CIMMS FOCUSED WORKSHOP

on

Multiscale Techniques for Dynamic Interfaces

Program & Abstracts

Beckman Institute Auditorium
Pasadena, California
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Sponsored by the
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FOCUSED CIMMS WORKSHOP 2003
Multiscale Techniques for Dynamic Interfaces

Beckman Institute Auditorium, Caltech

Program

FRIDAY March 7, 2003

MORNING SESSION CHAIR:  Steve Shkoller
09:00–09:10  Welcome, Opening Remarks:
Jerry Marsden
09:10–10:00  John Lowengrub, University of California, Irvine,
10:00–10:10  Short Break
10:10–11:00  James Glimm, Stony Brook University,
Numerical and Theoretical Methods for Determination of Fluid Mixing
11:00–11:30  Discussion
11:30–12:20  Tom Hou, California Institute of Technology,
Singularity Formation in 3-D Vortex Sheets
12:20–14:00  Lunch

AFTERNOON SESSION CHAIR:  Tom Hou
14:00–14:50  Jack Xin, University of Texas, Austin,
Dynamics of Basilar Membrane and Signal Processing of Sounds
14:50–15:00  Short Break
15:00–15:50  Phillip Colella, Lawrence Berkeley Laboratory,
Volume-of-Fluid Discretization Methods for PDE in Irregular Domains
15:50–16:20  Discussion
16:20–17:10  Shiyi Chen, John Hopkins University,
Hybrid Methods for Micro- and Nano-Fluidics
17:10–18:00  Discussion
18:00  Close for the day
SATURDAY March 8, 2003

**Morning Session Chair:** Shiyi Chen

09:00–09:50  *George Papanicolaou*, Stanford University  
Multiscale Methods for Stochastic Equations and Applications to Imaging  
in Cluttered Media

09:50–10:00  **Short Break**

10:00–10:50  *Tom Beale*, Duke University,  
Computing with Singular and Nearly Singular Integrals

10:50–11:20  **Discussion**

11:20–12:10  *Steve Shkoller*, University of California, Davis,  
On the motion of an elastic solid in an incompressible fluid

12:10–14:00  **Lunch**

**Afternoon Session Chair:** John Lowengrub

14:00–14:50  *Mike Shelley*, Courant Institute,  
Flexible Bodies Interacting with Moving Fluids

14:50–15:00  **Short Break**

15:00–15:50  *Stan Osher*, University of California, Los Angeles,  
The Level Set Method—what’s in it for you?

15:50–16:30  **Wrap up Panel Discussion**  
*Steve Shkoller* (Chair) University of California, Davis  
*Kaushik Bhattacharya*, California Institute of Technology  
*Michael Ortiz*, California Institute of Technology  
*Jerry Marsden*, California Institute of Technology  
*Stan Osher*, University of California, Los Angeles  
*James Glimm*, Stony Brook University  
*Tom Hou*, California Institute of Technology

16:30  **Close of the Workshop**
Contents

Friday Speakers

John Lowengrub ...................................................... 2
James Glimm ......................................................... 2
Thomas Y. Hou ......................................................... 3
Jack Xin ................................................................. 4
Phillip Colella ......................................................... 4
Shiyi Chen ............................................................... 5

Saturday Speakers

George Papanicolaou .................................................. 6
J. Thomas Beale ........................................................ 7
Steve Shkoller .......................................................... 8
Mike Shelley ........................................................... 8
Stan Osher .............................................................. 9
Friday March 7, 2002

SPEAKERS
Modeling and Simulation of Multicomponent and Multiphase Fluid Flows

JOHN LOWENGRUB
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Abstract
In this talk, I will discuss a modeling effort to describe fluid flows with deformable interfaces. The model is physically based and is capable of describing a wide variety of interface phenomena including interface pinchoff and reconnection as well as phase-transitions. We will validate the model by comparisons to theory and experimental results for a number of different fluid flows. We discuss practical issues associated to the model as well as the incorporation and effectiveness of adaptive mesh refinement.

Numerical and Theoretical Methods for Determination of Fluid Mixing

JAMES Glimm  Applied Mathematics and Statistics
Stony Brook University
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Abstract
Fluid mixing is a close relative of fluid turbulence, in that both describe multiscale, chaotic fluid flow regimes. Of the two, the mixture problem, in which two distinct fluids become intertwined and entangled, is harder, as the number of important distinct flow regimes appears to be larger. We study flow regimes generated by acceleration processes, directed across a density gradient or fluid interface discontinuity and we study regimes in which the major flow structures are of substantial size (the chunk flow regime). Two idealized cases are steady acceleration (Rayleigh-Taylor, or RT mixing) and impulsive (Richtmyer-Meshkov, or RM mixing).

We present numerical and theoretical studies conducted jointly with coworkers on the growth of such mixing layers. We will present theoretical and numerical estimates of the growth rate which fall within the range of experimental values (for chaotic RT mixing), and we will offer an explanation (numerical mass diffusion) for the failure of many simulations to accomplish this goal. In view of the strong nonlinearity and complexity of the problem and its solutions, it is remarkable that the theoretical methods are powerful, and in fact may be somewhat in advance of the numerical methods.
Singularity Formation in 3-D Vortex Sheets

THOMAS Y. HOU Department of Applied Mathematics, 217-50
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Abstract

One of the classical examples of hydrodynamic instability occurs when two fluids are separated by a free surface across which the tangential velocity has a jump discontinuity. This is called Kelvin-Helmholtz instability. Kelvin-Helmholtz instability is a fundamental instability of incompressible fluid flow at high Reynolds number. The idealization of a shear layered flow as a vortex sheet separating two regions of potential flow has often been used as a model to study mixing properties, boundary layers and coherent structures of fluids. It is well known that small initial perturbations on a vortex sheet may grow rapidly due to Kelvin-Helmholtz instability. The problem is ill-posed in the Hadamard sense. Most analytical studies of vortex sheet singularity to date rely heavily on complexifying the interface variables. It is not clear how to generalize this technique to 3-D vortex sheets in a natural way.

In a joint work with G. Hu and P. Zhang, we study the singularity of 3-D vortex sheets using a new approach. First, we derive a leading order approximation to the boundary integral equation governing the 3-D vortex sheet. This leading order equation captures the most singular contribution of the integral equation. Moreover, after applying a transformation to the physical variables, we found that this leading order 3-D vortex sheet equation de-generates into a two-dimensional vortex sheet equation in the direction of the tangential velocity jump. This rather surprising result confirms that the tangential velocity jump is the physical driving force of the vortex sheet singularities. We show that the singularity type of the three-dimensional problem is similar to that of the two-dimensional problem. Moreover, we introduce a generalized Moore’s approximation to 3-D vortex sheets. This model equation captures the same singularity structure of the full 3-D vortex sheet equation, and it can be computed efficiently using Fast Fourier Transform. This enables us to perform well-resolved calculations to study the generic type of 3-D vortex sheet singularities. We will provide detailed numerical results to support the analytic prediction, and to reveal the generic form of the vortex sheet singularity. We will also discuss the possible implication of our 3-D vortex sheet results on the shear flow instability in 3-D Euler and Navier-Stokes equations.
Dynamics of Basilar Membrane and Signal Processing of Sounds

JACK XIN
Department of Mathematics
University of Texas at Austin
Austin, TX, 78750, USA
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Abstract
We present a class of nonlinear nonlocal cochlear models of basilar membrane in the cochlea for processing multi-frequency sound signals. The models are based on classical mechanics and neural phenomenology. Time domain computation reveals nonlinear phenomena such as the suppression effects among tones, also tones and banded noise. We also discuss upscaling of model output to perception and analytical construction of multi-tone solutions.

Volume-of-Fluid Discretization Methods for PDE in Irregular Domains

PHILLIP COLELLA  Applied Numerical Algorithms Group
MS 50A-1148, Lawrence Berkeley National Laboratory
1 Cyclotron Rd., Berkeley, CA 94720
colella@davis.lbl.gov

Abstract
We give an overview of a set of numerical methods being developed for solving classical PDE in irregular geometries, or in the presence of free boundaries. In this approach, the irregular boundary is represented on a rectangular grid by specifying the intersection of each grid cell with the region on either side of the boundary. Such a description leads to natural extensions of conservative finite difference methods to the case of irregular boundaries. Some of the issues that arise include consistency and stability of such methods, methods for representing the surface geometry, and the interaction of the discretization method with the boundary conditions, particularly in the case of free boundaries.
Hybrid Methods for Micro- and Nano-Fluidics

SHIYI CHEN
Department of Mechanical Engineering
The Johns Hopkins University
105 Latrobe Hall
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Baltimore, MD, 21218, USA
syc@pegasus.me.jhu.edu

Abstract
In this talk, I will present a hybrid multiscale numerical method for micro- and nano-fluidics based on constrained dynamics which employs continuum description in one region and atomistic description in the another. The method is used to simulate time-dependent Couette flow, channel flow with rough wall and driven cavity. The simulated results from the multiscale method agree quantitatively with analytical solutions and predictions from full molecular dynamics simulations. Future applications of the multiscale method will be elaborated.
Saturday March 8, 2002

SPEAKERS
Multiscale Methods for Stochastic Equations and Applications to Imaging in Cluttered Media

GEORGE PAPANICOLAOU
Department of Mathematics
Stanford University
Stanford, CA, 94305, USA
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Abstract
I will present a resolution study for detection and imaging in random media based on a multiscale asymptotic analysis of the stochastic Schroedinger equation.

Computing with Singular and Nearly Singular Integrals

J. THOMAS BEALE
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Duke University
Durham, NC, 27708, USA
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Abstract
We will describe a simple, direct approach to computing a singular or nearly singular integral, such as a harmonic function given by a layer potential on a curve or surface. The value is found by a standard quadrature, using a regularized form of the singularity, with correction terms added for the errors due to regularization and discretization. These corrections are found by local analysis near the singularity. This technique might be useful in fluid calculations with moving interfaces, since a pressure term due to a boundary force can be written as a layer potential. We have used this approach to design a convergent boundary integral method for time-dependent, doubly periodic, three-dimensional water waves. The accurate evaluation of a layer potential near the curve or surface on which it is defined is not routine, since the integral is nearly singular. In work with M.-C. Lai, we solve boundary value problems in 2D by computing the integral at grid points near the curve in the manner described and using these to find values elsewhere. A similar approach works in 3D, with the surface integrals computed in overlapping coordinate grids on the surface. To solve a boundary value problem, we first need to solve an integral equation for the strength of a dipole layer on the surface. We have proved that the solution of the discrete integral equation converges to the exact solution.
On the Motion of an Elastic Solid in an Incompressible Fluid

STEVE SHKOLLER  Department of Mathematics  
University of California, Davis  
Davis, CA, 95616, USA  
shkoller@math.ucdavis.edu

Abstract  
We prove the existence of a unique solution for the motion of an elastic solid inside of an incompressible fluid on a short time interval.

Flexible Bodies Interacting with Moving Fluids

MICHAEL SHELLEY  Department of Mathematics and Neural Science  
The Courant Institute at New York University  
New York, NY, 10012, USA  
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Abstract  
I will discuss some important prototypical examples of flexible bodies interacting with fluid flows. One concerns understanding the reduction in drag, at high Reynolds number, that can accompany body deformability. I will connect this to the development of new numerical methods for this class of problems. The second problem is motivated by chemical engineering and biology. I will discuss my recent work on developing tractable numerical methods for studying the nonlocal hydrodynamics of slender elastic bodies at very low Reynolds number.
The Level Set Method — what’s in it for you?

STAN OSHER
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University of California, Los Angeles
Los Angeles, CA, USA
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Abstract
The level set method for capturing moving fronts was introduced in 1987 by Osher and Sethian. It has proven to be phenomenally successful as a numerical device. For example, typing in "Level Set Methods" on Google’s search engine gives roughly 3800 responses. Applications range from capturing multiphase fluid dynamical flows, to special effects in Hollywood to visualization, image processing, control, epitaxial growth, computer vision and many more. In this talk we shall give an overview of the numerical technology and a few new applications.
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**Beckman Institute Auditorium, Caltech**

<table>
<thead>
<tr>
<th>Time</th>
<th>FRIDAY March 7</th>
<th>Time</th>
<th>SATURDAY March 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00</td>
<td><strong>Welcome &amp; Opening Remarks</strong></td>
<td>09:00</td>
<td><strong>George Papanicolaou</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Jerry Marsden</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>09:10</td>
<td><strong>John Lowengrub</strong></td>
<td>09:50</td>
<td><strong>Short Break</strong></td>
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<tr>
<td>10:00</td>
<td><strong>Short Break</strong></td>
<td>10:00</td>
<td><strong>Tom Beale</strong></td>
</tr>
<tr>
<td>10:10</td>
<td><strong>James Glimm</strong></td>
<td>10:50</td>
<td><strong>Discussion</strong></td>
</tr>
<tr>
<td>11:00</td>
<td><strong>Discussion</strong></td>
<td>11:20</td>
<td><strong>Steve Shkoller</strong></td>
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<tr>
<td>11:30</td>
<td><strong>Tom Hou</strong></td>
<td></td>
<td></td>
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<tr>
<td>12:20</td>
<td><strong>Lunch</strong></td>
<td>12:10</td>
<td><strong>Lunch</strong></td>
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<tr>
<td>14:00</td>
<td><strong>Jack Xin</strong></td>
<td>14:00</td>
<td><strong>Mike Shelley</strong></td>
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<td>14:50</td>
<td><strong>Short Break</strong></td>
<td>14:50</td>
<td><strong>Short Break</strong></td>
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<tr>
<td>15:00</td>
<td><strong>Phillip Colella</strong></td>
<td>15:00</td>
<td><strong>Stan Osher</strong></td>
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<tr>
<td>15:50</td>
<td><strong>Discussion</strong></td>
<td>15:50</td>
<td><strong>Panel Discussions</strong></td>
</tr>
<tr>
<td>16:20</td>
<td><strong>Shiyi Chen</strong></td>
<td>16:30</td>
<td><strong>Close of Workshop</strong></td>
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<td>17:10</td>
<td><strong>Discussion</strong></td>
<td></td>
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<tr>
<td>17:45</td>
<td><strong>Close for the Day</strong></td>
<td></td>
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