Trade-Offs, Strategies and Negotiation in Engineering Design

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CIMMS Focused Workshop on Uncertainty Management in Engineering Design
May 23, 2002
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- William Law
- Kevin Otto
- Kris Wood
- Ford Motor Company
- Volkswagen, Wolfsburg
Traditional Engineering Design

Example: Vehicle Structure Design

The straightforward problem:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Current</th>
<th>Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_B$ (N/mm)</td>
<td>2480</td>
<td>2730</td>
</tr>
<tr>
<td>$K_T$ (N-m/deg)</td>
<td>4920</td>
<td>5420</td>
</tr>
<tr>
<td>$m$ (kg)</td>
<td>160</td>
<td>144</td>
</tr>
</tbody>
</table>
We’re changing everything.

Again.

100%
Electronic design

100%
Electronic pre-assembly

100%
Faster data transmission

100%
Brighter high beams

46%
More bending rigidity

37%
More torsional stiffness

24%
More standard horsepower

10%
Better fuel economy

10%
More trunk space
Load Test Results:

**TORSIONAL STIFFNESS TEST (WINDSHIELD AND REAR HATCH INTACT)**

**TEST DATE:** July 11

Indicator radius: 1.42 m (56 in)
moment arm: 1.65 m (65 in)

**Weight (lb), Deflection (0.001 in)**

<table>
<thead>
<tr>
<th>Weight (lb)</th>
<th>Deflection (0.001 in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>29</td>
<td>43</td>
</tr>
<tr>
<td>62</td>
<td>95</td>
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<tr>
<td>91</td>
<td>137</td>
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<tr>
<td>124</td>
<td>176</td>
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<tr>
<td>153</td>
<td>225</td>
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<tr>
<td>198</td>
<td>275</td>
</tr>
<tr>
<td>219</td>
<td>318</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Moment (N-m)</th>
<th>Deflection (mm)</th>
<th>Twist (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>128.99</td>
<td>212.84</td>
<td>1.09</td>
<td>0.04407</td>
</tr>
<tr>
<td>275.78</td>
<td>455.03</td>
<td>2.41</td>
<td>0.09736</td>
</tr>
<tr>
<td>404.77</td>
<td>667.87</td>
<td>3.48</td>
<td>0.14041</td>
</tr>
<tr>
<td>551.56</td>
<td>910.06</td>
<td>4.47</td>
<td>0.18208</td>
</tr>
<tr>
<td>680.54</td>
<td>1122.90</td>
<td>5.72</td>
<td>0.23060</td>
</tr>
<tr>
<td>845.12</td>
<td>1394.45</td>
<td>6.95</td>
<td>0.28184</td>
</tr>
<tr>
<td>974.11</td>
<td>1607.28</td>
<td>8.08</td>
<td>0.32591</td>
</tr>
</tbody>
</table>

**Regression Analysis:**

- **Moment (N-m) vs. Twist (deg)**
  - **y = mx + c**
  - **slope: 4960.74 N-m/deg**
  - **y - intercept:** -10.16 N-m

**BENDING STIFFNESS TEST (REAR HATCH AND WINDSHIELD INTACT)**

**TEST DATE:** July 16

**Deflection (in), Load (lbf)**

<table>
<thead>
<tr>
<th>Deflection (in)</th>
<th>Load (lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>64</td>
</tr>
<tr>
<td>0.007</td>
<td>124</td>
</tr>
<tr>
<td>0.013</td>
<td>188</td>
</tr>
<tr>
<td>0.031</td>
<td>450</td>
</tr>
<tr>
<td>0.036</td>
<td>514</td>
</tr>
<tr>
<td>0.046</td>
<td>638</td>
</tr>
</tbody>
</table>

**Regression Analysis:**

- **Deflection (mm) vs. Load (N)**
  - **y = mx + c**
  - **slope:** 2424.16 N/mm
  - **y - intercept:** 51.79 N

- **Deflection (mm) vs. Load (N)**
  - **y = mx + c**
  - **slope:** 2484.80 N/mm
  - **y - intercept:** 153.84 N

**Graphs:**

- **Moment (N-m) vs. Twist (deg)**
- **Deflection (mm) vs. Load (N)**
Problem: There’s more to the problem!

- Information is *imprecise*
- Designer judgement and experience
- Some targets are *not explicit* (informal):
  - style
  - manufacturability
  - availability
- Uncontrolled variations (noise)
- Negotiations:
  - targets may change
  - how do targets interact?
Difficulties in Making the Decision Computable

**Fuzziness:** Specifications are imprecise.

**Unmodelled Criteria:** Many (often crucial) decisions depend on unquantified or unmeasured criteria.

**Set-based design:** Sets provide rapid design exploration; facilitate concurrency and support iteration.

**Rationality:** Need to reconcile competing, incommensurate objectives in a rational way.

**Cost:** Computation cost to effectively explore a large design space can be significant.
Difficulties in Making the Decision Computable

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Most of engineering, particularly design, can best be represented with some level of imprecision or approximation.

“Fuzziness is more than the exception in engineering design problems: usually there is no well-defined best solution or design.”

Imprecision is the representation an incomplete design description: ranges of possibilities resulting from choices not yet made.

Preliminary design information is necessarily imprecise.

Designers need to evaluate designs early in the design process.

Designers need to rapidly explore large design spaces early in the design process.

Need to trade-off Precision vs. Computation Cost.
Functional Requirement $\mu_p(\vec{p})$:
the customer’s **expressed** preference(s) on $\vec{p}$,
based on *performance considerations*.

Performance considerations are customer specifications and requirements.

Design Preference $\mu_d(\vec{d})$:
the customer’s **un-expressed** preference(s) on $\vec{d}$,
based on *design considerations*.

Design considerations are the aspects of performance **not quantified** by preferences on $\vec{p}$.
Fuzziness: Imprecise Specification

\[ \mu_p \]

\[ 0 \]

\[ 1 \]

\[ 500 \text{ km} \]

\[ \text{range} \]

\[ \text{crisp} \]

\[ \text{fuzzy} \]
Fuzziness: Imprecise Design Variable

\[ \mu_{d_1} \]

\[ \alpha_M \]

\[ \alpha_1 \]

\[ d_{1\alpha_1} \] \hspace{1cm} d_{1\alpha_M} \hspace{1cm} d_{1\alpha_M} \hspace{1cm} d_{1\alpha_1} \]

\[ d_{1\text{min}} \] \hspace{1cm} d_{1\text{min}} \hspace{1cm} d_{1\text{max}} \hspace{1cm} d_{1\text{max}} \]
Calculating a comprehensive design problem

Use performance preferences in place of crisp targets:

- Bending Stiffness (N/mm)
  - 2000, 3000, 4000

- Torsional Stiffness (N-m/deg)
  - 5000, 6000, 7000, 8000

- Weight (kg)
  - 120, 140, 160, 180
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Calculating a comprehensive design problem
Use designer preferences to incorporate unmodelled concerns
Difficulties in Making the Decision Computable

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Set-based Design

Traditional evaluation tools suppress imprecision, and evaluate designs one at a time:

An alternative approach is to evaluate sets of designs:
Set-based Design

- Set refinement is a natural (& beneficial) way to develop designs. [Ward and Liker 1995]
- Intrinsically slower but has potential for time savings. [Eppinger 1991]
- Facilitates concurrency, communication, iterative redesign.
- There is a trade-off: not natural for optimization.

With this negotiation model:

- Level-set information is found with minimal extra computation.
- Sufficient conditions for propagation of set-based information.
The Basic Idea:

1. Designs are described by variables $\vec{d} \in DVS$; Measured performances are given by $\vec{p} \in PVS$.

2. Specify preferences ($\mu \in [0, 1]$) on both $DVS$ and $PVS$.

3. Map preferences from the design space ($DVS$) to the performance space ($PVS$) using the performance model $f$, and the extension principle [Zadeh 1975]:

$$
\mu_d(p) = \sup_{\vec{d} \mid f(\vec{d}) = p} \min_i \left( \mu_d(d_i) \right)
$$

4. Aggregate results to determine overall preference.

[The Method of Imprecision (MoI): Wood and Antonsson 1988, Otto 1992]
\[ A(x_1) = a_1 \quad \mu_B(y_1) = a_1 \]
\[ A(x_2) = a_2 \quad \mu_B(y_2) = a_2 \]

\[ f: A \rightarrow B \]

\[ \mu_A(x_1) = a_1 \]
\[ \mu_A(x_2) = a_2 \]
The Method of Imprecision (MoI)

Find the sets of designs $\vec{d}^*$ with the best performance:

$$f(\vec{d}^*) = \vec{p}^*$$

where $\vec{p}^*$ maximizes

$$\mathcal{P} \left( \mu_d(\vec{d}), \mu_p(\vec{p}) \right) = \mathcal{P} \left( \mu_{d_1}(d_1), \ldots, \mu_{d_q}(d_q), \mu_{p_1}(p_1), \ldots, \mu_{p_n}(p_n) \right)$$

$\vec{d}$ $q$-vector of design variables

$\vec{p}$ $n$-vector of performance variables

$\mu_d(d_i)$ preference for $i$th design variable

$\mu_p(p_j)$ preference for $j$th measured aspect of performance

$f$ mapping from design to performance

(analytic, black box, FEA, etc.)

$\mathcal{P}$ aggregation function — calculates overall preference
How should multiple preferences be aggregated?

min

geometric mean

arithmetic mean

max
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Rationality and the Multi Attribute Decision Making problem (or, Why not Economics?):

- Designs are almost always judged on several criteria.
- These criteria compete (stiff vs. light) but do not compare (Newton/mm vs. kg?)
- Economics approach reduces everything to $$; introducing various difficulties:
  - Constraints
  - Non-probabilistic uncertainty
  - Practical issues in preference elicitation


Aggregation Operator Axioms

At each point \( \bar{x} \) the following hold:

1. **Monotonicity:**
   \[
   \mathcal{P}(\mu_1, \mu_2; \omega_1, \omega_2)(\bar{x}) \leq \mathcal{P}(\mu_1, \mu'_2; \omega_1, \omega_2)(\bar{x}) \quad \forall \mu_2(\bar{x}) \leq \mu'_2(\bar{x})
   \]
   \[
   \mathcal{P}(\mu_1, \mu_2; \omega_1, \omega_2)(\bar{x}) \leq \mathcal{P}(\mu_1, \mu_2; \omega_1, \omega'_2)(\bar{x}) \quad \forall \omega_2 \leq \omega'_2; \quad \mu_1(\bar{x}) < \mu_2(\bar{x})
   \]

2. **Symmetry:**
   \[
   \mathcal{P}(\mu_1, \mu_2; \omega_1, \omega_2)(\bar{x}) = \mathcal{P}(\mu_2, \mu_1; \omega_2, \omega_1)(\bar{x})
   \]

3. **Continuity:**
   \[
   \mathcal{P}(\mu_1, \mu_2; \omega_1, \omega_2)(\bar{x}) = \lim_{\mu_2' \to \mu_2} \mathcal{P}(\mu_1, \mu_2'; \omega_1, \omega_2)(\bar{x})
   \]
   \[
   \mathcal{P}(\mu_1, \mu_2; \omega_1, \omega_2)(\bar{x}) = \lim_{\omega_2' \to \omega_2} \mathcal{P}(\mu_1, \mu_2; \omega_1, \omega_2')(\bar{x})
   \]

4. **Idempotency:**
   \[
   \mathcal{P}(\mu, \mu; \omega_1, \omega_2)(\bar{x}) = \mu(\bar{x}) \quad \forall \omega_1 + \omega_2 > 0
   \]

5. **Annihilation:**
   \[
   \mathcal{P}(\mu, 0; \omega_1, \omega_2)(\bar{x}) = 0 \quad \forall \omega_2 \neq 0
   \]

6. **Self-scaling weights:**
   \[
   \mathcal{P}(\mu_1, \mu_2; \omega_1 t, \omega_2 t)(\bar{x}) = \mathcal{P}(\mu_1, \mu_2; \omega_1, \omega_2)(\bar{x}) \quad \forall \omega_1 + \omega_2, t > 0
   \]

7. **Zero weights:**
   \[
   \mathcal{P}(\mu_1, \mu_2; \omega_1, 0)(\bar{x}) = \mu_1(\bar{x}) \quad \forall \omega_1 \neq 0
   \]
Trade-off Strategies and Weights

Trade-off strategies reflect the compensation among goals inherent in the problem (independent of weights).

- A *compensating* strategy is used when higher preference in one variable may compensate for lower preference in another.

- A *non-compensating* strategy is appropriate when the overall performance of a design is limited by its lowest-performing attribute.

Weights reflect relative importance of attributes.

A strategy–weight pair is required to define the aggregation of two independent attributes.
Early MoI Aggregation Functions

[Otto and Antonsson 1990]

non-compensating: \( \mathcal{P}(\mu_1, \ldots, \mu_n; \omega_1, \ldots, \omega_n) = \min(\mu_1, \ldots, \mu_n) \)

and compensating:

\[
\mathcal{P}(\mu_1, \ldots, \mu_n; \omega_1, \ldots, \omega_n) = (\mu_1^{\omega_1} \cdots \mu_n^{\omega_n})^{\frac{1}{\omega_1 + \cdots + \omega_n}}
\]

Other functions (QFD [Hauser and Clausing 1988], AHP [Saaty 1980], fuzzy logic):

- weighted sums:

\[
u(x^1, \ldots, x^N) = \sum_{i=1}^{N} \omega_i \frac{u(x^i)}{N}.
\]

- T-norms (< \text{min}) model intersection (logical AND).
- T-conorms (> \text{max}) model union (logical OR).
**Weighted Means** are a class of functions

\[ P(\mu_1, \mu_2; \omega_1, \omega_2) \]

that satisfy all the M₀I axioms except annihilation and also have strict monotonicity.

**Theorem** The properties of the weighted mean are necessary and sufficient for the function \( P(\mu_1, \mu_2; \omega_1, \omega_2) \) to be of the form

\[ P(\mu_1, \mu_2; \omega_1, \omega_2) = g \left( \frac{\omega_1 g^{-1}(\mu_1) + \omega_2 g^{-1}(\mu_2)}{\omega_1 + \omega_2} \right) \]

where \( \exists \mu_a, \mu_b \) such that

\[ \mu_a \leq \mu_1, \mu_2 \leq \mu_b ; \, \omega_1, \omega_2 \geq 0 ; \, \omega_1 + \omega_2 > 0 \]

and \( g \) is a strictly monotonic, continuous function with inverse \( g^{-1} \).

A parameterized family of aggregation functions

\[ g(t) = t^s, \text{ where } s \text{ is a real parameter, generates the aggregation function:} \]

\[ P_s(\mu_1, \mu_2; \omega_1, \omega_2) = \left( \frac{\omega_1 \mu_1^s + \omega_2 \mu_2^s}{\omega_1 + \omega_2} \right)^{\frac{1}{s}} \]

A little algebra shows the following:

\[ P_0 = \lim_{s \to 0} P_s = \text{geometric mean} \]
\[ P_{-\infty} = \lim_{s \to -\infty} P_s = \text{min} \]
\[ P_{\infty} = \lim_{s \to +\infty} P_s = \text{max} \]
\[ P_1 = \frac{\omega_1 \mu_1 + \omega_2 \mu_2}{\omega_1 + \omega_2} = \text{arithmetic mean} \]
Example

A company makes two products, 1 and 2:

- Product 1 yields $2 profit but requires $1 in imports.
- Product 2 can be exported for $2 revenue but makes only $1 profit.

\[ x_1 = \text{Number of Product 1 produced.} \]
\[ x_2 = \text{Number of Product 2 produced.} \]

Objective: “Maximize” balance of trade \((z_1)\) and profits \((z_2)\):

\[ \tilde{z}(\vec{x}) = \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \]

subject to a number of constraints:
Decision Space with Pareto-optimal Region:

Constraints:

C1: $-x_1 + 3x_2$  
C2: $x_1 + 3x_2$  
C3: $4x_1 + 3x_2$  
C4: $3x_1 + x_2$  
C5: $x_1$  
C6: $x_2$
Performance preferences:

\[ z_1 \]

\[ \begin{align*}
\mu & | -5.0 & 0.0 & 5.0 & 10.0 & 15.0 \\
\text{balance of trade} & | 0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0
\end{align*} \]

\[ z_2 \]

\[ \begin{align*}
\mu & | 5.0 & 10.0 & 15.0 & 20.0 & 25.0 \\
\text{profit} & | 0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0
\end{align*} \]

Note: \( \tilde{z} \left( \frac{0}{7} \right) = \left( \frac{14}{7} \right), \quad \tilde{z} \left( \frac{9}{3} \right) = \left( \frac{-3}{21} \right) \)
Some solutions:

Zimmermann uses the $\min(P_{-\infty})$:

$$\bar{x} = (5.03, 7.32)$$ is best to two decimal places

$$\bar{x} = (5, 7)$$ is the best integer value

For the geometric mean ($P_0$):

$$\bar{x} = (5.70, 7.10)$$ is best to two decimal places

$$\bar{x} = (6, 7)$$ is the best integer value
### Optimal points for different aggregation functions \( \mathcal{P}_s \)

<table>
<thead>
<tr>
<th>( s = 0 )</th>
<th>( s = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>g.mean</td>
<td>a.mean</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
    & s = -\infty \\
    & s = \infty
\end{align*}
\]

#### Diagram

- **Undominated points**
- **Admissible region**
- **Inadmissible region**

- **Points**: C1, C2, C3, C4

- **Axes**: \( x \) and \( y \)

- **Values**: (-10, -5, 0, 5, 10) and (0.0, 0.5, 1.0)

- **Lines**:
  - \( x = (0,7) \)
  - \( x = (3,8) \)
  - \( x = (5,7) \)
  - \( x = (6,7) \)
The decision depends on the aggregation.

Use indifference points to determine strategies and weights.
Definition:

A **Negotiation** (or a trade-off) is the choice of the compensation parameter (or strategy) $s$ to aggregate multiple preferences, and the weights $\omega$.

For any undominated (Pareto-optimal) design, there is a choice of $s$ and $\omega$ that selects that point over all other undominated points.

This is not true for methods that use fixed strategies (weighted sum, for example).
Choosing a strategy and a pair of weights

State of the art:

- specify weights directly, with fixed granularity (AHP, QFD)
- extract eigenvectors from matrix, normalize (AHP)
- no choice of strategies (usually arithmetic mean)

This negotiation model calculates a strategy and a weight pair using indifference points. For two preferences:

- Get $\mu = 0.5$, $\mu = 1.0$ for each attribute.
- We know $\mathcal{P}(\mu_1, \mu_2) = 0.5$ for $\mu_1 = \mu_2 = 0.5$.
- Ask, “At what value of $\mu_1$ is there indifference between $\mathcal{P}(\mu_1, 1.0)$ and $\mathcal{P}(0.5, 0.5) = 0.5$?”
- Simple procedure (with some numerical solving) returns $s$ and $\frac{\omega_1}{\omega_2}$.
1. Determine \(x\) and \(y\) such that
\[
P_s(x, 1) = P_s(1, y) = 0.5.
\]
2. Let \(b = \frac{\omega_y}{\omega_x}\).
3. If \(x = y\), then \(b = 1\):
   (a) If \(x = 0.5\), then \(s = -\infty\).
   (b) If \(x = 0.25\), then \(s = 0\).
   (c) If \(x > 0.25\), then
       \(s \in (-\infty, 0)\). Solve
       \[x^s + 1 = 2(0.5)^s\]
       numerically.
   (d) If \(x < 0.25\), then \(s \in (0, \infty)\). Solve
       \[x^s + 1 = 2(0.5)^s\]
       numerically.
4. If \(x \neq y\), then \(b \neq 1\). Note that if \(s = 0\),
   \[x^m = 0.5 = y^{1-m} \Rightarrow y^{1-\log_x 0.5} = 0.5\]
   (a) If \(y^{1-\log_x 0.5} = 0.5\), then
       \(s = 0\), and \(b = \frac{1-\log_x 0.5}{\log_x 0.5}\)
   (b) If \(y^{1-\log_x 0.5} > 0.5\), then
       \(s < 0\).
       If \(y^{1-\log_x 0.5} < 0.5\), then
       \(s > 0\).
       Solve numerically for \(s\) from
       \[
       \left(\frac{1+by^s}{1+b}\right)^{\frac{1}{s}} = \left(\frac{x^s+b}{1+b}\right)^{\frac{1}{s}} = 0.5
       
       \]
       which reduces to
       \[
       (x^s - 0.5^s)(y^s - 0.5^s) = (1 - 0.5^s)^2
       \]
       From \(s\), determine \(b\).
It should be noted that this method will never return an answer \( s > 1 \), which is nice since it avoids “supercompensating” functions. It should also be noted that if either \( x \) or \( y \) is close to 0, the \((s, b)\) pair is quite sensitive to small differences in \( x \) and \( y \). In these cases, it might be preferable to elicit other indifference points to determine \( s \) and \( b \).

Saaty’s AHP could be used in pairwise comparison at the start, as a check. There is (presently) no comparable normalization scheme for strategies \( s \).
Choosing a strategy and a pair of weights, Example

Bending stiffness vs. torsional stiffness:
\[
\mu_1(3600) = \mu_2(7200) = 1 \\
\mu_1(3000) = \mu_2(6400) = 0.5 \\
\mathcal{P}_s(0.5, 1) \approx \mathcal{P}_s(1, 0.5) \\
\approx \mathcal{P}_s(0.5, 0.5) = 0.5
\]
Conclude:
\[ s = -\infty, \omega_2 = \omega_1 \]

B-pillar location (style) vs. other design preferences:
\[
\mathcal{P}_s(0.4, 1) = \mathcal{P}_s(1, 0.3) = 0.5 \\
\text{Conclude:} \\
\quad s = -1.4, \omega_2 = 0.6\omega_1
\]

Quantified vs. Unquantified:
\[
\mathcal{P}_s(0.3, 1) = \mathcal{P}_s(1, 0.2) = 0.5 \\
\text{Conclude:} \\
\quad s = -0.02, \omega_2 = 0.8\omega_1
\]
Uncontrolled Variations (Noise)

- For typical noise, the preference of a design parameter set is weighted by its probability of occurring through the probabilistic uncertainty.

- Given a probability space (NPS), the expected preference of a point $d \in \text{DPS}$ is defined by:

$$E[\mu(d)] = \int_{\text{NPS}} \mu(d) \delta Pr$$
Uncontrolled Variations (Noise)

- If the NPS $\simeq \mathbb{R}$, then a probability density function $pdf(n)$ would be used.

- The integral of the equation above then becomes:

$$E[\mu(d)] = \int_{NPS|d} \mu(d) \, pdf(n|d) \, \delta(n|d)$$

- Since the distributions over $n$ can vary with $d$, the notation $n|d$ is used.
Uncontrolled Variations (Noise)
Difficulties in Making the Decision Computable

**Fuzziness:** Specifications are imprecise.

**Unmodelled Criteria:** Many (often crucial) decisions depend on unquantified or unmeasured criteria.

**Set-based design:** Sets provide rapid design exploration; facilitate concurrency and support iteration.

**Rationality:** Need to reconcile competing, incommensurate objectives in a rational way.

**Cost:** Computation cost to effectively explore a large design space can be significant.
Computation Cost Issues:

- Real problems have “many” (10+) design variables.
- Real problems can have expensive analysis functions.

Some solutions:

- Apply approximation methods to analysis function \( f \) [Law 1996]
- Technical (not substantial) restrictions on \( \mu \)
- \( \alpha \)-cut representations
Set-Based Computation Cost

Wrapper around existing analysis methods \((f)\).

The *Level Interval Algorithm (LIA)* discretizes design preference into \(\alpha\)-cut intervals:

\[
\begin{align*}
\alpha_M & \quad \mu_{d_1} \\
\alpha_1 & \quad d_{1\min} \quad d_{1\max} \\
\end{align*}
\]
Construct a linear approximation to $\tilde{f}$ over $D^d_e$ where possible:

[Law and Antonsson 1995, 1996]

$$f'(\vec{d}) = f(\vec{d}_{ctr}) + \Delta + A[\vec{d} - \vec{d}_{ctr}]$$

Where a linear approximation fails (e.g., non-monotonicity), find extremal points (e.g., using Thompson’s method), then build a linear approximation between extremal points.
A linear approximation $\tilde{f}$ fulfills several purposes:

1. It removes near-linear design variables from the search space for optimization.

\[ f(d_1, d_2) \approx f(0, d_2) + k d_1 \]
2. It allows the geometry of $P_{\alpha_k}^d$ to be interpolated between extremal points.
3. It provides a means to backwards map $\mu_p(\vec{p})$ onto the DVS.
Design of Experiments (DOE) is used to construct $\vec{f}'$.

Central Composite Design:

Resolution: III vs. IV.
Computation Cost:

For a Central Composite Design.
Return to the VW Example
Hierarchical aggregation

9.9 Bending
9.9 Torsion
13.8 Mass
1 min

1 Stiffnesses
s = 0
0.7

1 Measured performance

2 B-pillar
3.3 Floor pan
3.3 A-pillar
1 Floor sill

1 B-pillar location

1 Designer prefs
s = -1.4
0.6

1 Manufacturer prefs
s = -0.2
0.3

1 Engineering prefs

0.6 s = -1.4
1

1 Design preferences
s = 0
1

1 Overall preference

1.3
s = 0
1

0.6
The new maximization problem:

$$\mu_0(\tilde{d}) = \mathcal{P}_0(\mu_d, \mu_p; 1, 1.3)$$

where

$$\mu_d = \mathcal{P}_{-1.4}\left(\mathcal{P}_0\left(\mathcal{P}_{-1.4}(d_2, d_4; 1, 0.6), \mathcal{P}_{-0.2}(d_1, d_3; 1, 0.3); 1, 1\right), d_5; 0.6, 1\right)$$

$$\mu_p = \mathcal{P}_0\left(\mathcal{P}_{-\infty}(f_1, f_2; 1, 1), f_3; 1, 0.7\right)$$

Normalized weights:

<table>
<thead>
<tr>
<th>$\omega_{d1}$</th>
<th>$\omega_{d2}$</th>
<th>$\omega_{d3}$</th>
<th>$\omega_{d4}$</th>
<th>$\omega_{d5}$</th>
<th>$\omega_{f1}$</th>
<th>$\omega_{f2}$</th>
<th>$\omega_{f3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>2</td>
<td>1</td>
<td>3.3</td>
<td>16.1</td>
<td>9.9</td>
<td>9.9</td>
<td>13.8</td>
</tr>
</tbody>
</table>
Some results:

<table>
<thead>
<tr>
<th>${d_1, d_2, d_3, d_4, d_5}$</th>
<th>$K_T$</th>
<th>$K_B$</th>
<th>$m$</th>
<th>$\mu_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_p$</td>
<td>$\mu_p$</td>
<td>$\mu_p$</td>
<td></td>
</tr>
<tr>
<td>(1.0, 0.9, 0.9, 1.0, 50)</td>
<td>2832</td>
<td>5836</td>
<td>147</td>
<td>0.44 (*)</td>
</tr>
<tr>
<td></td>
<td>0.23</td>
<td>0.14</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>(1.1, 1.3, 1.2, 1.4, 100)</td>
<td>3365</td>
<td>6029</td>
<td>170</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.77</td>
<td>0.25</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(0.7, 0.9, 0.8, 1.0, 0)</td>
<td>2803</td>
<td>5730</td>
<td>144</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.08</td>
<td>0.78</td>
<td></td>
</tr>
</tbody>
</table>

minimum assumption:

| (1.1, 0.9, 0.8, 1.2, 150)     | 2869   | 5933  | 156 | 0.2     |

minimum/geometric mean, equal weights:

| (0.9, 1.0, 0.9, 1.2, 50)      | 2901   | 5876  | 156 | 0.36    |
| (0.7, 0.9, 0.9, 1.3, 50)     | 2912   | 5820  | 157 | 0.36    |
Approximations

These results come from a coarse but complete search ($5^5 = 3125$ runs, or about 50 hours). But:

- At 5 minutes a run, that would be 11 days.
- At 7 points per axis, 1 minute a run, that’s 12 days. [Miller, 1965]
- At 7 points per axis, 5 minutes a run, that’s 58 days.

Approximation (with DOE) gives results within 4% in only 21 runs, or about 20 minutes.

Note that recalculation with new strategies and weights is of negligible cost.
A pillar thickness

$\mu_{\text{max}}$ achievable = 0.440

$\mu_o$ selected = 0.306

B pillar thickness

Floor sill thickness

Floor pan thickness

B pillar location

$\mu_o$ selected = 0.306
A pillar thickness

\[ \mu_{\text{max} \text{ achievable}} = 0.440 \]

B pillar thickness

\[ \mu_{\text{o selected}} = 0.440 \]

Floor sill thickness

Floor pan thickness

B pillar location

65
A pillar thickness

$\mu_{\text{max}} \text{ achievable} = 0.440$

$\mu_{\text{o}} \text{ selected} = 0.384$

B pillar thickness

Floor sill thickness

Floor pan thickness

B pillar location

Design preferences
A pillar thickness

\( \mu_{\text{max}} \) achievable = 0.440

\( \mu_{\text{selected}} \) selected = 0.394

Floor sill thickness
design preferences

Floor pan thickness

B pillar location
A pillar thickness
$\mu_{\text{max}}$ achievable = 0.440

B pillar thickness
$\mu_{\text{selected}}$ selected = 0.440

Floor sill thickness

Floor pan thickness

B pillar location

Design preferences
\( \mu_0 = 0.44019 \)
Discussion of VW example

- Calculated overall preference $\mu_o$ depends on:
  - specified design preferences
  - specified performance preferences
  - performance analysis $f$ (here, the FEM)
  - trade-off strategies and weights

- Trends:
  - overall performance varies with design preferences; exceptions are floor pan thickness ($d_4$) and A-pillar thickness ($d_1$).
  - styling dominates this decision problem (high weight).

- Choice of strategy affects the rank order of candidate designs.

- Explicit negotiation dependencies
Conclusions

Demonstration of a formalism for representing and manipulating imprecise descriptions of engineering designs, constraints and specifications that:

- incorporates uncomputed performance by direct specification of preferences.
- reconciles competing attributes explicitly and rationally.
- identifies and incorporates different trade-off strategies.
- effects of uncontrolled variations (noise) can be incorporated.
- set-based approach facilitates concurrency in design.
- provides a trade-off between computation cost and accuracy.

Now being applied to example design problems from industry.
A framework for making preferences, trade-offs and negotiations explicit:

- Decision *framework* rather than decision *making*.
- Support for iteration.
- Allows engineers to attach “soft” requirements (their own and others’) to an engineering model:
- Helps identify the rationale for decisions; can lessen political influence in design negotiations.
- Gives a graphical display of preference in many dimensions.
Sponsors:

- National Science Foundation
- NASA’s Jet Propulsion Laboratory
- DARPA/NSF OPAAL Program