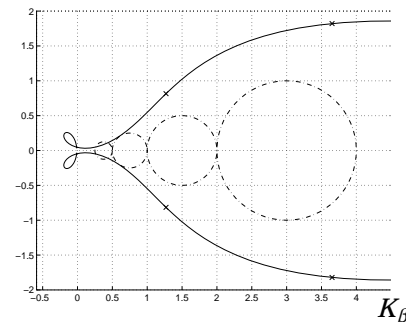


Figure 5. Inverse Circle Criterion with Circles Corresponding to the Tower Blockage

By applying an extended stability theorem for systems with periodic coefficients, as stated by Willems [8], the stability can be shown for this more conservative system.

PROPOSITION 1—WILLEMS, 1968

Let $M(s) = c(sI - \bar{A})^{-1}b$ with \bar{A} HURwitz. The system

$$\dot{x} = (\bar{A} + b\phi(t)c)x$$

is asymptotically stable for every piecewise continuous T -periodic $\phi(t) \in [\alpha, \beta]$ provided that there exist a rational function $F(s)$ such that for all ω

$$0 \leq \text{Re } F(j\omega)$$

$$F(j\omega) = F\left(j\omega + j\frac{2\pi}{T}\right)$$

$$0 < \text{Re } \left\{ F(j\omega) \frac{\beta M(s) + 1}{\alpha M(s) + 1} \right\}$$

□

Another way of stating the conditions is the following.

1. the Inverse Nyquist locus of $M(s)$ encircles the point α on the negative real axis of the Nyquist plane ρ times where ρ is the number of open-loop poles of $M(s)$ in $\text{Re}(s) > 0$,
2. there exist a circle C through the points $-1/\alpha$ and $-1/\beta$, such that the Nyquist locus of $M(s)$ for $\omega \geq 0$ does not intersect it,
3. C does not enclose any multiple of the periodic frequency (ω_0) of $\phi(t)$ for $\omega \leq 0$.

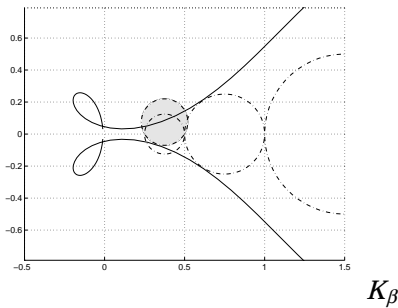


Figure 6. Inverse Nyquist Plot and Frequency Analysis of Periodic Uncertainty (notice that the axis in my plot is scaled to correspondence with K_β see Equation 6)

From Figure 6, the stability of the periodic uncertainty can be shown for small values of K_β by constructing similar smaller circles to C .

Some interesting remarks can be added to the conclusion from this theorem. First, the frequency of the periodic uncertainty is of great importance. This can be illustrated by examining the system operating at the constant wind at the operating point ($K_\beta = 2$). Let the uncertainty be assumed to swing the same amount up and down, thus, centering the circle at $K_\beta = 2$. We then see that for $\omega_0 = 2P$ the circle can be made very big

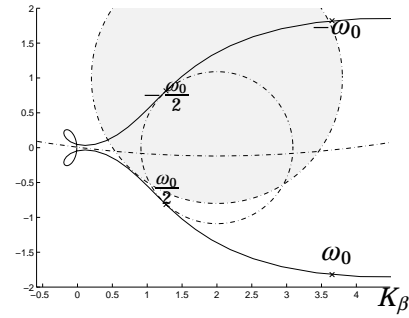


Figure 7. Inverse Nyquist Plot and Frequency Analysis of Periodic Uncertainty

(see Figure 7) but for lower frequencies the uncertainty sector (or circle) decrease.

Thus by restricting the uncertainty to be periodic the uncertainty sector can be increased with increasing frequency of the periodicity (see figure 7).

3. Robust Performance

Certainly, stability is only a necessary condition for smooth operation of the power plant. In this section, the analysis is pushed further, to get a bound on the variations in β . For this purpose, we consider variations

$$K_\beta(t) \in \left[\frac{2}{3} \bar{K}_\beta, \frac{4}{3} \bar{K}_\beta \right]$$

for different values of \bar{K}_β . The model is written on the form

$$\begin{bmatrix} \dot{v} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ U \end{bmatrix}$$

$$w(t) = \phi(t)v(t)$$

where $\phi(t) \in [-1, 1]$ correspond to the allowed interval of K_β . See Figure 8.

The gain of the operator $U \mapsto \dot{\beta}$ is used as a measure of how fast pitch-movement is needed to compensate for wind variations. For the nominal plant (i.e. $\phi(t) = 0$) the transfer-function from u to $\dot{\beta}$ is given by $M_{22}(s)$ and the maximum amplitude can then be found to be 0.11 (rad/s)/(m/s) occurring at a frequency of approximately 4 rad/s.

The assumption that $|\phi(t)| < 1$ for all t can be written as a quadratic inequality

$$\int |w|^2 d\omega \leq \int |v|^2 d\omega \quad (7)$$

where $w(t) = \phi(t)v(t)$ (see Figure 8). Hence if one can find $x, \gamma \geq 0$ such that

$$x\|v\|^2 - x\|w\|^2 + \|\beta\|^2 - \gamma^2\|U\|^2 \leq 0 \quad (8)$$

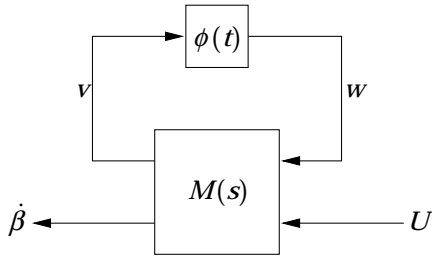


Figure 8. Robust Performance Analysis

for all $v, w, \dot{\beta}, U$ satisfying the system equations, then it follows that

$$\int |\dot{\beta}|^2 d\omega \leq \gamma^2 \int |U|^2 d\omega \quad (9)$$

and γ serves as a bound for the gain of the operator $U \mapsto \dot{\beta}$.

This approach is known as the \mathcal{S} -procedure and the crucial problem is to find valid $x, \gamma \geq 0$. For this purpose, the inequality (8) can be written as

$$x \|M_{11}w + M_{12}U\|^2 - x \|w\|^2 + \|M_{21}w + M_{22}U\|^2 - \gamma^2 \|U\|^2 \leq 0 \quad \forall w, U$$

or equivalently

$$\begin{bmatrix} M(j\omega) \\ I \end{bmatrix}^* \text{diag} \{x, 1, -x, -\gamma^2\} \begin{bmatrix} M(j\omega) \\ I \end{bmatrix} \leq 0 \quad \forall \omega$$

This problem is of a form which can easily be solved numerically utilizing modern optimization algorithms in MATLAB's LMI-toolbox.

Notice that the inequality (8) does not take into account the periodicity of the uncertainty.

When minimizing γ for the robust performance of the input output pair U to $\dot{\beta}$, the solution is $\gamma = 0.19$ (rad/s)/(m/s) which gives us the upper limit of the amplification. Thus, we know that the gain is at least 0.11 and at most 0.19. Recall that the lower bound is found by looking at the maximum amplitude of $M_{22}(s) \forall s$.

How much pitch movement can we tolerate? The dynamics of the wind tell us that wind-change in itself is not a big problem compared to the uncertainty of the tower blockage.

Robust Performance under Tower Blockage

When analyzing for the perturbation of the tower blockage, the circles used is corresponding to the blockage. (see Figure 5). The interval of γ is then determined for each of the circles and the result is presented in Figure 9 Thus, since the tower blockage occurs with a frequency of 5.2 rad/s, γ in the range of 0.4 (rad/s)/(m/s)

will force the servo to oscillate very much. In fact $\gamma \in [0.11, 0.42]$ will force the system to oscillate at the frequency $2P$ with amplitudes of $0 - 6^\circ$. The amplitude interval due to the tower-blockage is a function of the nominal value of K_β . Figure 9 shows that the robust performance is best around the nominal value $K_\beta = 2$ which also is the value the controller is designed for.

When evaluating these values of the gain from $\|U\|$ to $\|\dot{\beta}\|$, it is found to be too large, thus creating unacceptable oscillations at the frequency of $2P$.

Notice that γ increases rapidly as the $|\Delta\phi|_{max}$ increases (linearly with \overline{K}_β).

4. A more robust design

Since so much of the noise is frequency dependent with the frequency $2P$, a penalty on β for this frequency is wanted. This is done by introducing an inverse notch-filter (see Figure 10) on of frequency $2P$ to the $\dot{\beta}$. This filter is implemented in the state-space model, and the feedback is designed with weight on $\dot{\beta}$ by LQG.

We then follow the same analysis step for this new design and look at the gain (γ) from $\|U\|$ to $\|\dot{\beta}\|$ for the tower-blockage perturbation. The new intervals for γ are plotted in Figure 11.

The gain has decreased with a factor of more than 8 for operating points up to $K_\beta = 2.5$, and the oscillations is not longer a big problem. For comparison the gain interval is plotted as a function of the operating point. From Figure 11 we see that the inverted notch-filter has less effect when $|\Delta\phi|_{max}$ is high. Thus, the new design is not improved by more than a factor 2 for wind-speeds in the range of 27 m/s.

5. Conclusions

When deriving conclusions from the findings a great deal of engineering judgment is needed. One example of this is the study of stability for small K_β . When comparing the values of $K_\beta = 0.5$ with the corresponding wind speed, the wind speeds are in the range of < 7 m/s.

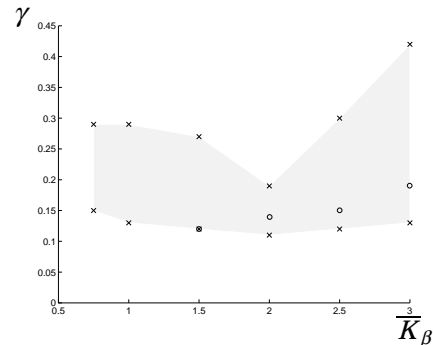


Figure 9. The interval of the gain from $\|U\|$ to $\|\dot{\beta}\|$ as a function of K_β . The o 's correspond to simulated values

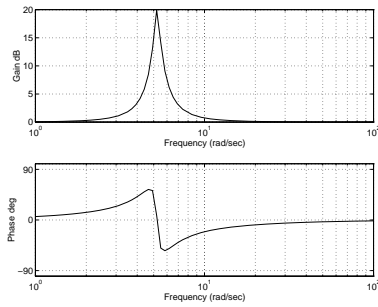


Figure 10. Inverse Notch-filter for Penalizing the 2P Frequency

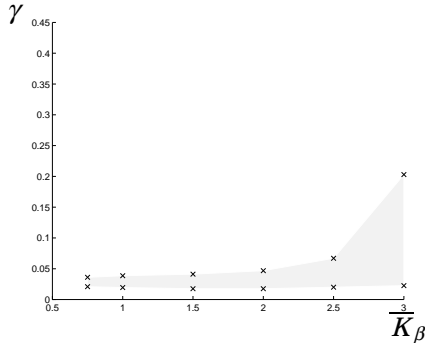


Figure 11. The interval of the gain from $\|U\|$ to $\|\beta\|$ as a function of K_β

The stability in this region is not very critical since this is barely enough to make the turbine rotate (the operating wind range is 7 – 27 m/s).

Through this paper, much work have been devoted to periodic uncertainties. In Figure 12 we see how these uncertainties can be allowed to vary more when their frequency increases. This is, in our opinion, rather interesting. Since the system itself behaves in a periodic fashion many of the uncertainties will also behave periodically (i.e. tower blockage).

Notice that uncertainties in the torque with a frequency twice the frequency of the turbine rotation (which is the case for the tower blockage) not is very critical for the system stability. It is also interesting to notice that $\omega_0 = P$ (the frequency of the turbine) only leaves room for a small circle, and the worst range of frequencies is $\omega_0 \in [0, P]$.

This analysis is interesting and positive for the

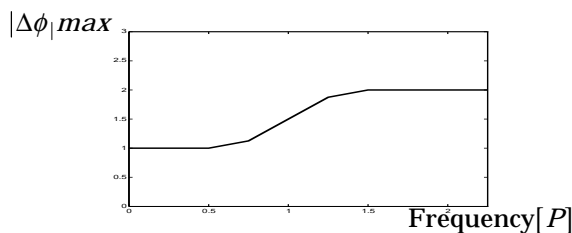


Figure 12. Inverse Nyquist plot and frequency analysis of periodic uncertainty

stability of the Wind Power Plant since there is no indication in the simulations of the plant that there are any elements that will introduce uncertainties in this frequency range. It would be even more positive if $\omega_0 = P$ not was in the range of low tolerance since it is natural that many uncertainties will occur with the same frequency as the turbine (i.e. imbalance in the blade weight of the blades, uncertainties of the gears due to wear).

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