Department of Mathematics
The University of Texas at Austin

Modeling Ear Hair Cells and Sound Compression in Hearing Aids

Jack Xin

Mathematics and ICES

Collaborator

Yingyong Qi, Qualcomm

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Outline

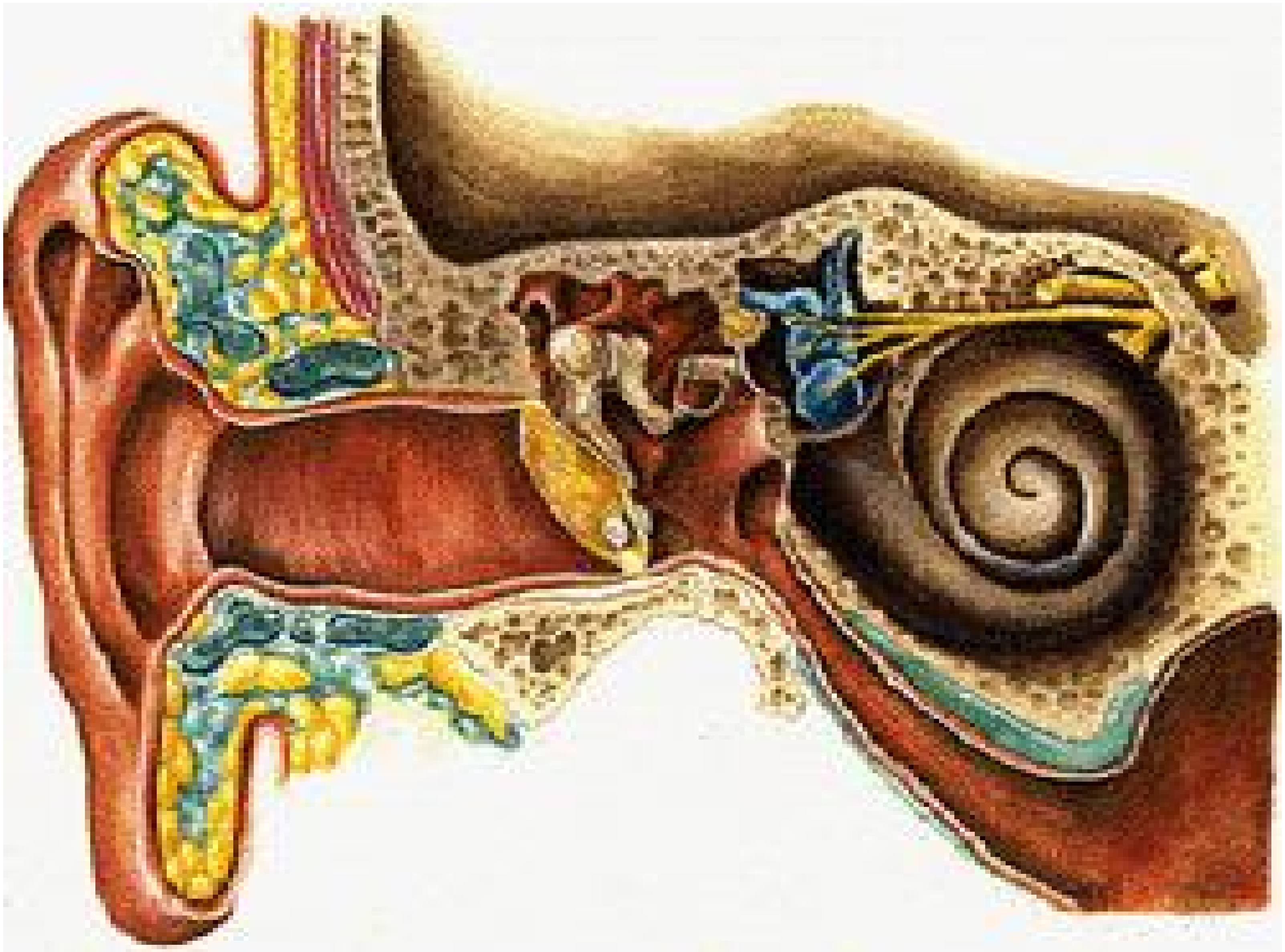
- **Background on Inner Ear & Auditory Pathways.**

- **Basilar Membrane (BM), Outer Hair Cells (OHC) & Cilia.**

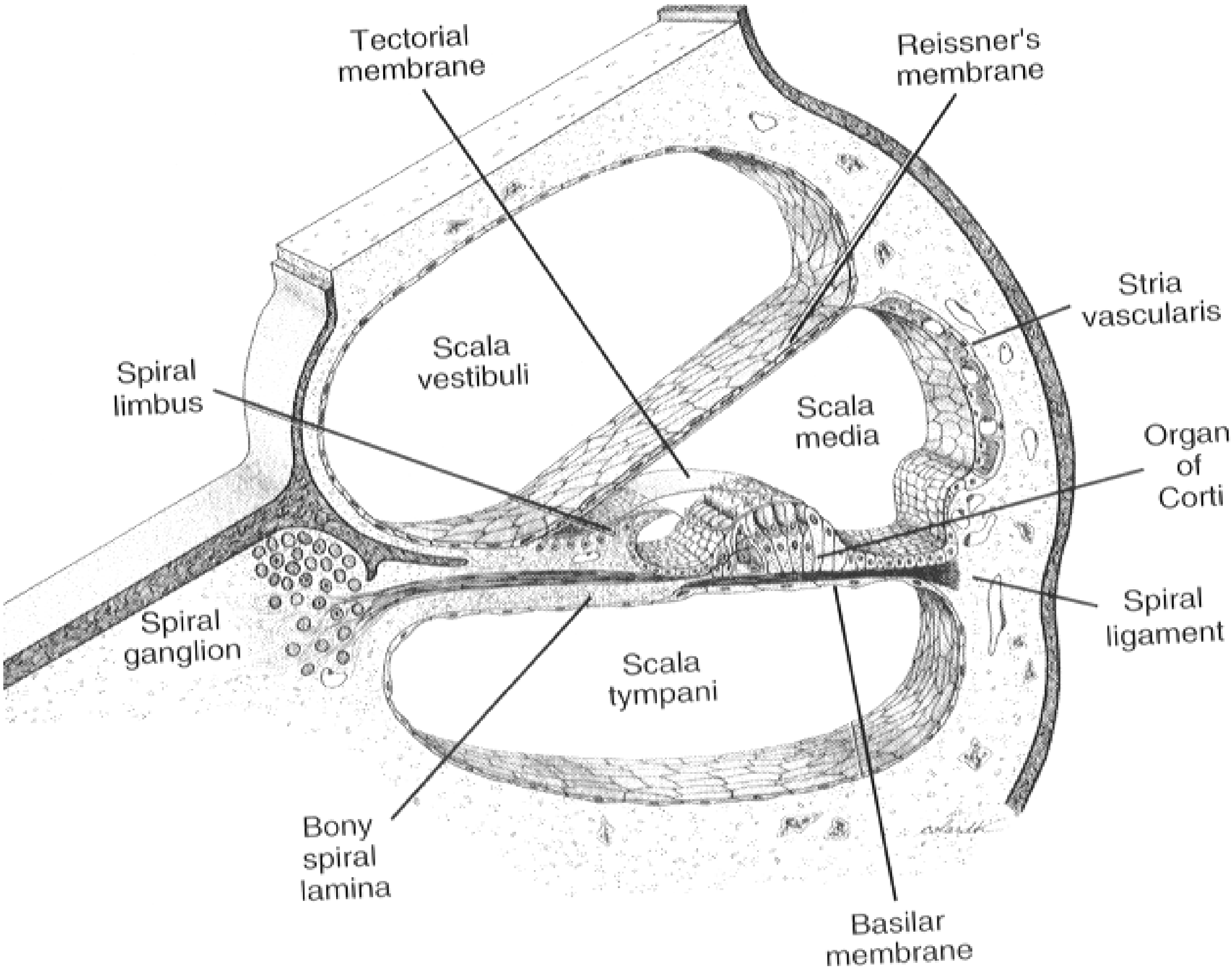
- **Feedforward Micromechanic Model of OHC.**

- **Model Application to Hearing Aids: gain curves, comparing with NAL-NL1 data (National Acoustic Lab, Australia).**

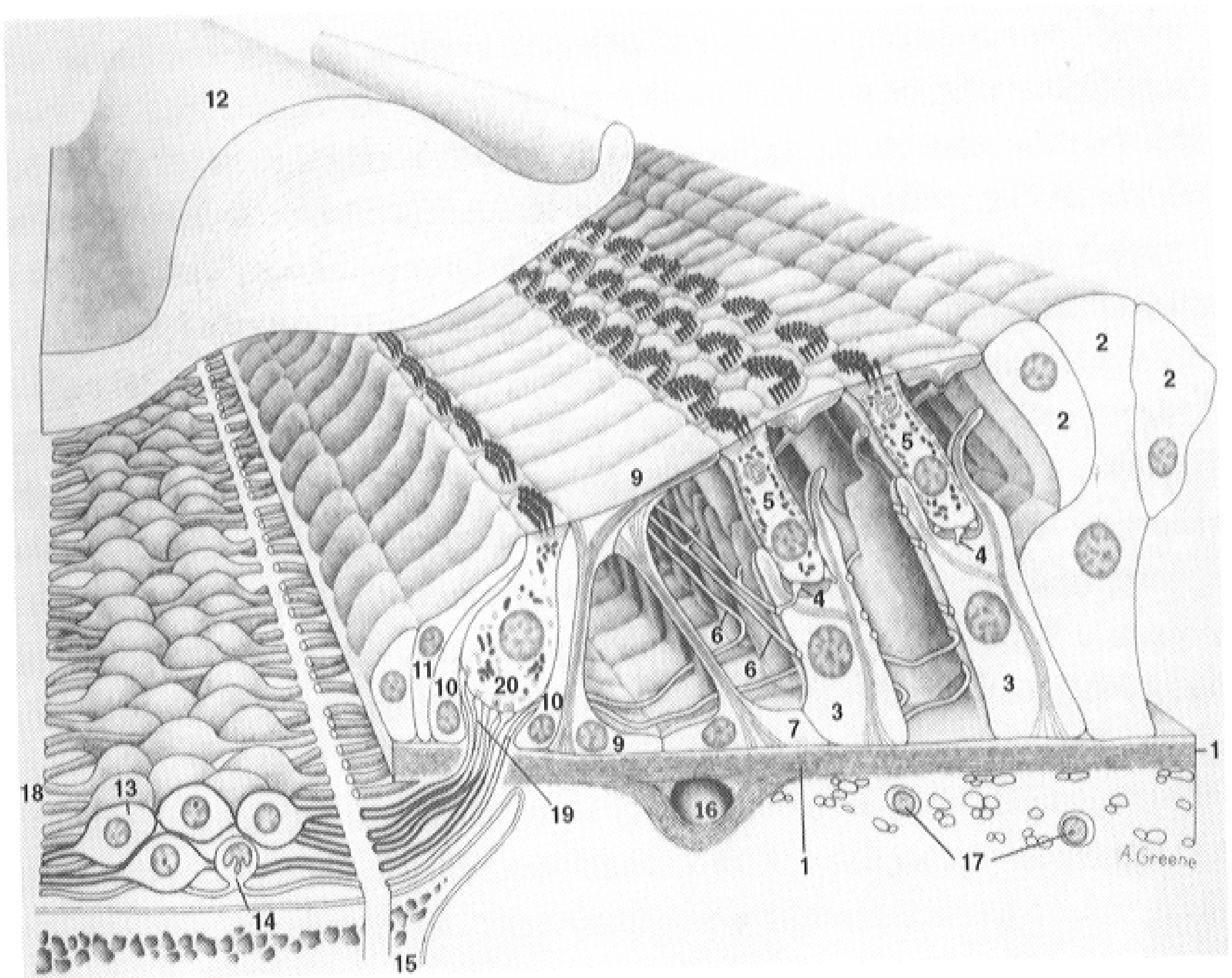
Inner Ear (Cochlea)



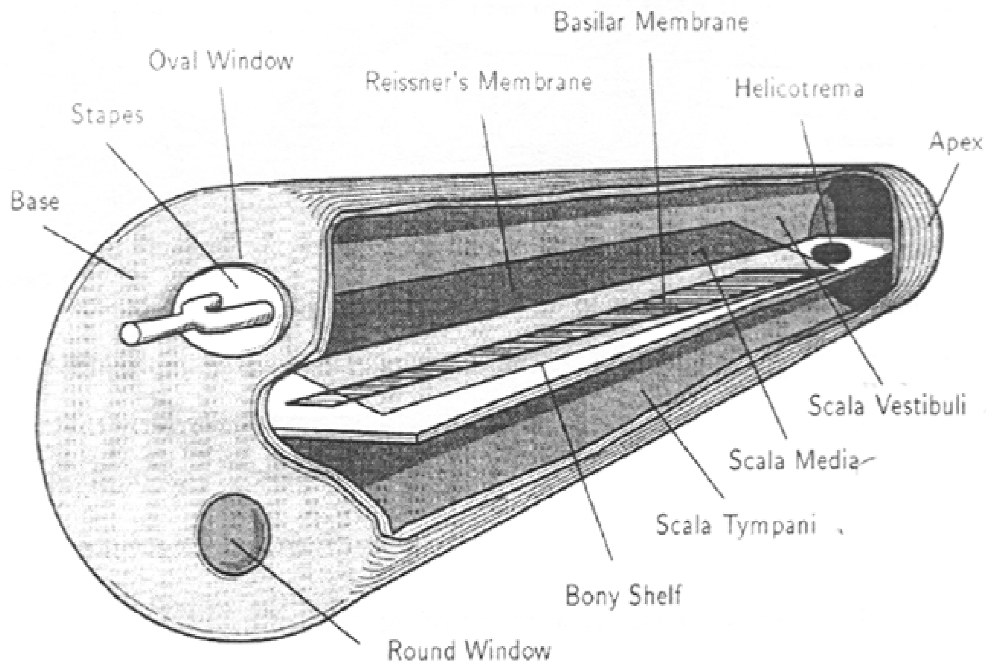
Cross Section View



Corti: BM1, OHC5, Cilia 12

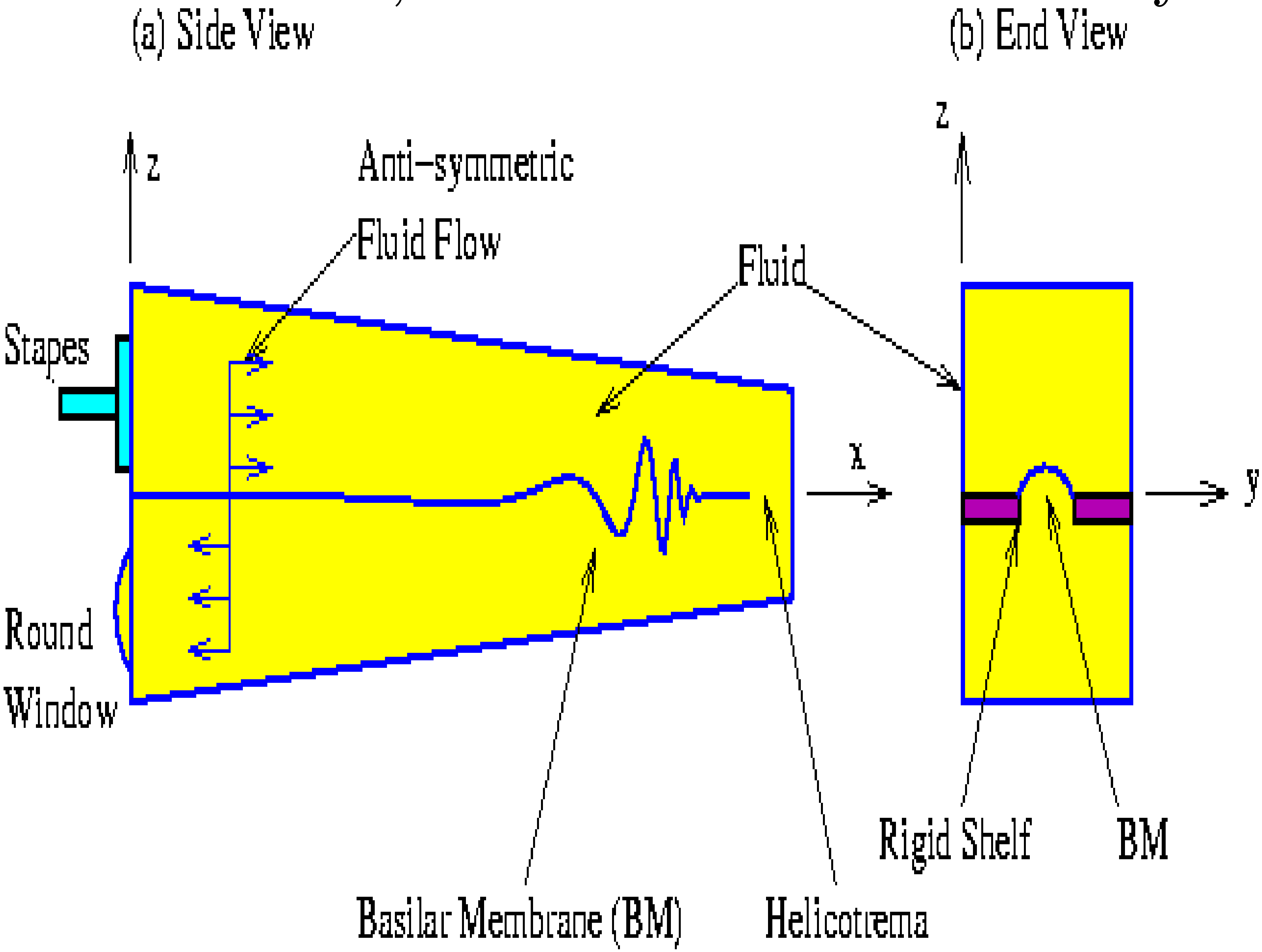


Schematic Uncoiled Cochlea



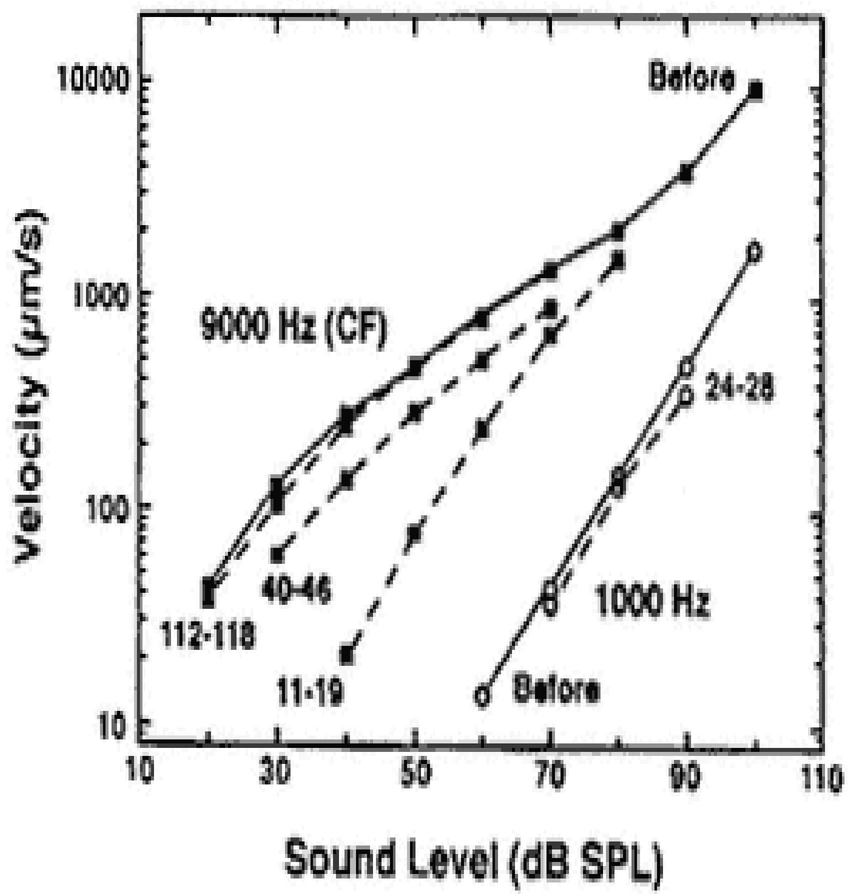
Side Views

Stokes Fluid, BM moves with fluid velocity.

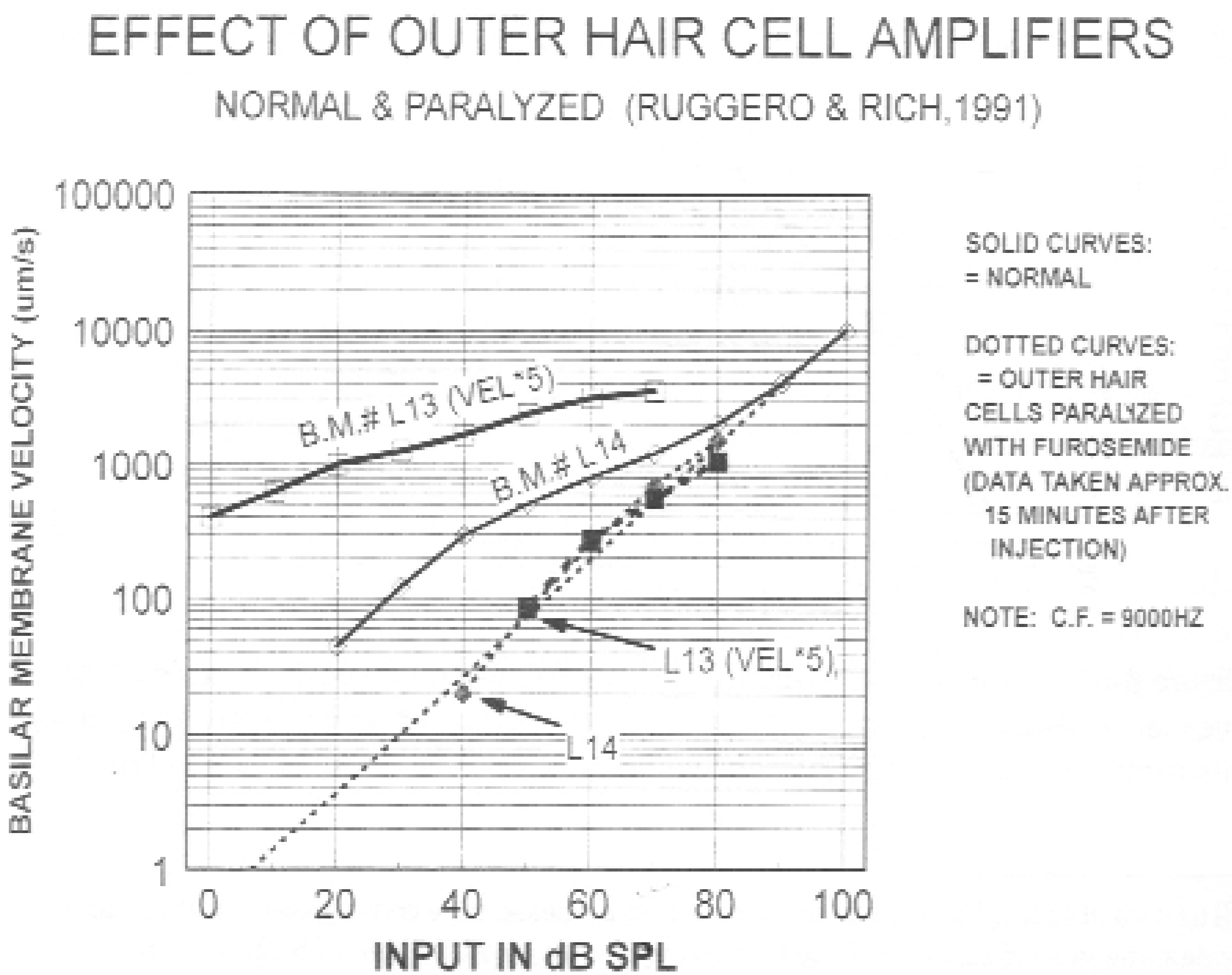


OHC Active Function

Input-output functions on chinchilla BM before and after furosemide injection (Ruggero-Rich,91). Compression when stimulus is a CF tone (9 kHz), linear to a 1 kHz tone.

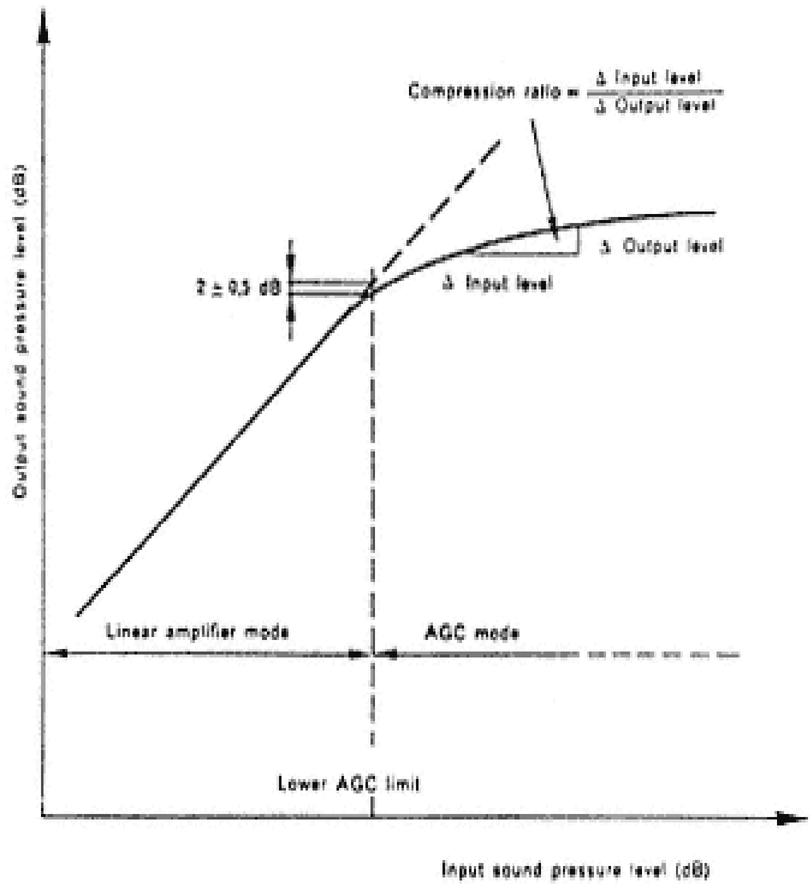


OHC Active Function

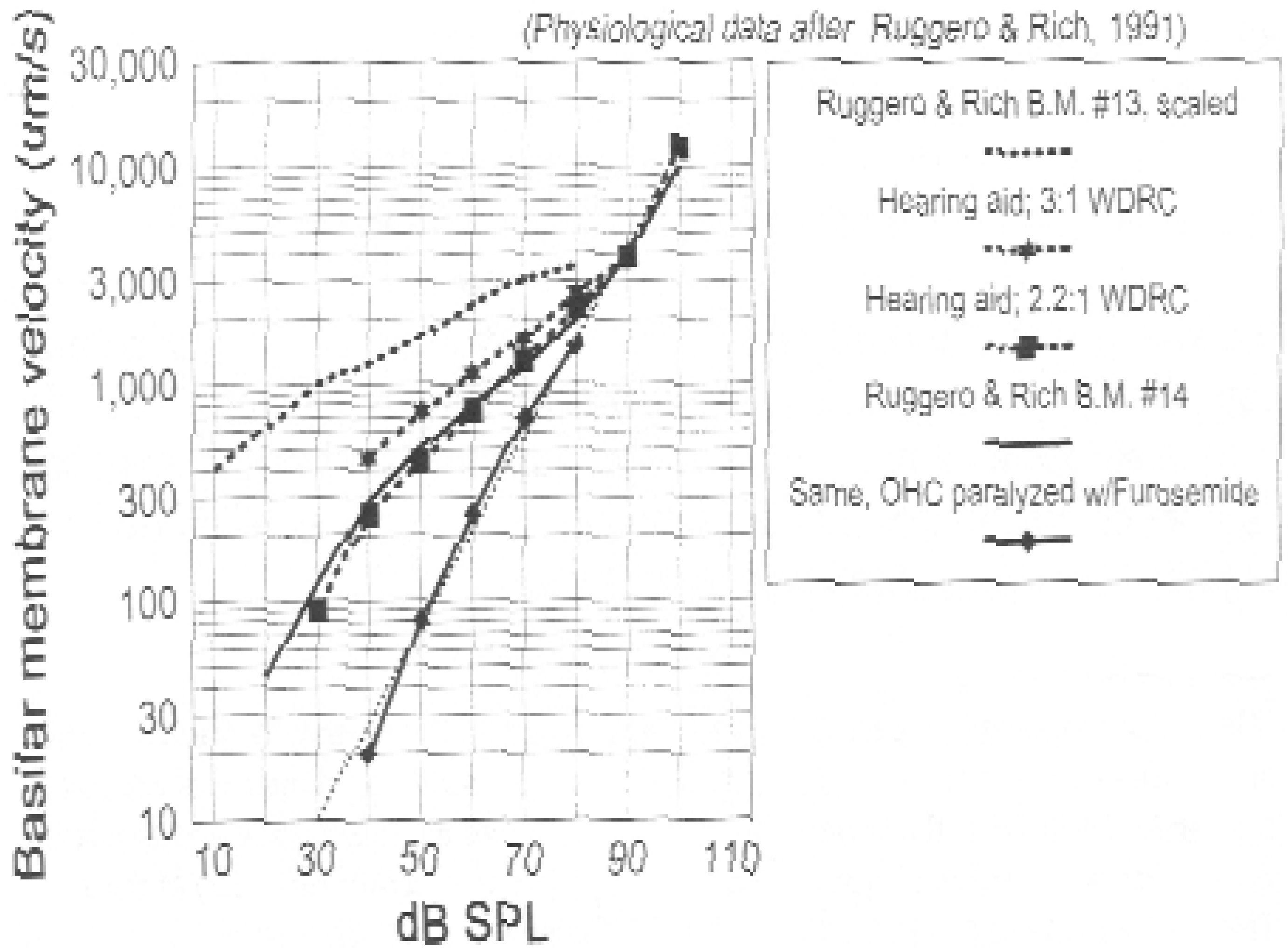


Compressive Amplification

Desirable to amplify weak sounds more than strong ones, so that wide range of input signals is compressed into a smaller range at output, a.k.a. Automatic Gain Control (AGC).

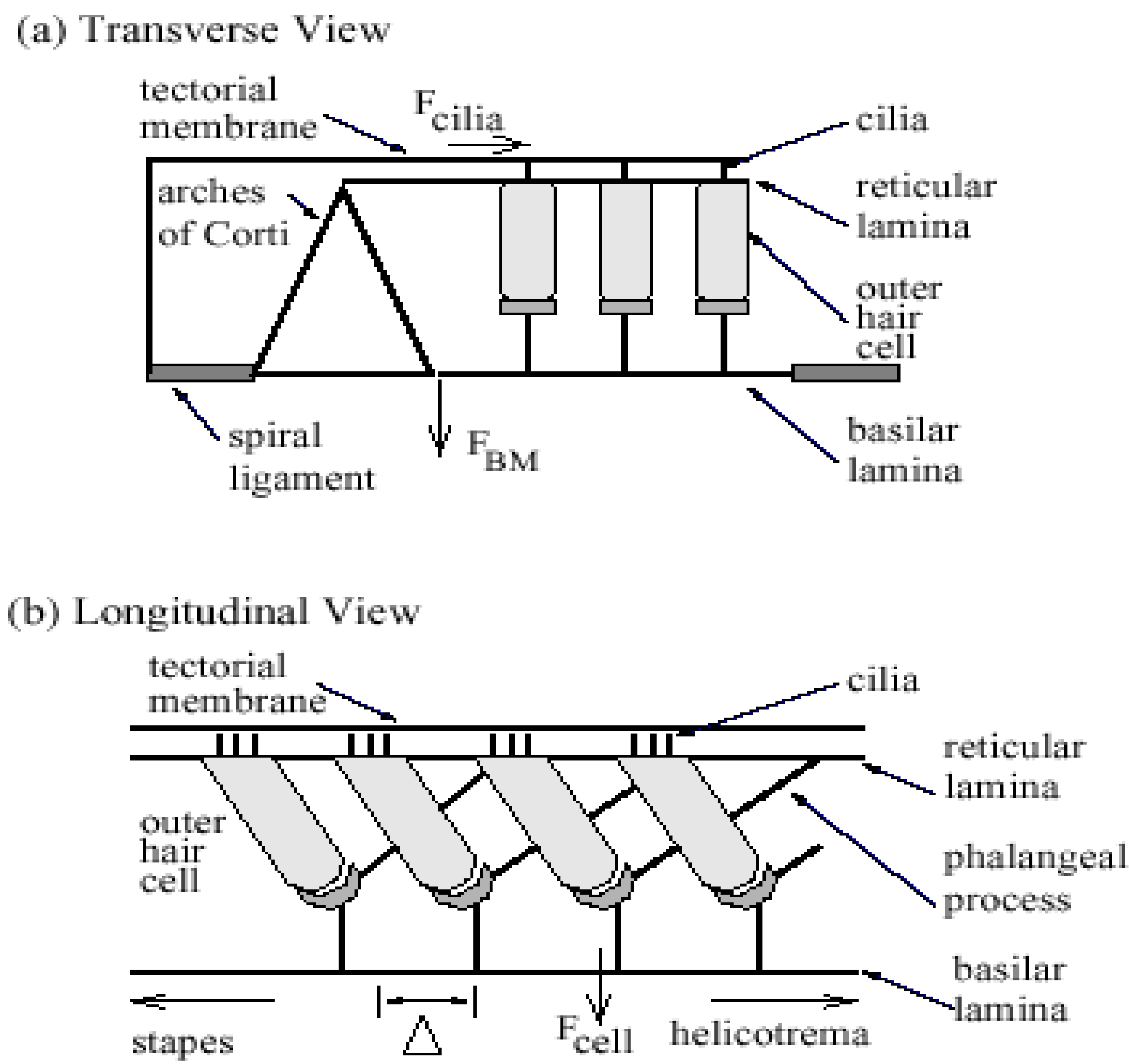


Compressive Amplification



Feedforward Model

Steele, Geisler, Lim (90's -00's).



Balance of Forces

$$F_{cilia}(x,t) = C_1(x)\ F_{BM}(x,t)$$

Both fluids and OHC act on BM:

$$F_{BM}(x,t) = (F_{BM}^{fld} +\ F_{BM}^{ohc})(x,t)$$

OHC longitudinal tilt $\Longrightarrow$ cilia force at x causes OHC to push BM at $x + \Delta$:

$$F_{BM}^{ohc}(x+\Delta,t) = C_2(x)\ F_{cilia}(x,t)$$

$$\Downarrow$$

$$F_{BM}^{ohc}(x+\Delta,t) = C_1\, C_2(x)\ (F_{BM}^{fld} +\ F_{BM}^{ohc})(x,t)$$

Model Equations (2-Dim)

Laplace Equation (Stokes Fluid) and BCs:

$$p_{xx} + p_{zz} = 0, \quad (x, z) \in (0, L) \times (0, H),$$

$$p_x\big|_{x=0} = -2\,\rho\,\xi(t), \;\; p\big|_{x=L} = 0,$$

$$p_z\big|_{z=0} = 2\,\rho\,u_{tt}, \quad p_z\big|_{z=H} = 0.$$

Pressure driven BM motion:

$$p + q = m(x)\,u_{tt} + r(x,t)\,u_t + s(x)\,u - \epsilon\,u_{xx}.$$

OHC force q:

$$q(x + \Delta, t) = \alpha\,(p(x,t) + q(x,t)).$$

Model Equations (2-Dim)

Function $\alpha = \alpha(x) \in (0,1)$, can be generalized to t dependent or a nonlinear function in u.

For $\Delta \ll 1$, away from boundaries:

$$q(x,t) \sim \frac{\alpha}{1-\alpha}\, p(x,t) - \frac{\alpha\Delta}{(1-\alpha)^2}\, p_x(x,t) + O(\Delta^2),$$

providing gain to p.

Solve p in terms of ξ, u_{tt} ($p = M\, u_{tt}$), write $q = M_{fw}(\Delta)\, p$, the closed u equation is:

$$(m + M + M_{fw}\, M)\, u_{tt} = p_0(x,t) - r(x,t)\, u_t - s(x)\, u + \epsilon\, u_{xx}.$$

Model Equations (2-Dim)

$M + M_{fw} M$: **compact nonnegative linear operator;**

p_0 **explicit, linear in** ξ**;**

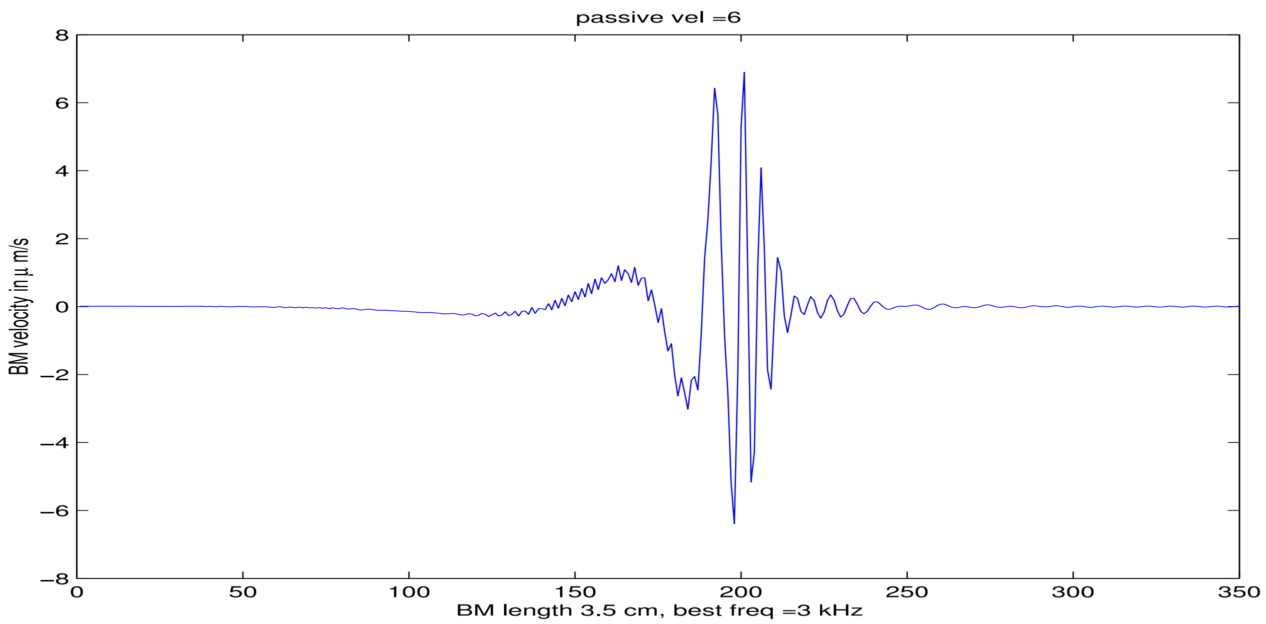
BM stiffness function $s(x)$ **exponentially decaying over 0 to L = 3.5 cm;**

$$r(x,t) = r_0 + \gamma\, |u| \star \exp\{-|x|/\lambda\}.$$

Discretize u **equation by a stable 2nd order method.**

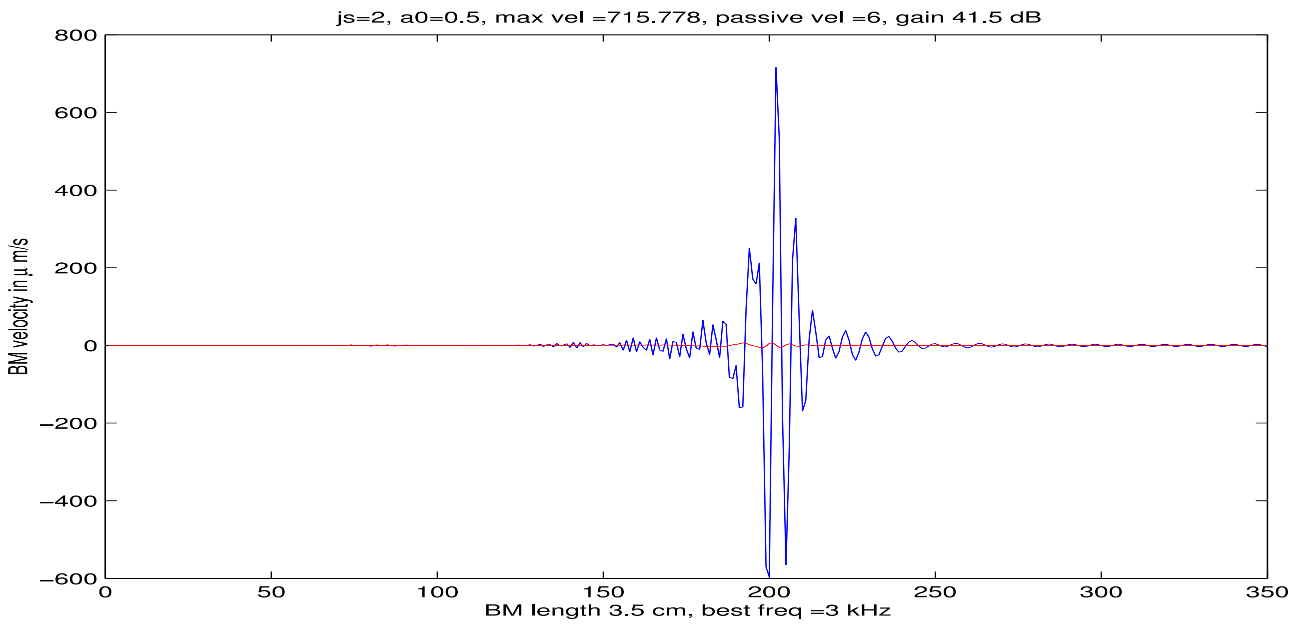
Feedforward Gain ≈ 40 dB

3 kHz tone w/o OHC.



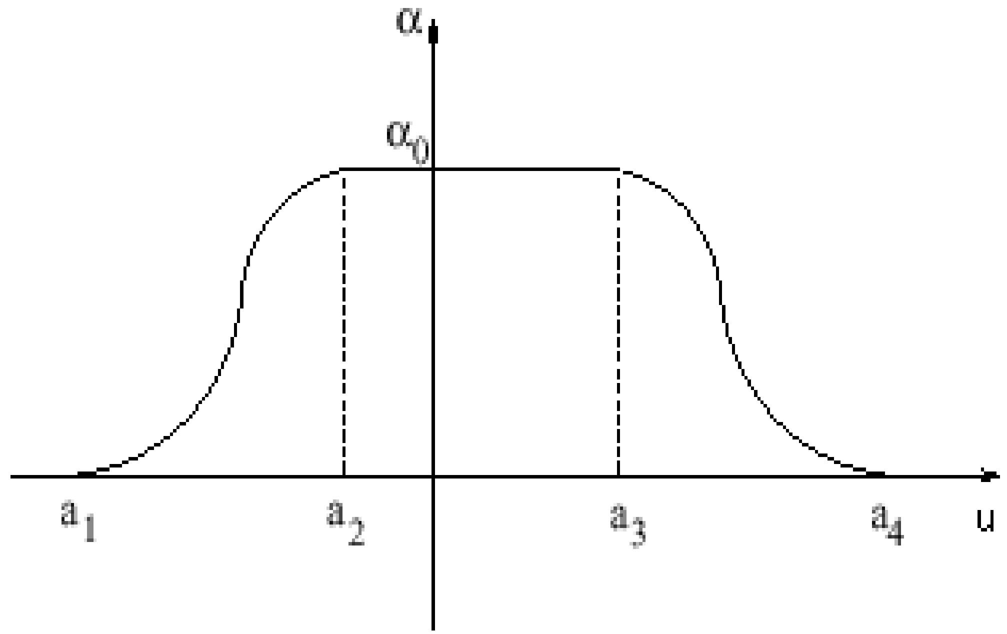
Feedforward Gain ≈ 40 dB

same 3 kHz tone w OHC (blue), w/o OHC (red).



Compressive Nonlinearity

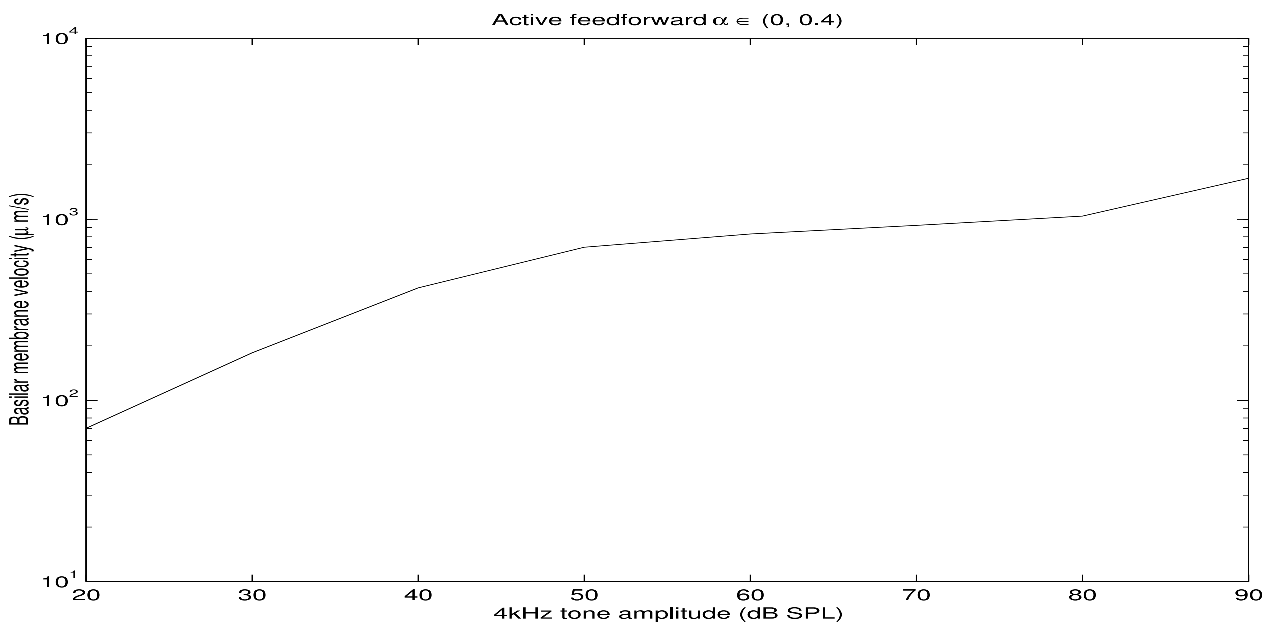
$a_i = a_i(x)$ **in general.**



For single tone input $A\sin(2\pi f\, t + \varphi)$**, compression can be introduced more directly by choosing** $\alpha = \alpha(A, f)$**,** α **in** A **behaves similarly.**

Model Input-Output Curve

input: 4kHz tone at diff dBs; output: BM max velocity.



Hearing Aid Modeling

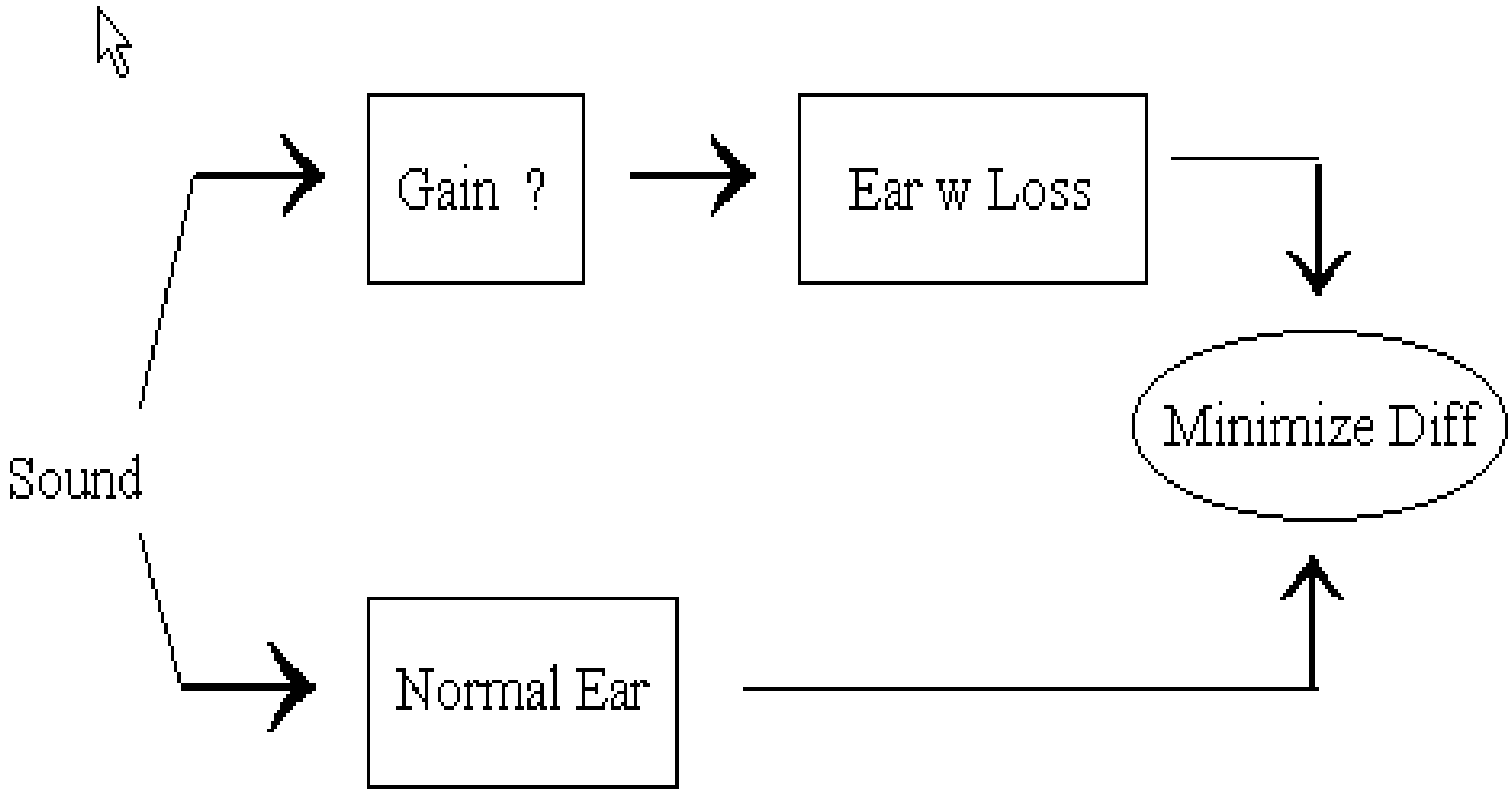
- Normal ear (NE) played by ear model with compressive feedforward OHC.

- Impaired ear (IE) with loss played by ear model without OHC, $\alpha = 0$ over some range of frequencies.

- Define loss using difference (dB) of NE and IE responses (BM velocity) at absolute hearing thresholds.

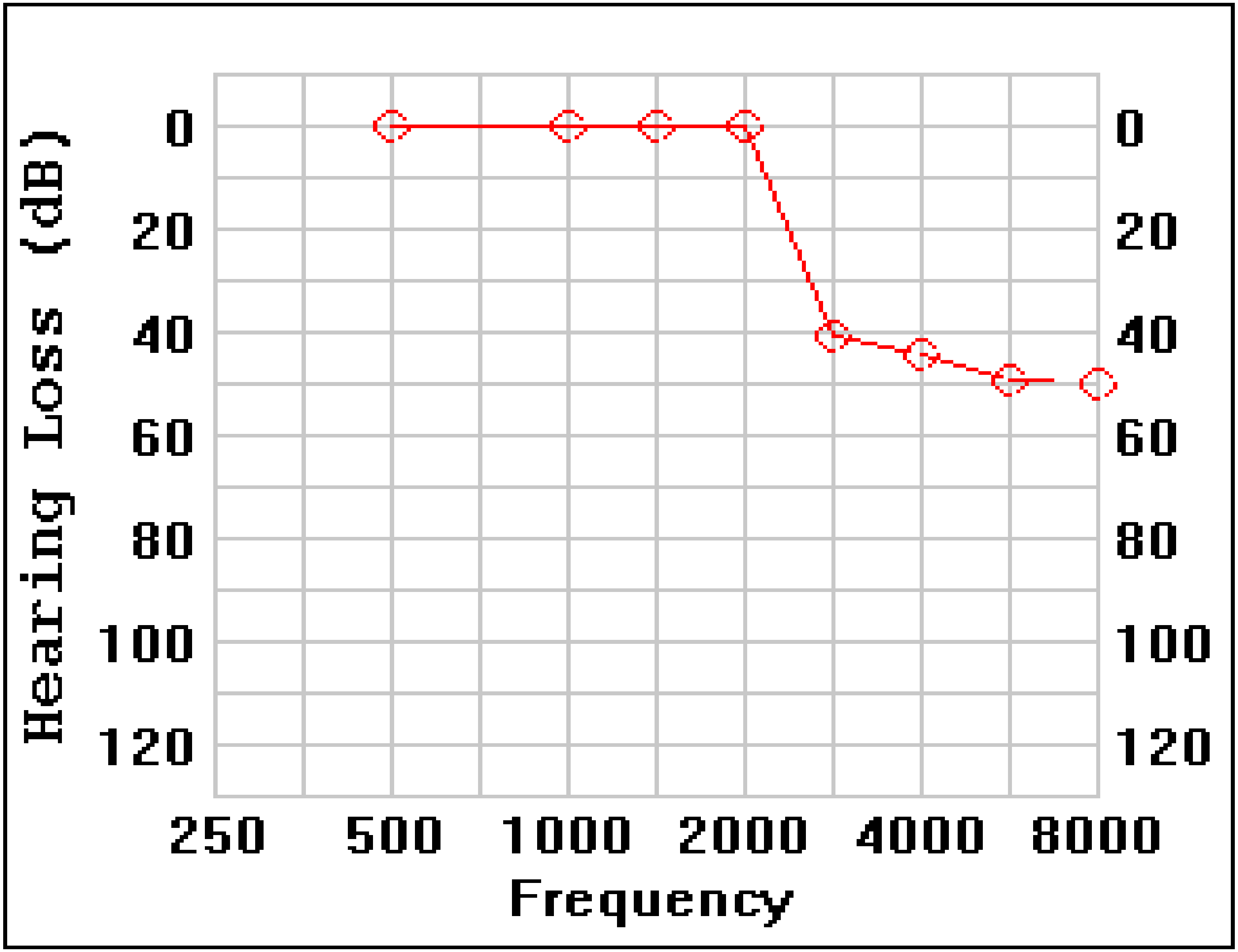
- Calculate gain needed for IE at higher input sound levels.

Schematic Gain Calculation

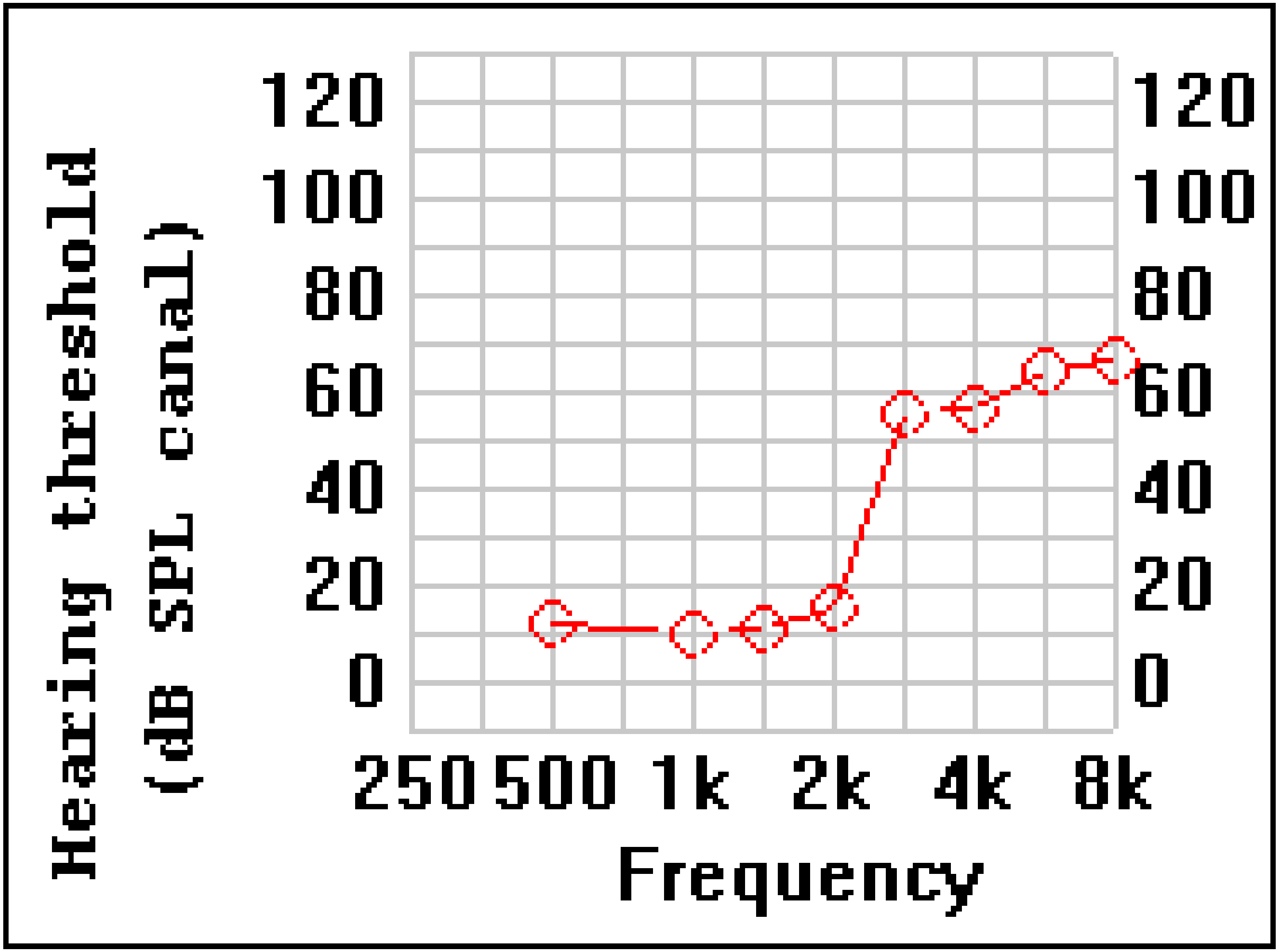


Audiogram

Loss at absolute thresholds.

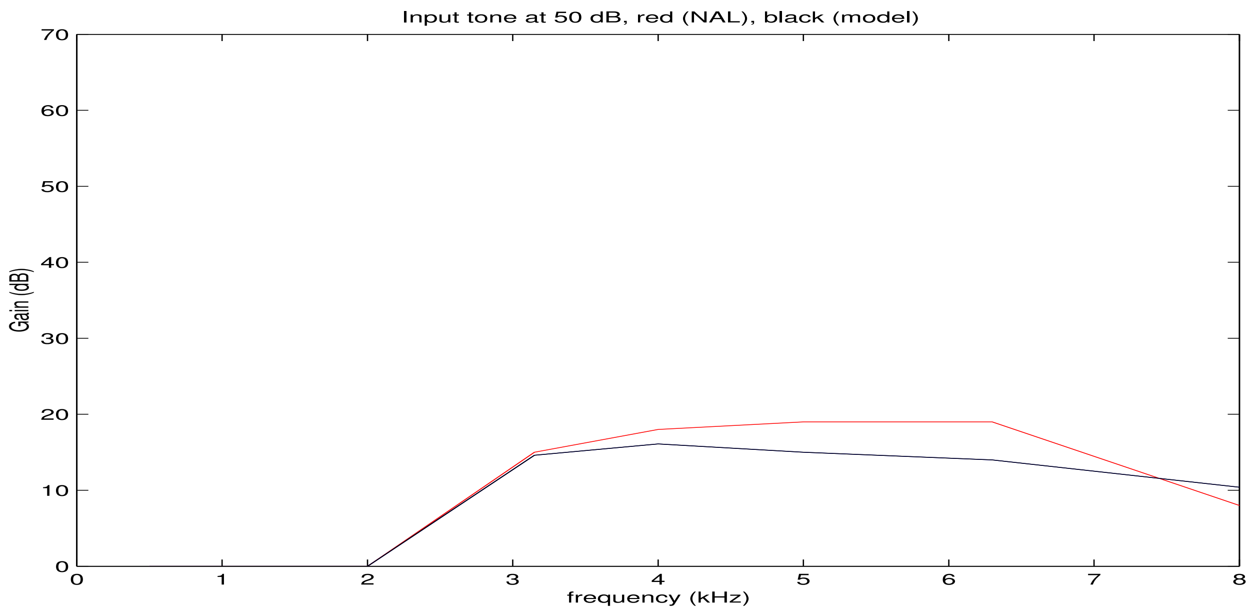


Loss Raises Thresholds



Gain Comparison at 50 dB

Red: NAL; Black: Model; simlr compression thresholds/ratios.



NAL formula

Maximize speech intelligibility index (SII):

$$SII = \sum_i I_i \cdot K_i \cdot L_i \cdot D_i,$$

i **number of frequency band,** $I_i \in (0, 1)$ **band weight;**

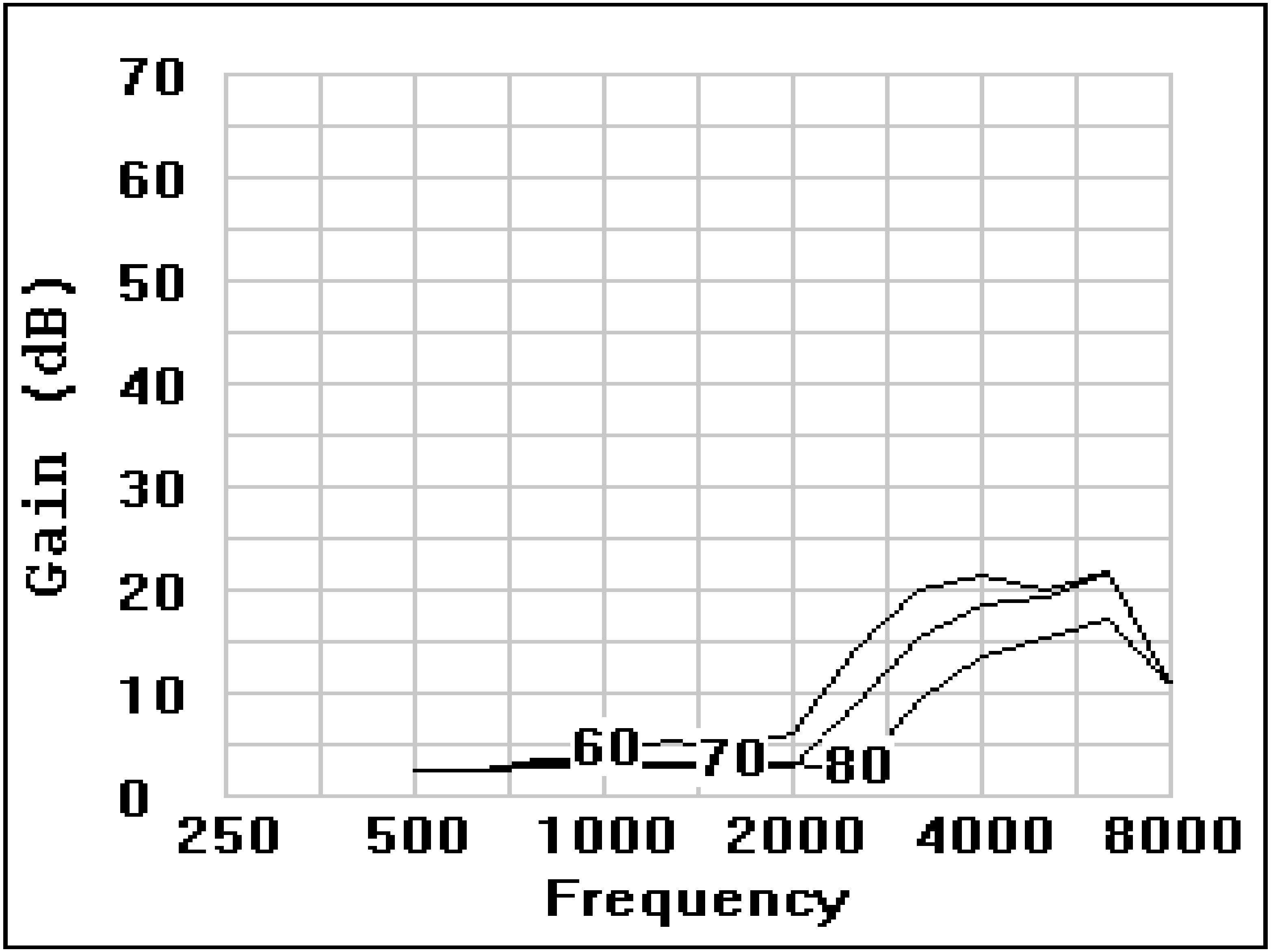
K_i **proportion of signal that is audible (inc. masking effect);**

$L_i \in (0, 1)$ **level distortion factor, smaller at high sound levels (similar to compression nonlinearity);**

$D_i \in (0, 1)$ **desensitization factor: correction for severe hearing loss (hearing more signal** $\longrightarrow$ **less understanding).**

NAL Gains

at 60, 70, 80 (dB).



Conclusions

- **OHC feedforward model helped to increase BM sensitivity.**

- **Computed (2)-dim models with compressive nonlinearities, and presented a model based scheme for finding hearing aids gain, results in qualitative agreement with NAL data.**

- **Optimization of model parameters needed.**

Ongoing and Future Works

- **Study compression thresholds/ratios, their dependence on frequency.**

- **Optimize SII over model output to select model parameters, and compare with NAL quantitatively.**

- **How to provide optimal gain for maximal intelligibility in the presence of noises ?**