Design of low energy space missions using dynamical systems theory

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Control & Dynamical Systems, Caltech
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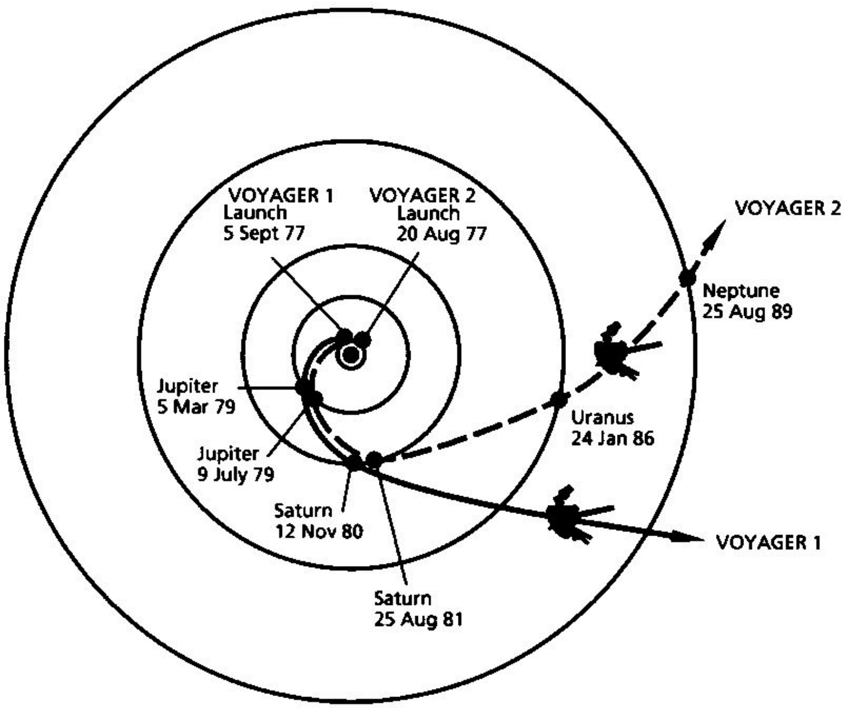
Low Energy Trajectory Design

- *Motivation: future missions*
- *What is the design problem?*
- *Solution space of 3-body problem*
- *Patching two 3-body trajectories: Mission to orbit multiple Jupiter moons*

Motivation

- Classical approaches to spacecraft trajectory design have been successful in the past: Hohmann transfers for *Apollo*, swingbys of planets for *Voyager*
- Costly in terms of fuel, e.g., large burns for orbit entry



Swingbys: Voyager Tour

Motivation

- **Low energy trajectories** $\longrightarrow$ large savings in fuel cost (as compared to classical approaches)
- Achieved using natural dynamics arising from the presence of a third body (or more)

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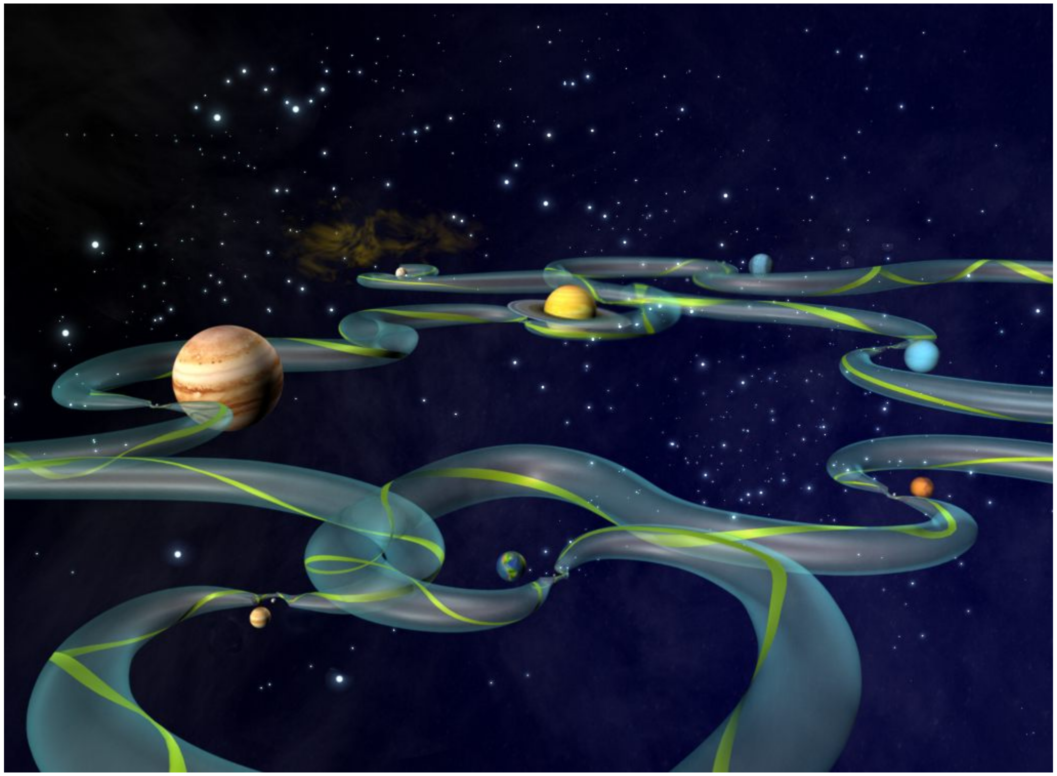
- **Low energy trajectories** $\longrightarrow$ large savings in fuel cost (as compared to classical approaches)
- Achieved using natural dynamics arising from the presence of a third body (or more)
- Low energy trajectory technology allows space agencies to envision missions in the near future involving long duration observations and/or constellations of spacecraft using little fuel

Motivation

- **Our approach:** Apply **dynamical systems techniques** to space mission trajectory design
- Find **dynamical channels** by considering phase space



Dynamical channels exist throughout the Solar System

Motivation

Current research importance

- development of some NASA mission trajectories, such as a lunar missions and **Jupiter Icy Moon Orbiter**

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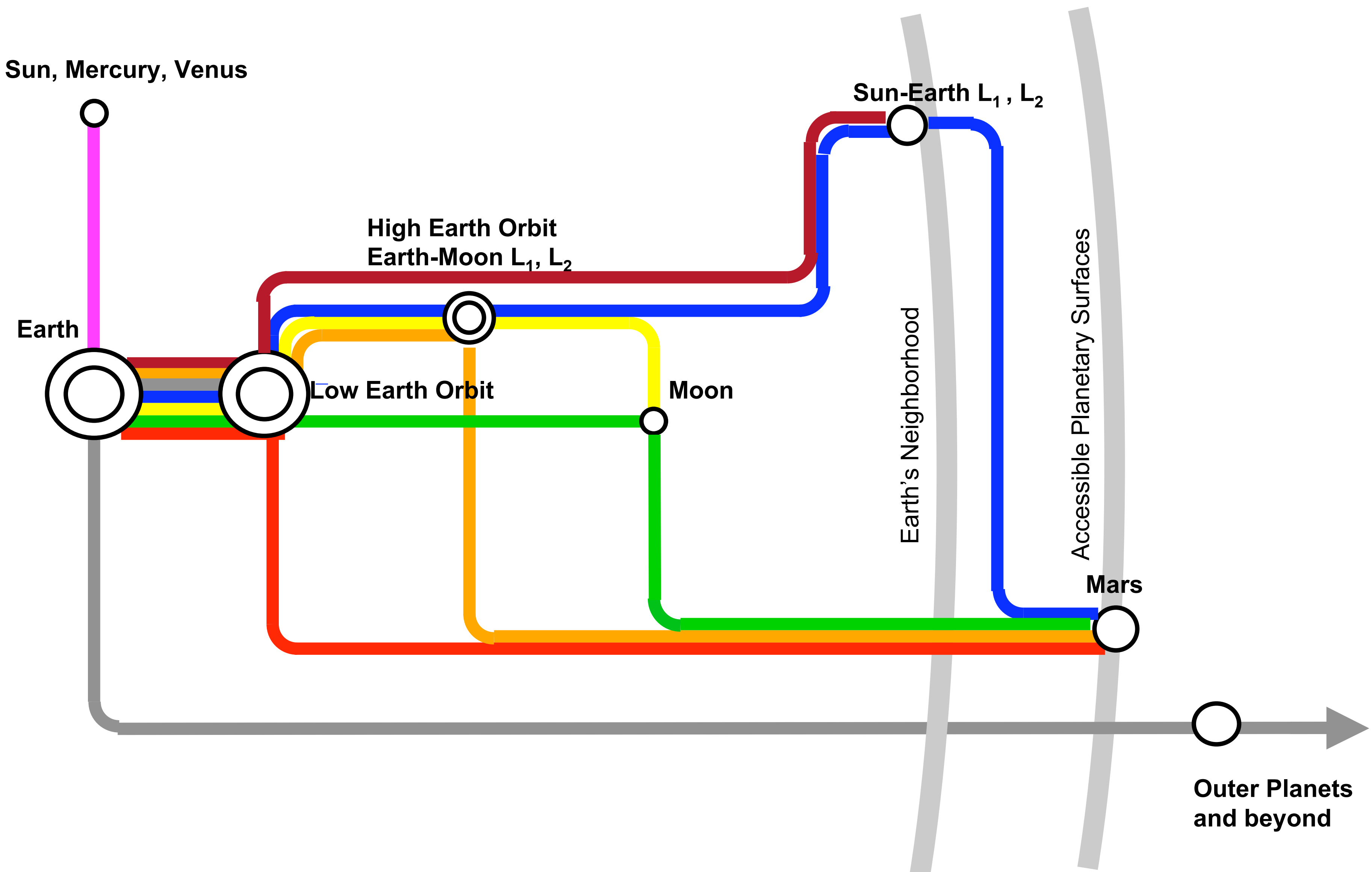
Current research importance

- development of some NASA mission trajectories, such as a lunar missions and **Jupiter Icy Moon Orbiter**
- **Spin-off:** results also apply to mathematically similar problems in chemistry, astrophysics, and fluid dynamics.
- Let's consider some missions...

Solar System Metro Map

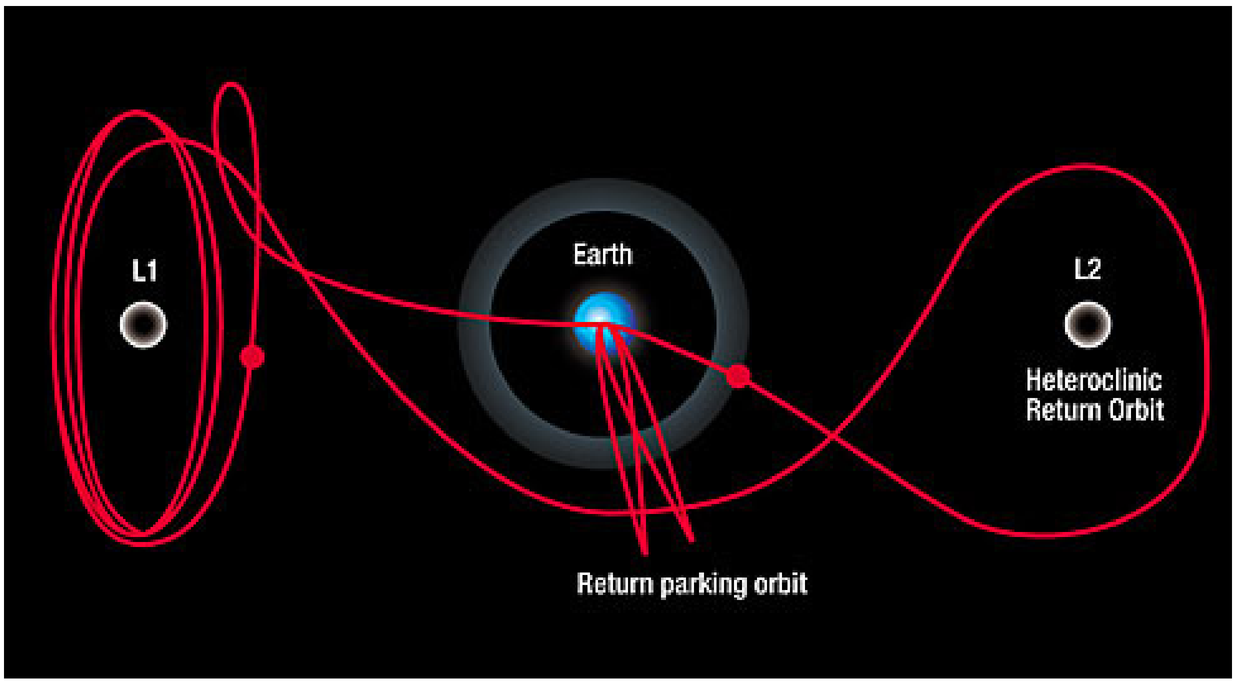


Source: Gary L. Martin, NASA Space Architect

Genesis Discovery Mission

- **Genesis** is collecting solar wind samples at the Sun-Earth L1 and will return them to Earth next year.
- First mission designed using dynamical systems theory.

Genesis Spacecraft



Genesis Trajectory

New Mission Architectures

Lunar L1 Gateway Station

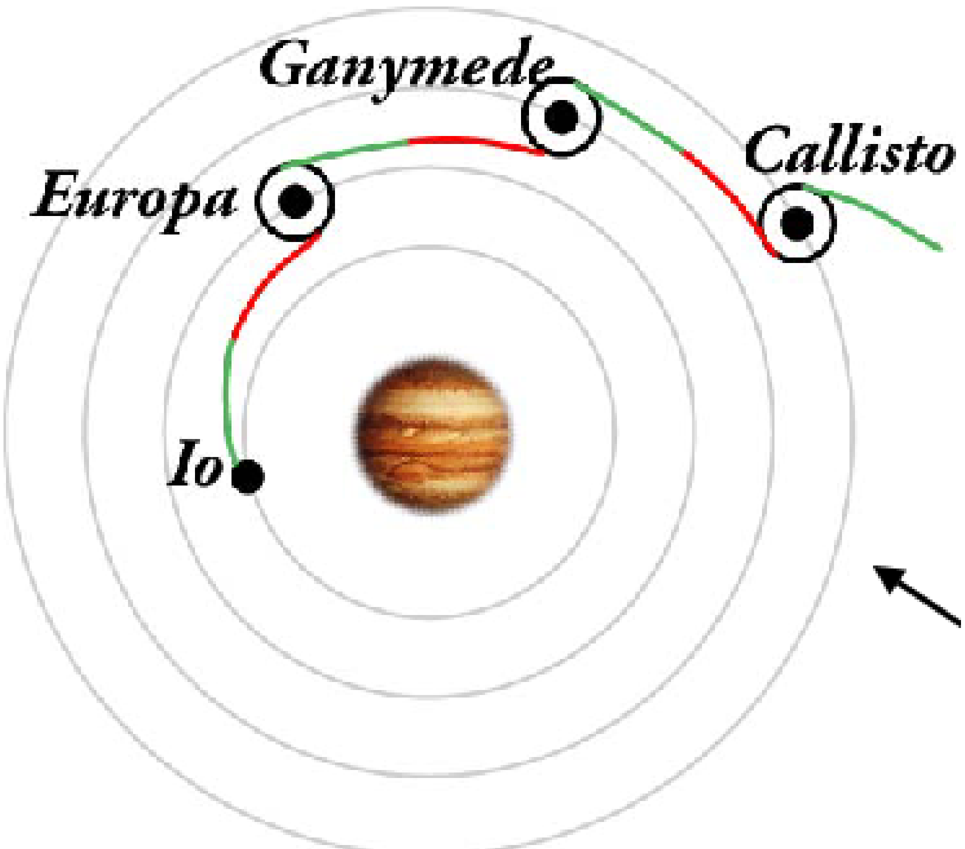
- transportation hub, servicing, commercial uses

Lunar L1 Gateway

Multi-Moon Orbiter

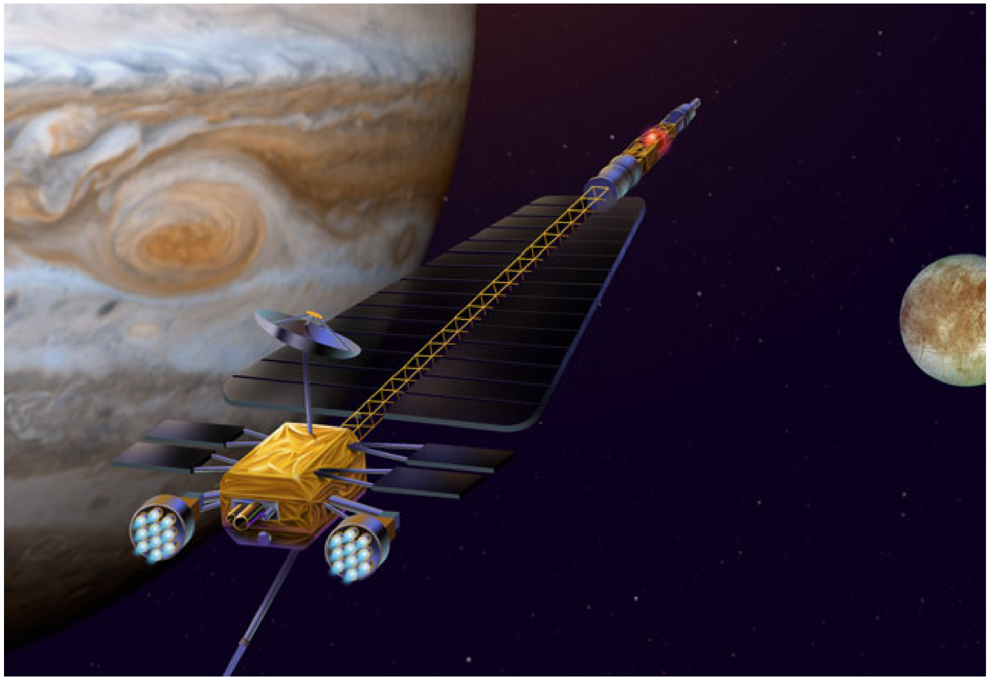
□ Multi-Moon Orbiter

- to Jovian, Saturnian systems (Koon, Lo, Marsden, SDR [2002])
- e.g., orbit Europa, Ganymede, and Callisto in one mission



Jupiter Icy Moons Orbiter

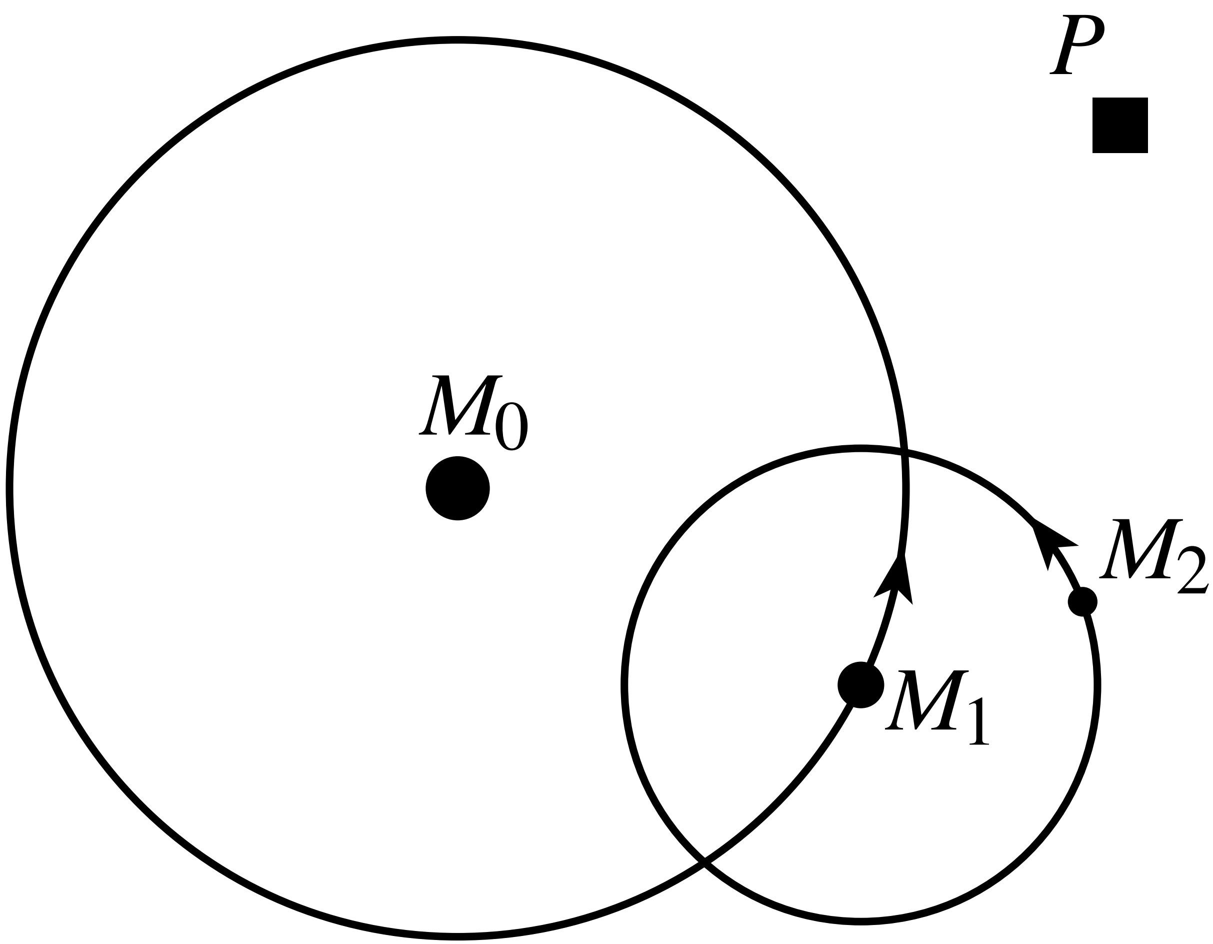
- NASA is considering a **Jupiter Icy Moons Orbiter**, inspired by the work on multi-moon orbiters
  - Earliest launch: 2011



Jupiter Icy Moons Orbiter

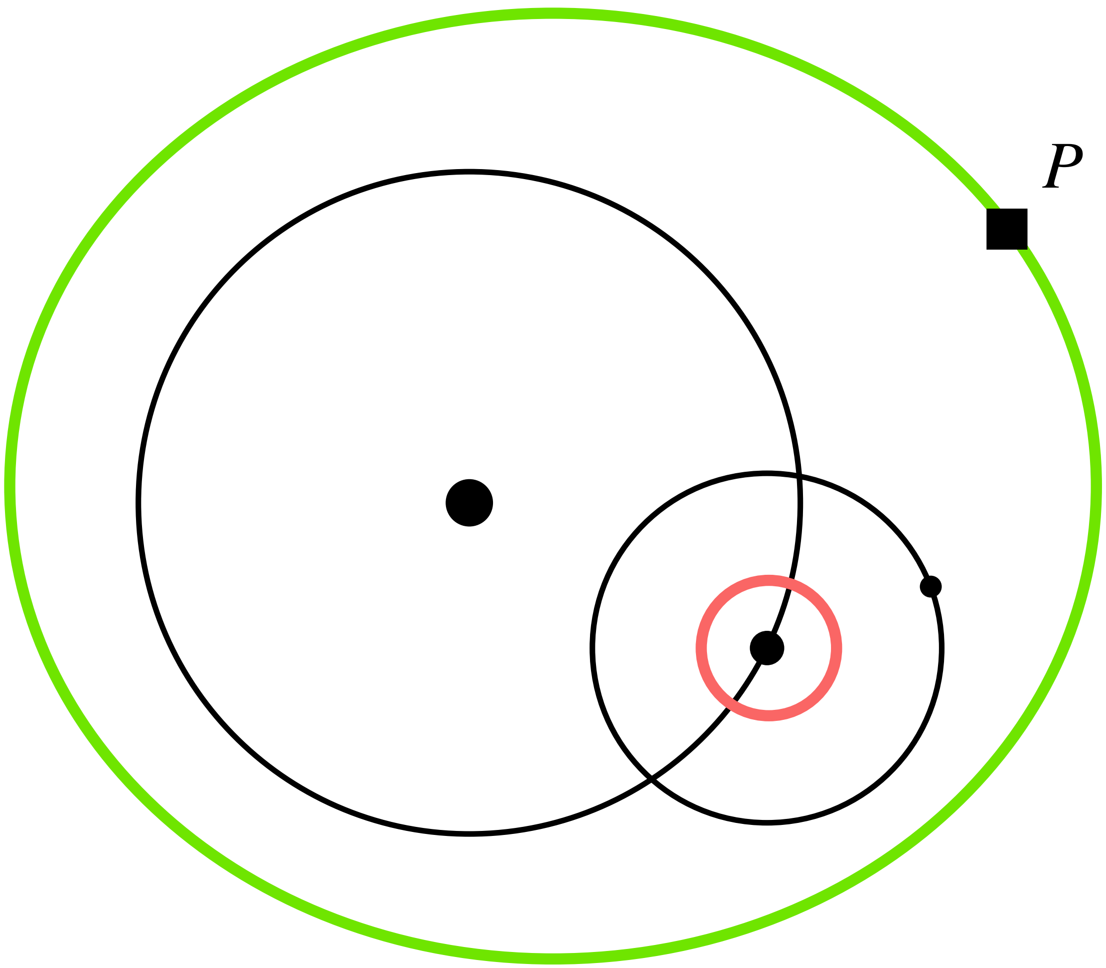
Design Problem Description

- Spacecraft P in gravity field of N massive bodies
- N massive bodies move in prescribed orbits



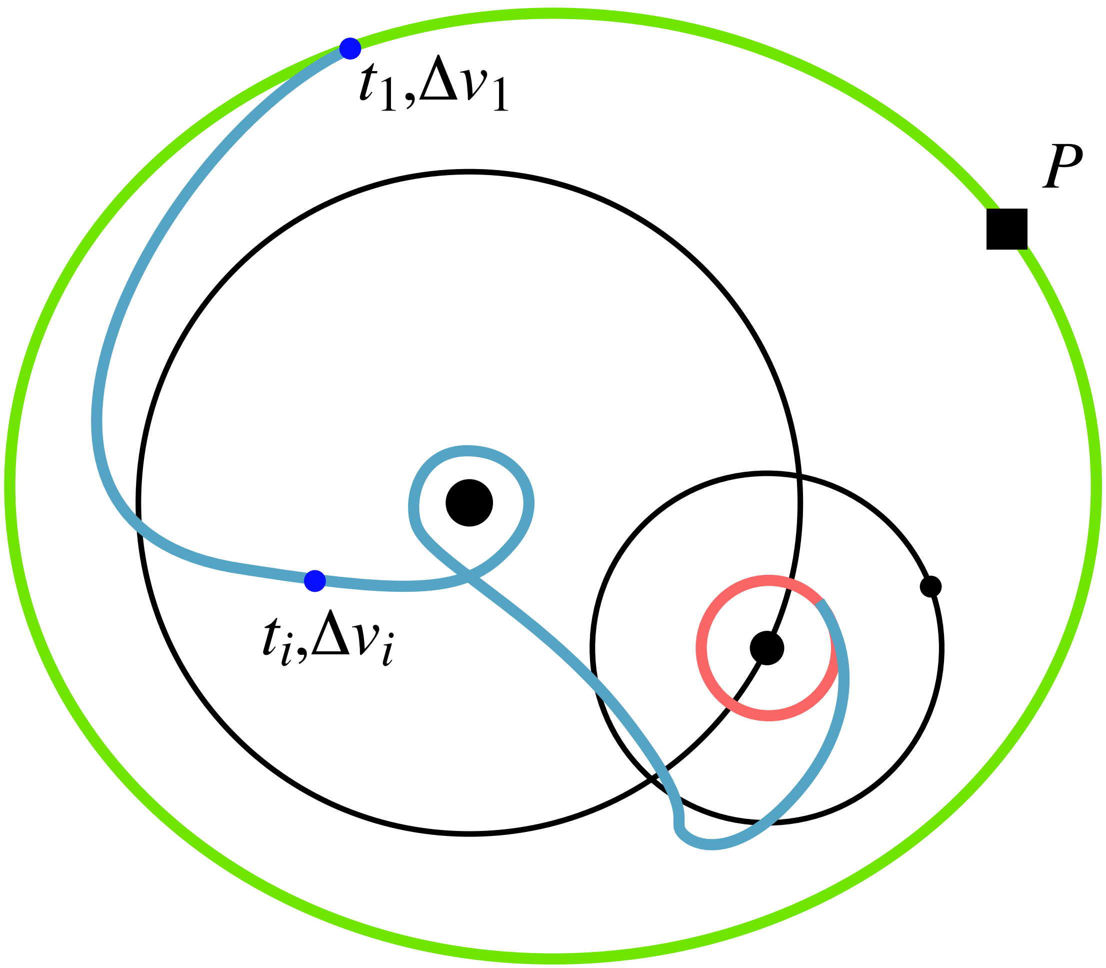
Design Problem Description

□ **Goal:** initial orbit $\longrightarrow$ final orbit



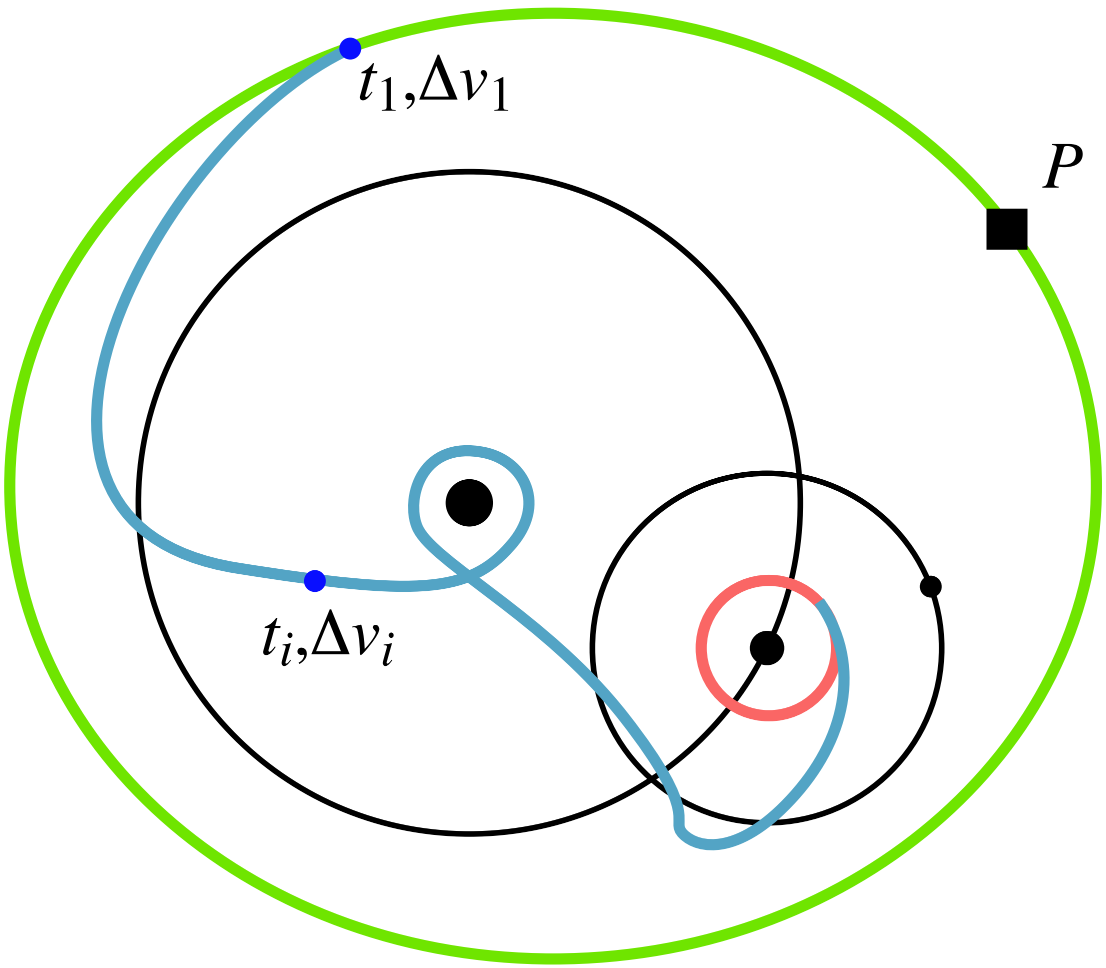
Design Problem Description

- **Impulsive controls:** instantaneous changes in spacecraft velocity, with norm Δv_i at time t_i



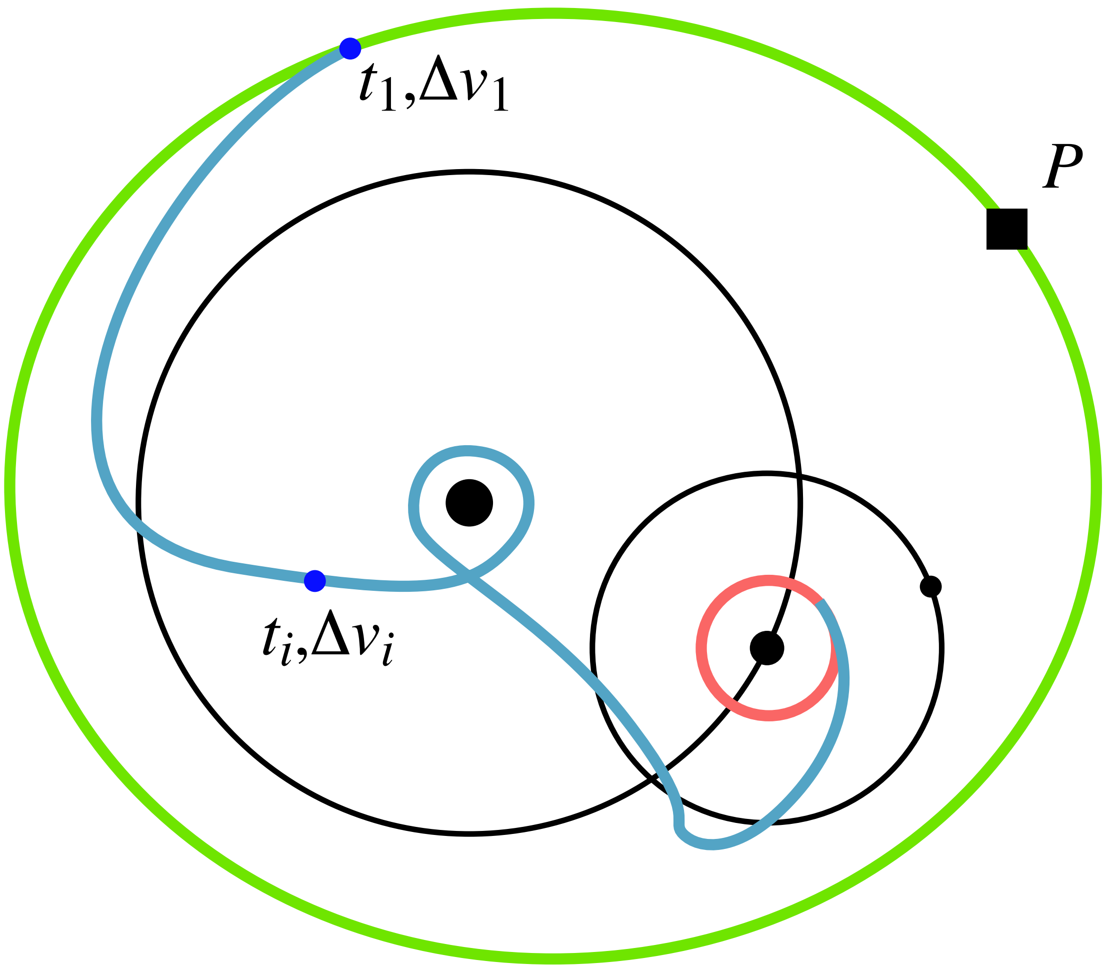
Design Problem Description

- corresponds to high-thrust engine burn maneuvers
- proportional to fuel consumption



Design Problem Description

- **Minimize Fuel/Energy:** find the maneuver times t_i and sizes Δv_i to minimize $\sum_i \Delta v_i =$ total ΔV



Tools Used in Solution

- **Hint:** Use natural dynamics as much as possible
  i.e., consider phase space geometry, integrals of motion, lanes of fast travel

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- **Hint:** Use natural dynamics as much as possible
  i.e., consider phase space geometry, integrals of motion, lanes of fast travel
- **Hierarchy of Models:** Start with simple models which capture essential features of natural dynamics
- Simple model solutions used as initial solutions in more realistic models

Tools Used in Solution

- **Patched 3-Body Approximation:** $N + 1$ body system decomposed into 3-body subsystems: spacecraft P + two massive bodies M_i & M_j

Tools Used in Solution

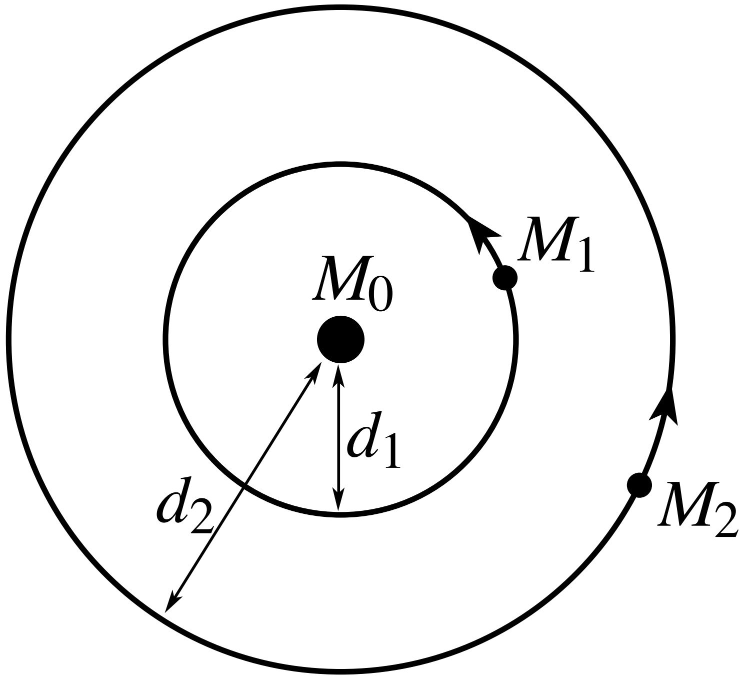
- **Patched 3-Body Approximation:** $N+1$ body system decomposed into 3-body subsystems: spacecraft P + two massive bodies M_i & M_j
- 3-body problem nonlinear dynamics
  - phase space $\longrightarrow$ tubes, resonance structures, ballistic capture
  - patched solutions $\longrightarrow$ **first guess solution** in realistic model
  - Optimization packages yield fast convergence to real solution

Tools Used in Solution

- **Patched 3-Body Approximation:** $N + 1$ body system decomposed into 3-body subsystems: spacecraft P + two massive bodies M_i & M_j
- 3-body problem nonlinear dynamics
  - phase space $\longrightarrow$ tubes, resonance structures, ballistic capture
  - patched solutions $\longrightarrow$ **first guess solution** in realistic model
  - Optimization packages yield fast convergence to real solution
- Further refinements
  - optimal control and parametric studies
  - impulsive burn maneuvers **or continuous low-thrust**

Patched 3-Body Approx.

- Consider spacecraft P in field of 3 massive bodies, M_0, M_1, M_2 e.g., Jupiter and two moons



Central mass M_0 and two massive orbiting bodies, M_1 and M_2

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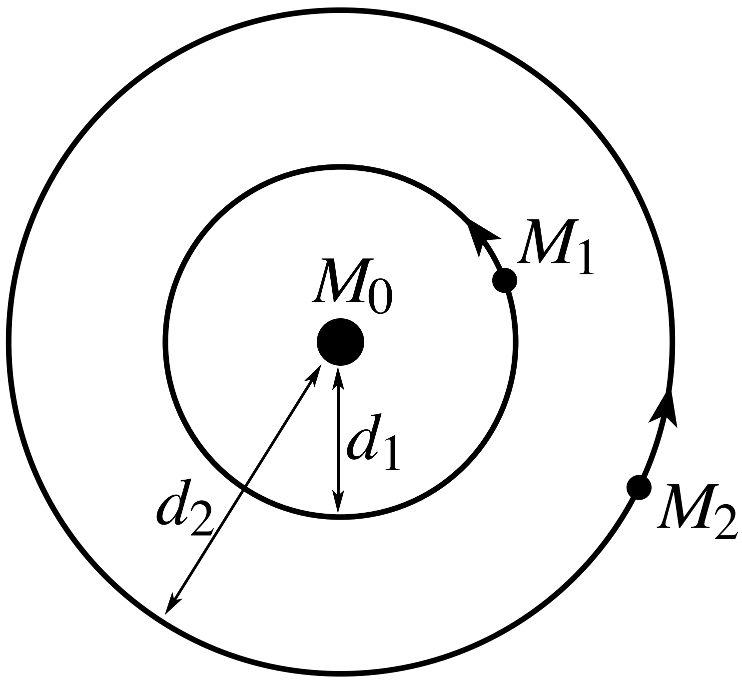

Central mass M_0 and two massive orbiting bodies, M_1 and M_2

- Assumption: Only one 3-body interaction dominates at a time (found to hold quite well)

Patched 3-Body Approx.

- **Initial approximation**

  4-body system approximated as two 3-body subsystems

- for $t < 0$, model as P-M_0-M_1

  for $t > 0$, model as P-M_0-M_2

  i.e., we “patch” two 3-body solutions

Patched 3-Body Approx.

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- for $t < 0$, model as P-M_0-M_1

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  i.e., we "patch" two 3-body solutions

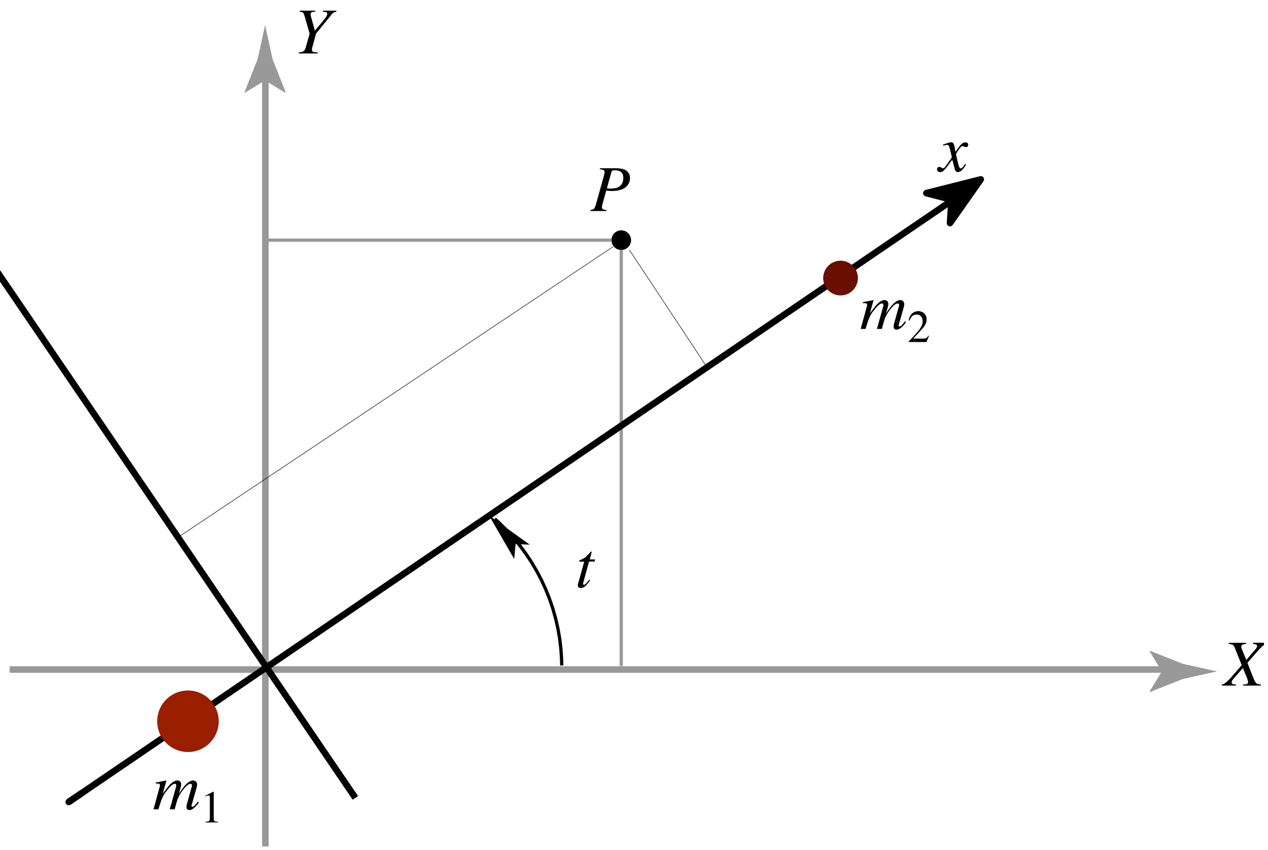
- 3-body solutions are now known quite well

  (Koon, Lo, Marsden, SDR [2000], ...)

  Consider the 3-body problem...

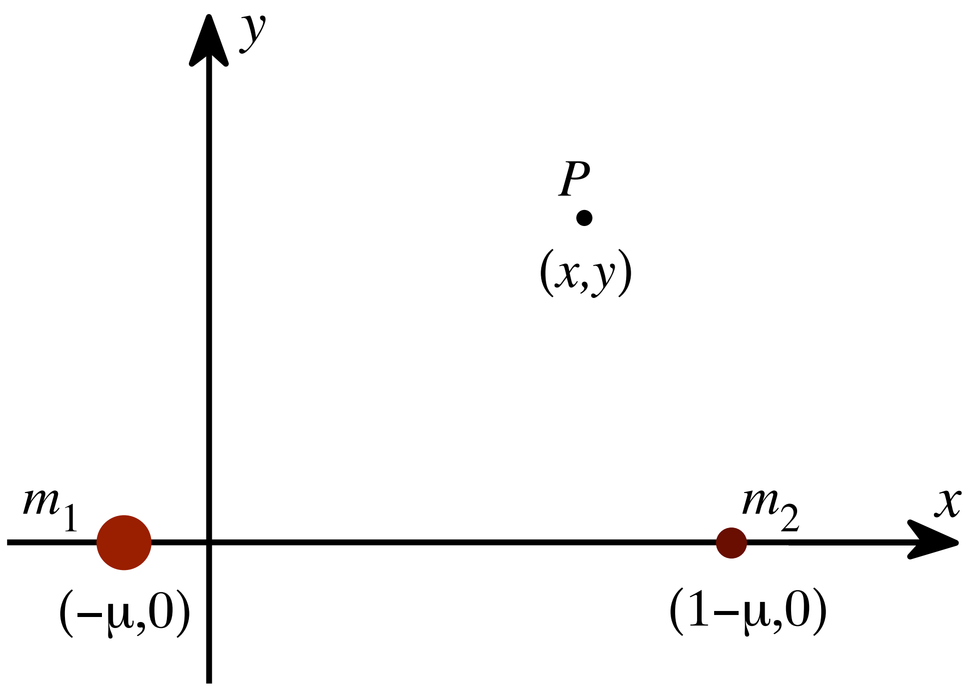
3-Body Problem

- **Planar, circular, restricted 3-body problem**
  - P in field of two bodies, m_1 and m_2
  - x-y frame rotates w.r.t. X-Y inertial frame

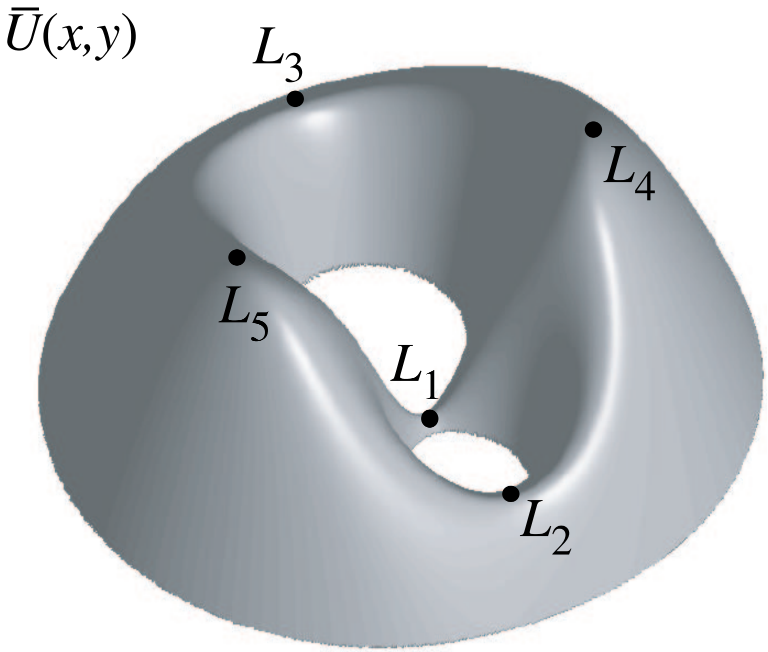


3-Body Problem

- Equations of motion describe P moving in an effective potential plus a coriolis force



Position Space



Effective Potential

Hamiltonian System

- Hamiltonian function

$$H(x, y, p_x, p_y) = \frac{1}{2}((p_x + y)^2 + (p_y - x)^2) + \bar{U}(x, y),$$

where p_x and p_y are the conjugate momenta, where

$$\bar{U}(x, y) = -\frac{1}{2}(x^2 + y^2) - \frac{\mu_1}{r_1} - \frac{\mu_2}{r_2}$$

where r_1 and r_2 are the distances of P from m_1 and m_2 and

$$\mu = \frac{m_2}{m_1 + m_2}$$

Motion within Energy Surface

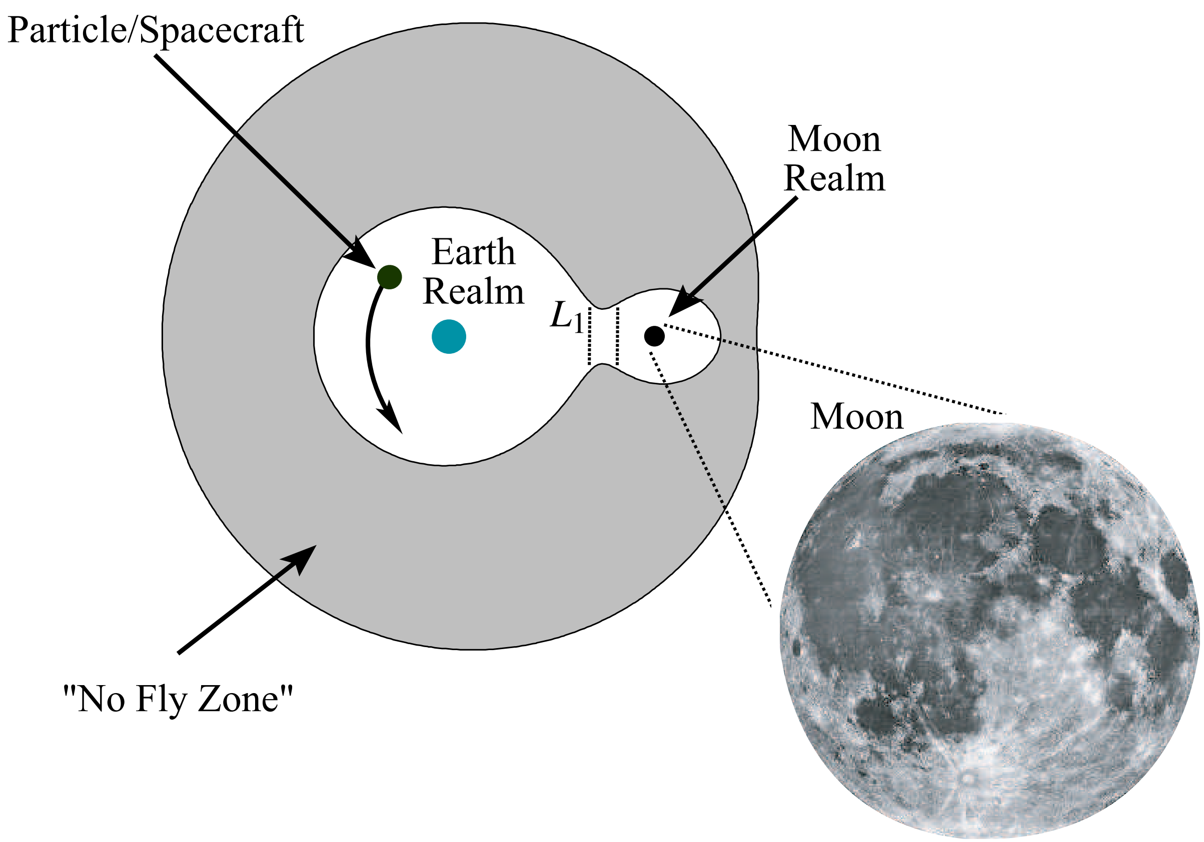
- For fixed μ, an energy surface of energy ε is

$$\mathcal{M}_\mu(\varepsilon) = \{(x, y, p_x, p_y) \mid H(x, y, p_x, p_y) = \varepsilon\}.$$

  In the 2 d.o.f. problem, these are 3-dimensional surfaces foliating the 4-dimensional phase space. In 3 d.o.f., 5D energy surfaces.

Realms of Possible Motion

- $\mathcal{M}_\mu(\varepsilon)$ partitioned into three **realms**
  e.g., Earth realm = phase space around Earth
- ε determines their connectivity



Multi-Scale Dynamics

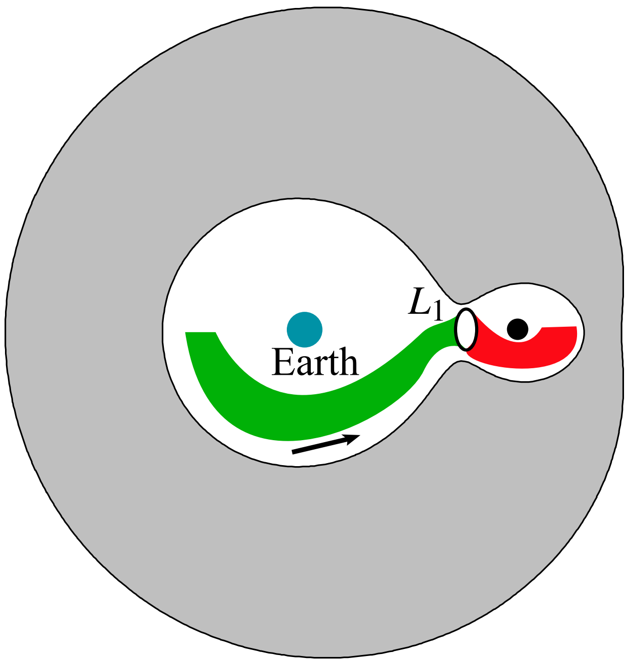
- **n $\geq$ 2 d.o.f. Hamiltonian systems** : for some regimes of motion (usually labeled "chaotic"), the phase space has structures mediating transport.

Multi-Scale Dynamics

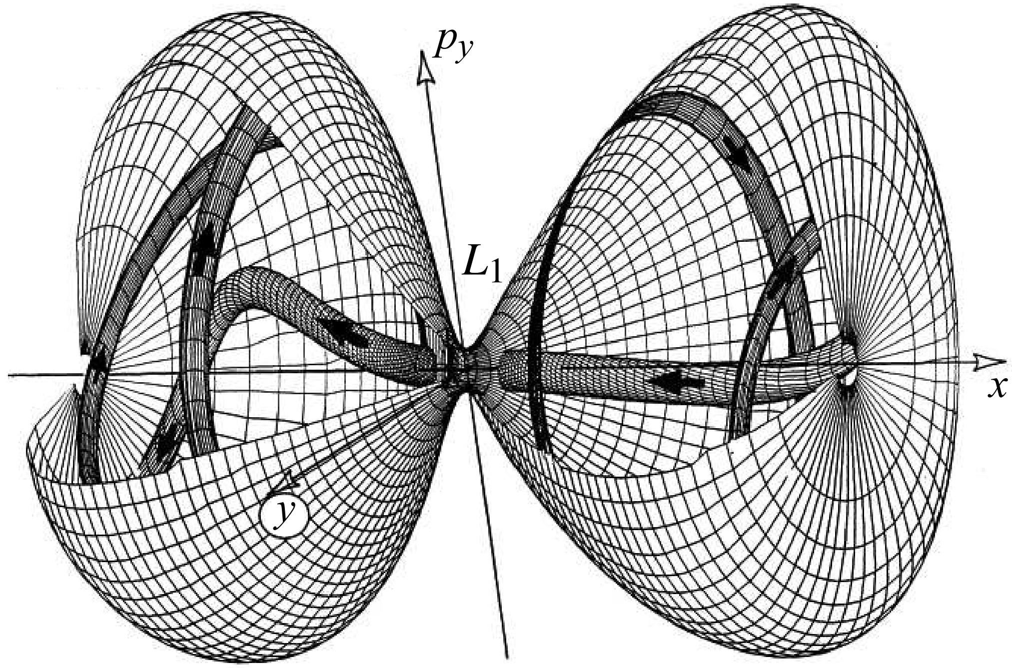
- **n $\geq$ 2 d.o.f. Hamiltonian systems** : for some regimes of motion (usually labeled "chaotic"), the phase space has structures mediating transport.
- Multi-scale approach:
- **Tube dynamics** : bottlenecks **in energy surface** motion across saddle of potential energy
- **Lobe dynamics** : bottlenecks **within chaotic sea** motion between stable resonance islands

Multi-Scale Dynamics

- Realms connected by tubes



Position Space

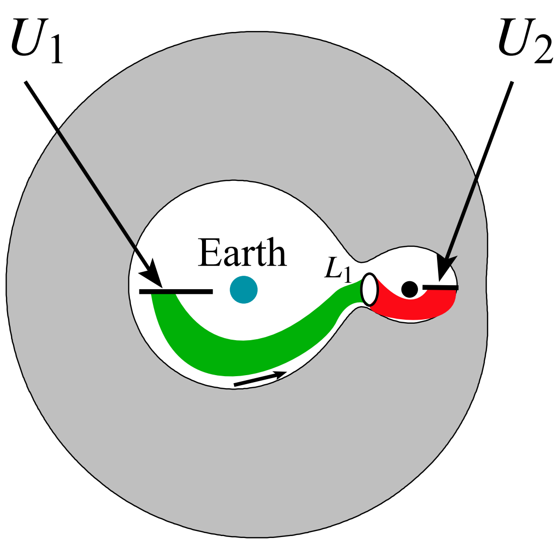


Phase Space (Position + Velocity)

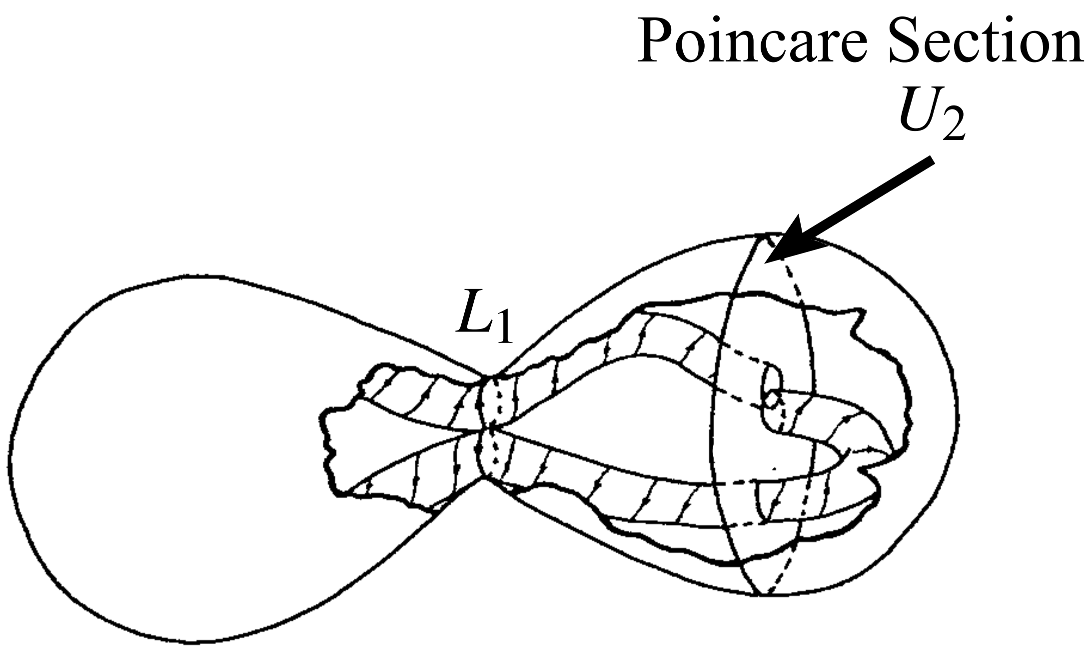
adapted from Topper [1997]

Multi-Scale Dynamics

- Poincaré section U_i in Realm i, $i = 1, \ldots, k$
- Lobe dynamics: evolution **on** U_i
- Tube dynamics: evolution **between** U_i



Position Space



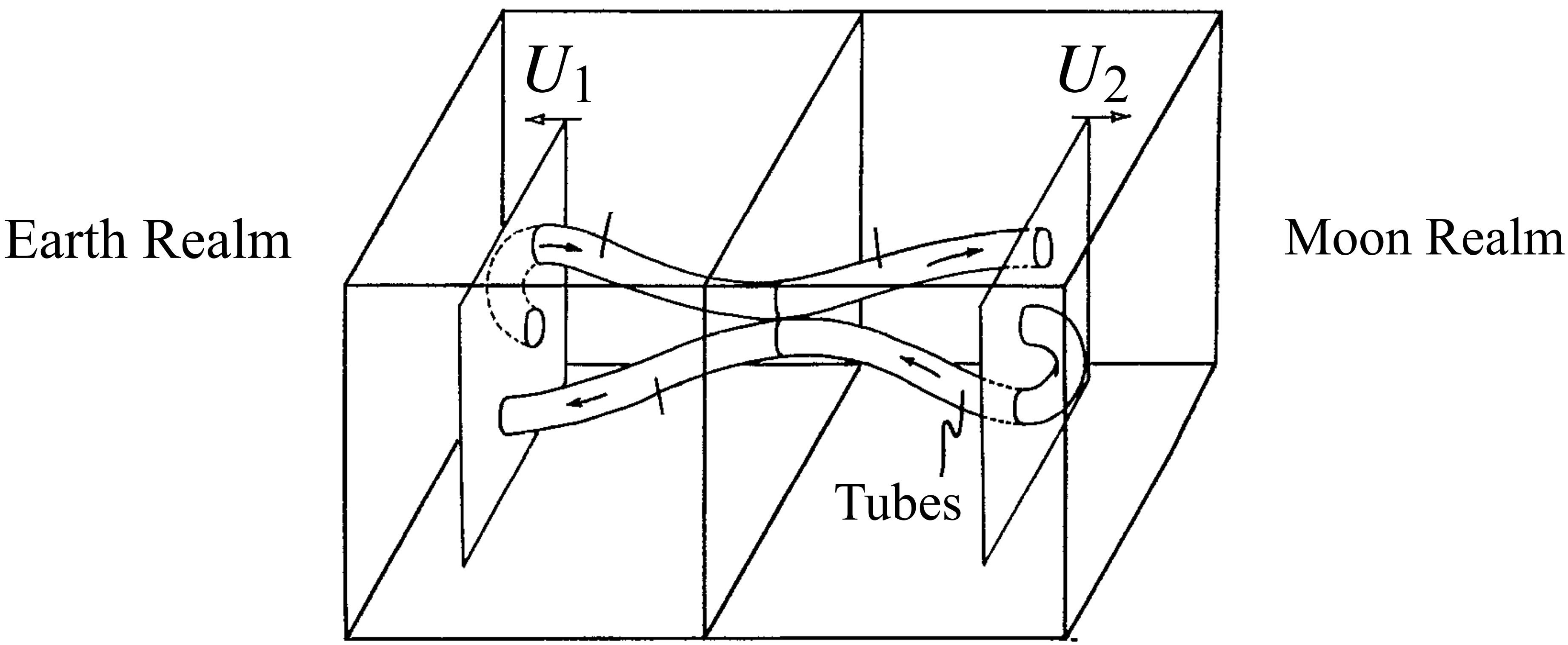
Phase Space

Tube Dynamics

- Motion on and between **Poincaré surfaces-of-section** (SOS) on $\mathcal{M}_\mu(\varepsilon)$:

$$U_i = \{(x, p_x) | y = \text{const} \in \text{Realm } i, p_y = g(x, p_x, y; \mu, \varepsilon) > 0\}.$$

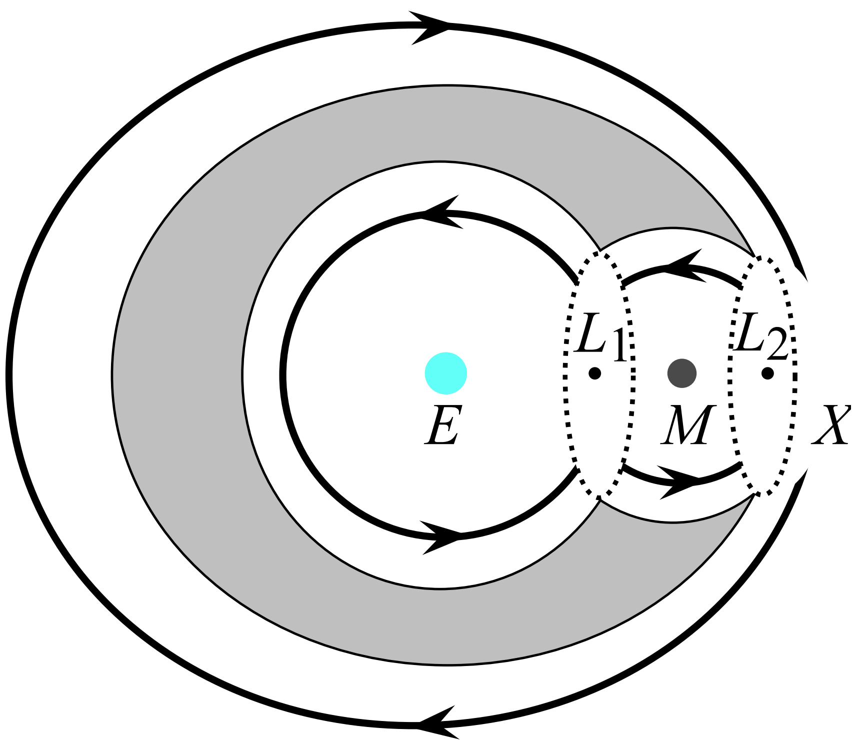
System reduced to area-preserving k-map dynamics between k SOS.



Poincaré surfaces-of-section U_1 & U_2 linked by tubes

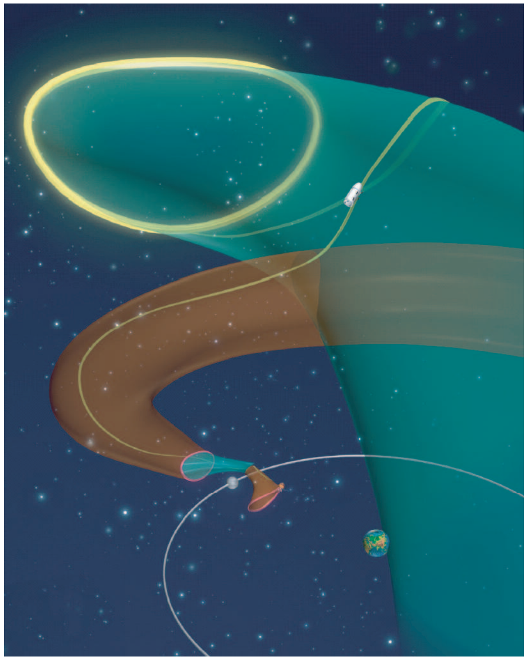
Tube Dynamics: Theorem

- *Theorem of global orbit structure*
  - says we can construct an orbit with any **itinerary**, eg $(\ldots, M, X, M, E, M, E, \ldots)$, where X, M and E denote the different realms (symbolic dynamics)
  - Main theorem of Koon, Lo, Marsden, SDR [2000]



Construction of Trajectories

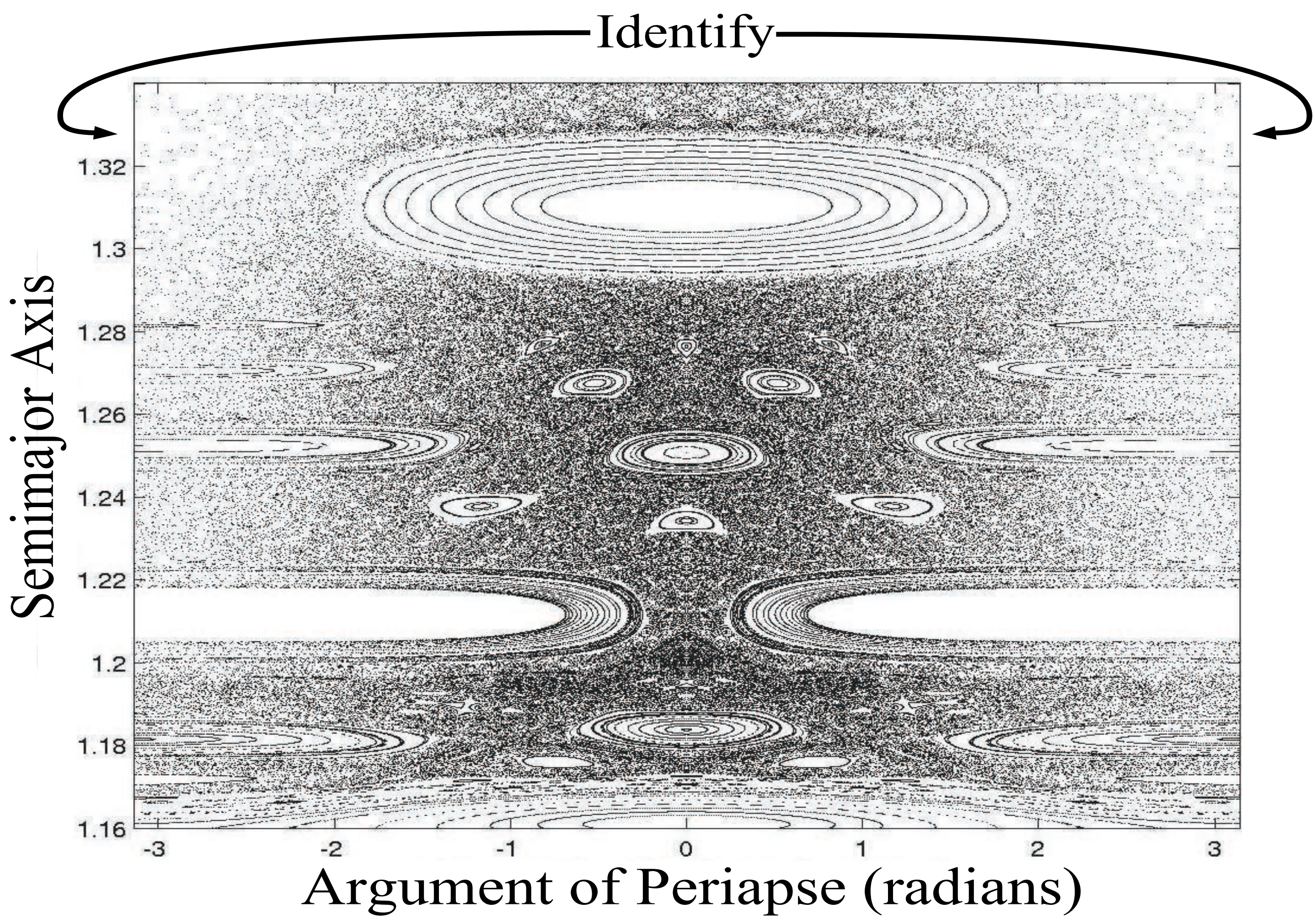
- Systematic construction of trajectories with desired itineraries – trajectories which use **little or no fuel.**
  - by linking tubes in the right order $\longrightarrow$ **tube hopping**
- Itineraries for multiple 3-body systems possible too.



Tube hopping

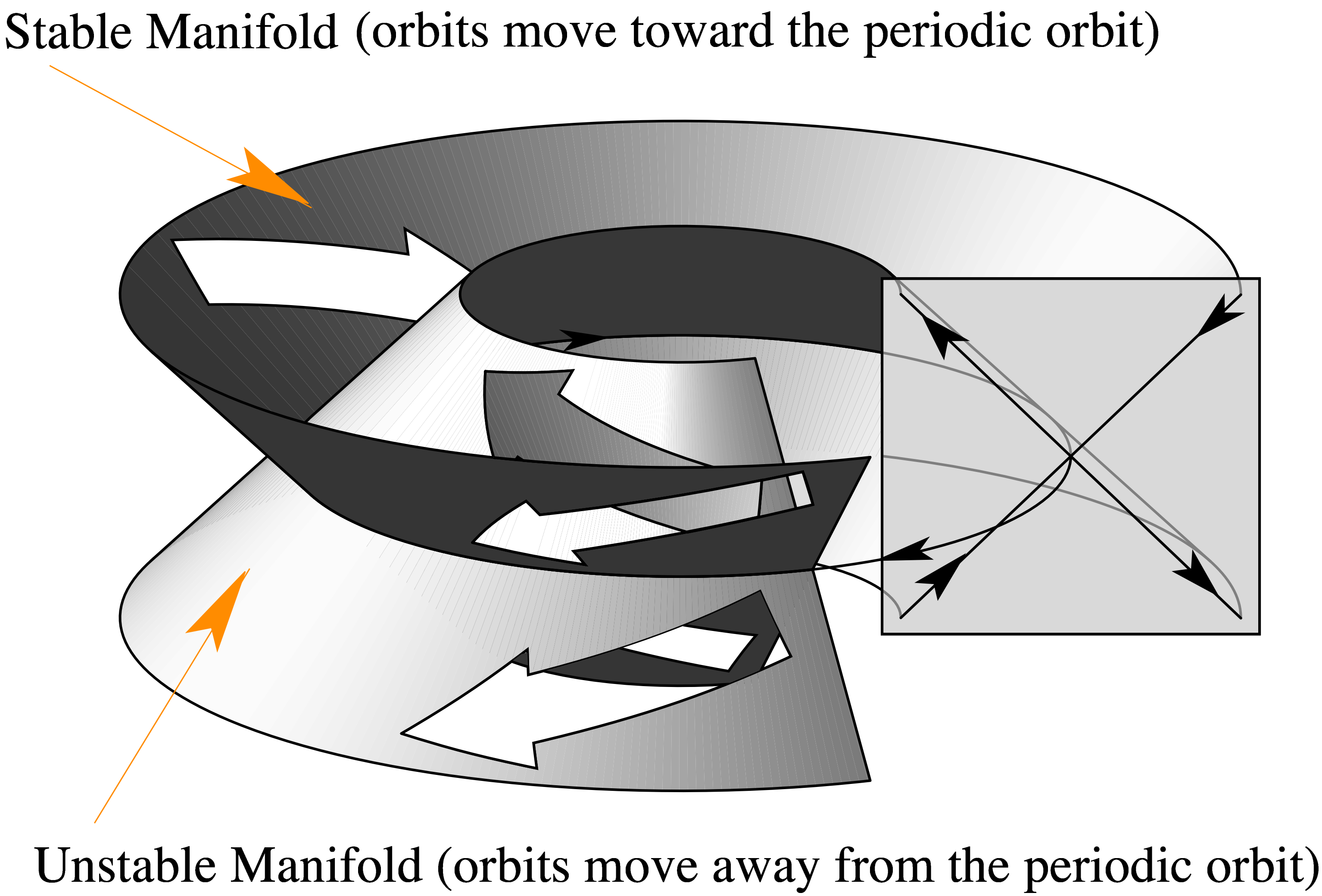
Lobe Dynamics

- Tubes do not give the full picture...
- In each realm: SOS reveals stable islands & irregular components. Large connected irregular component, the **"chaotic sea."**



Lobe Dynamics: Per. Orbits

- Unstable resonances: Periodic orbits form a **dynamical "back-bone,"** via their unstable and stable manifolds.



Unstable resonances and their manifolds.

Lobe Dynamics: Partition Σ

- Let $\Sigma = U_i$, then our Poincaré map is a diffeomorphism

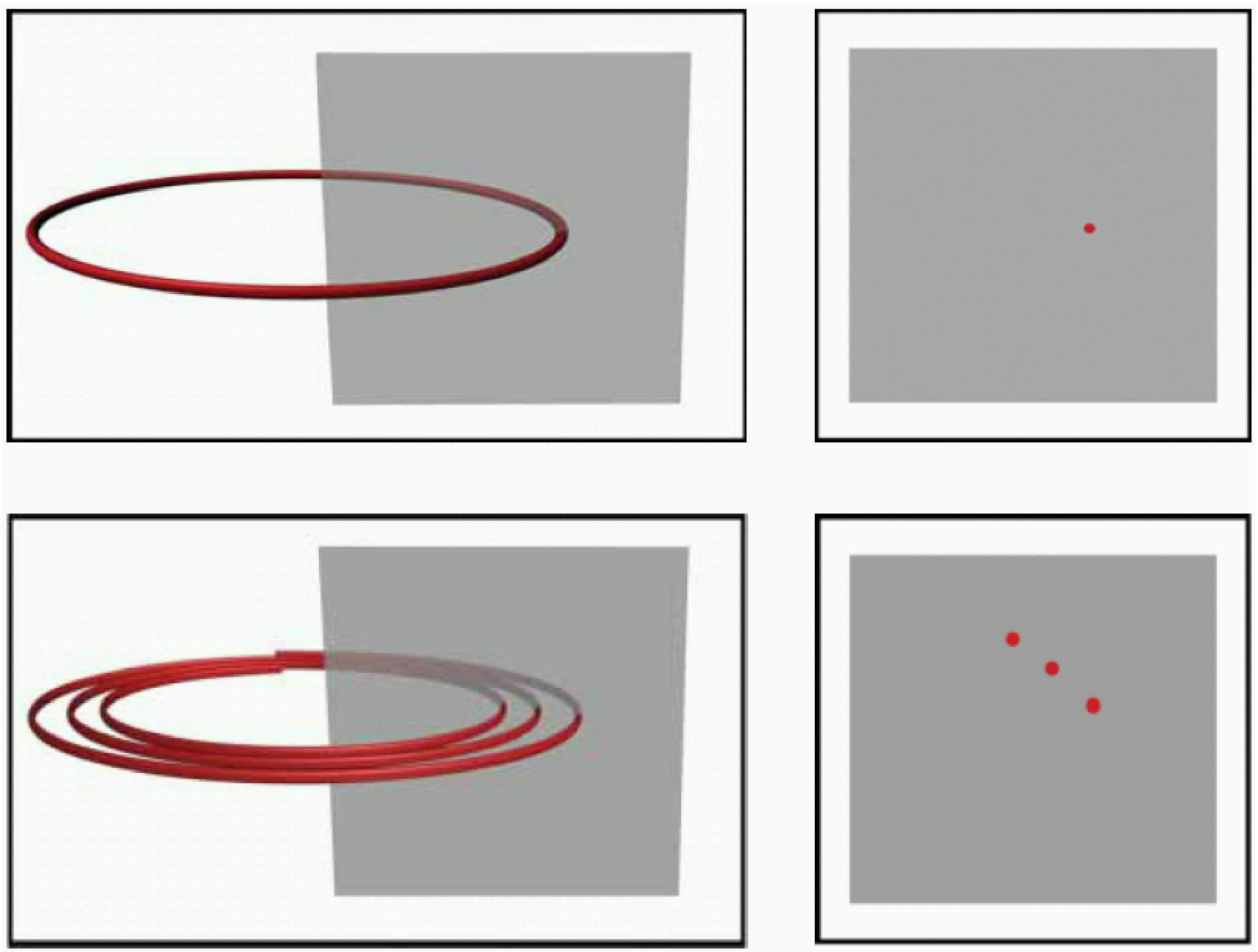
$$f : \Sigma \longrightarrow \Sigma,$$

- f is **orientation-preserving** and **area-preserving**

- Let $p_i, i = 1, ..., N_p$, denote a collection of saddle-type hyperbolic periodic points for f.

Lobe Dynamics: Partition Σ

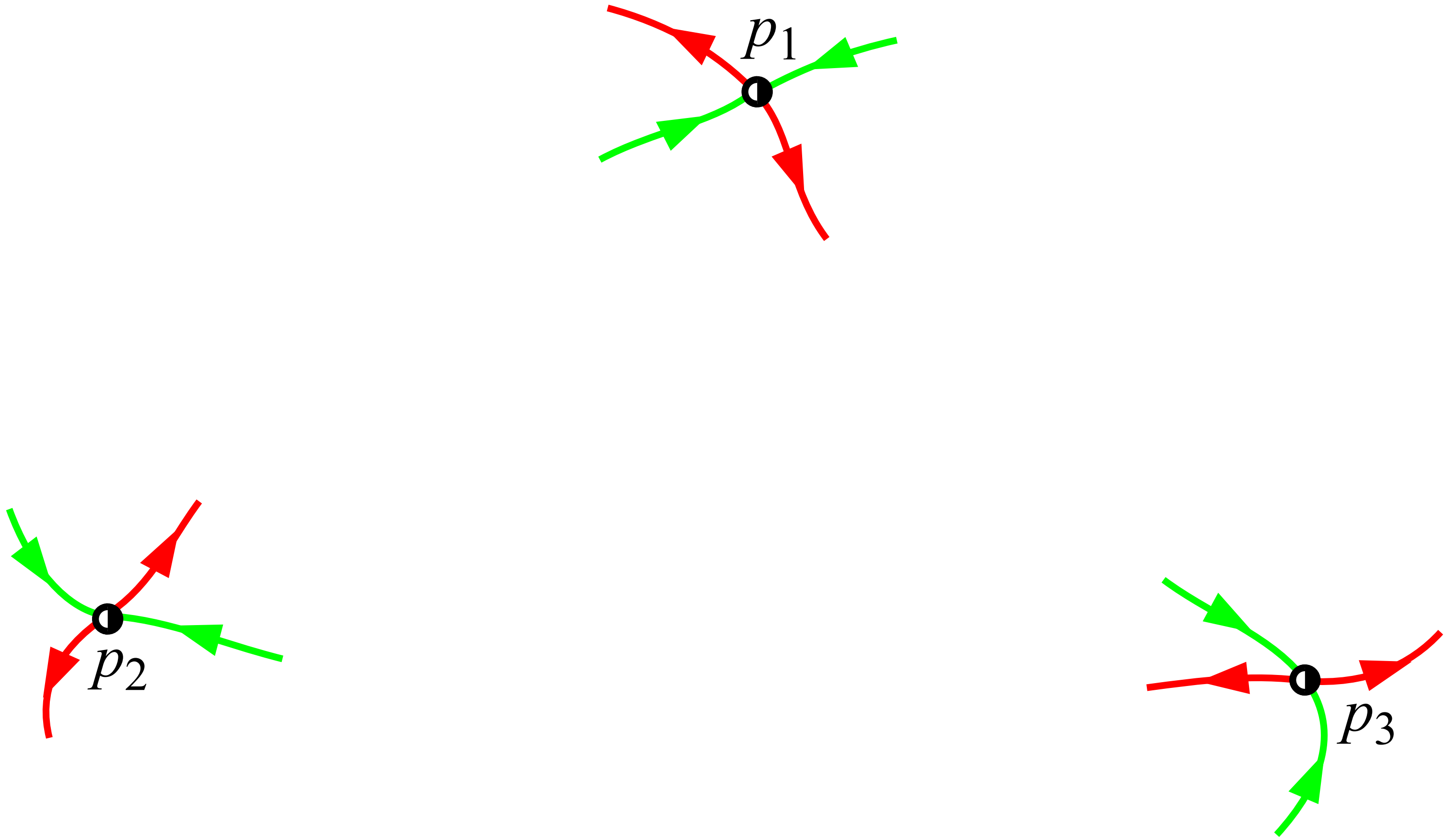
These are the unstable resonances reduced to Σ.



Poincaré surface of section

Lobe Dynamics: Partition Σ

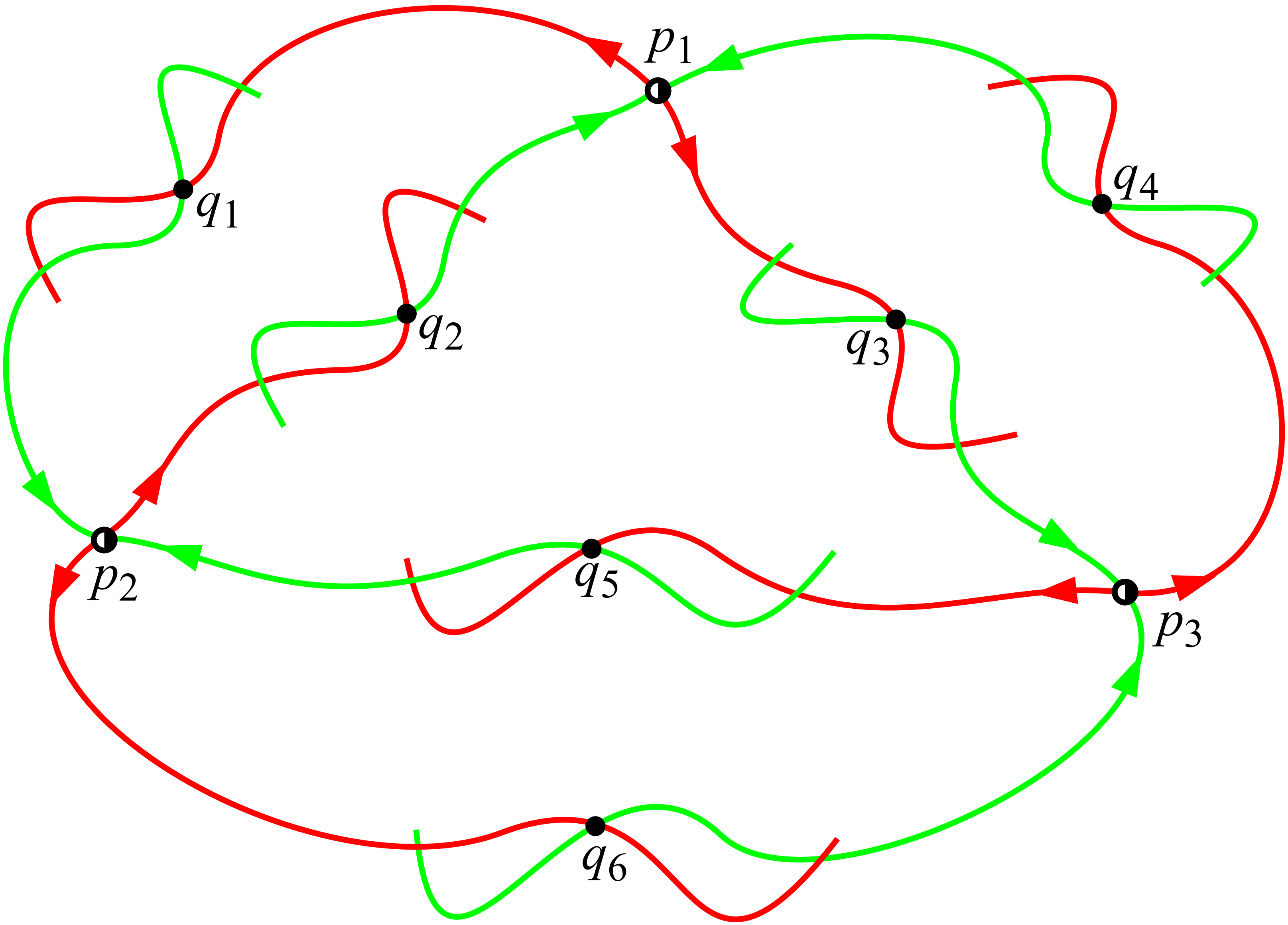
- Pieces of $W^u(p_i)$ and $W^s(p_i)$ partition Σ



Unstable and stable manifolds in **red** and **green**, resp.

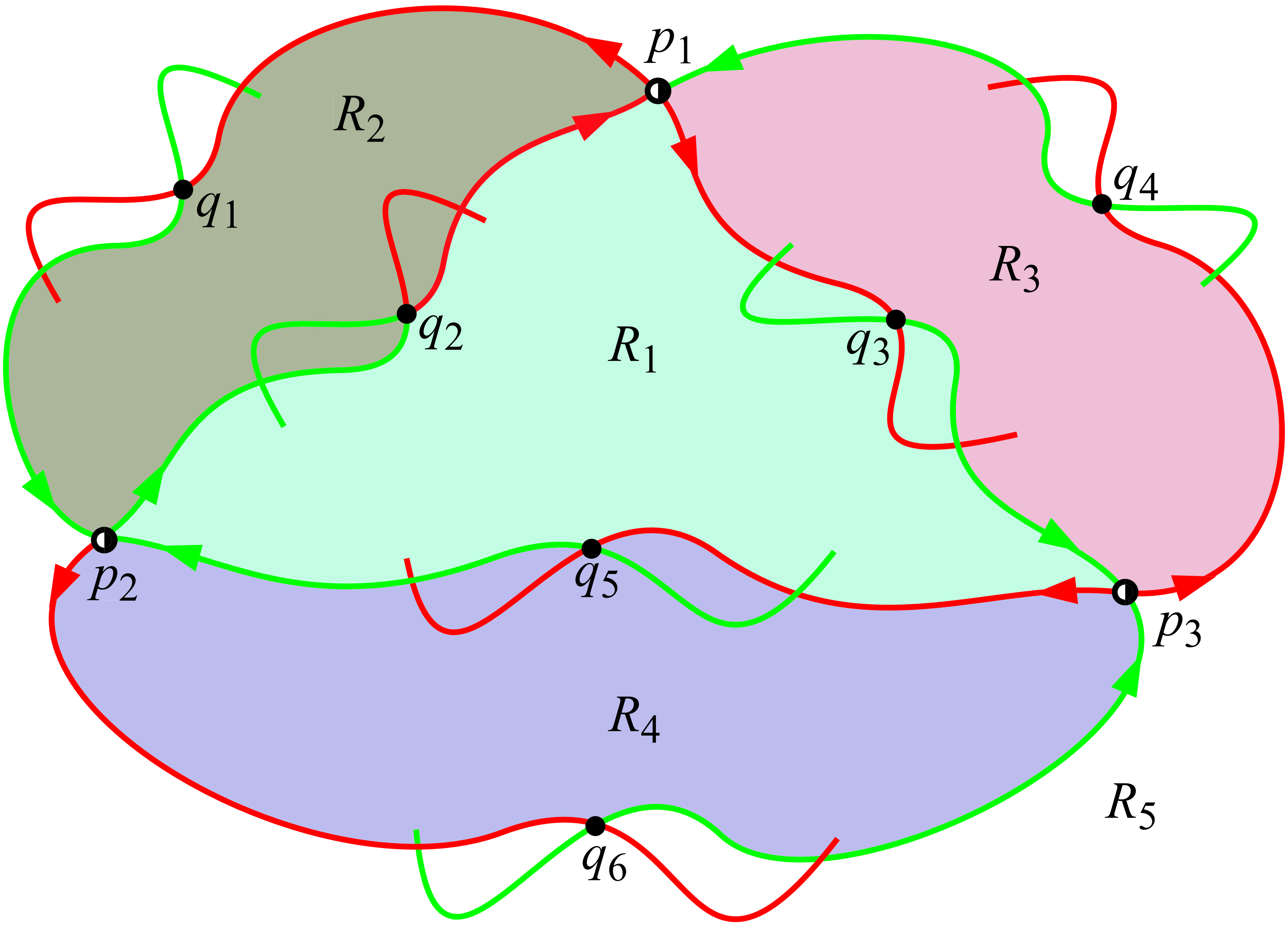
Lobe Dynamics: Partition Σ

- Intersection of unstable and stable manifolds define **boundaries**.



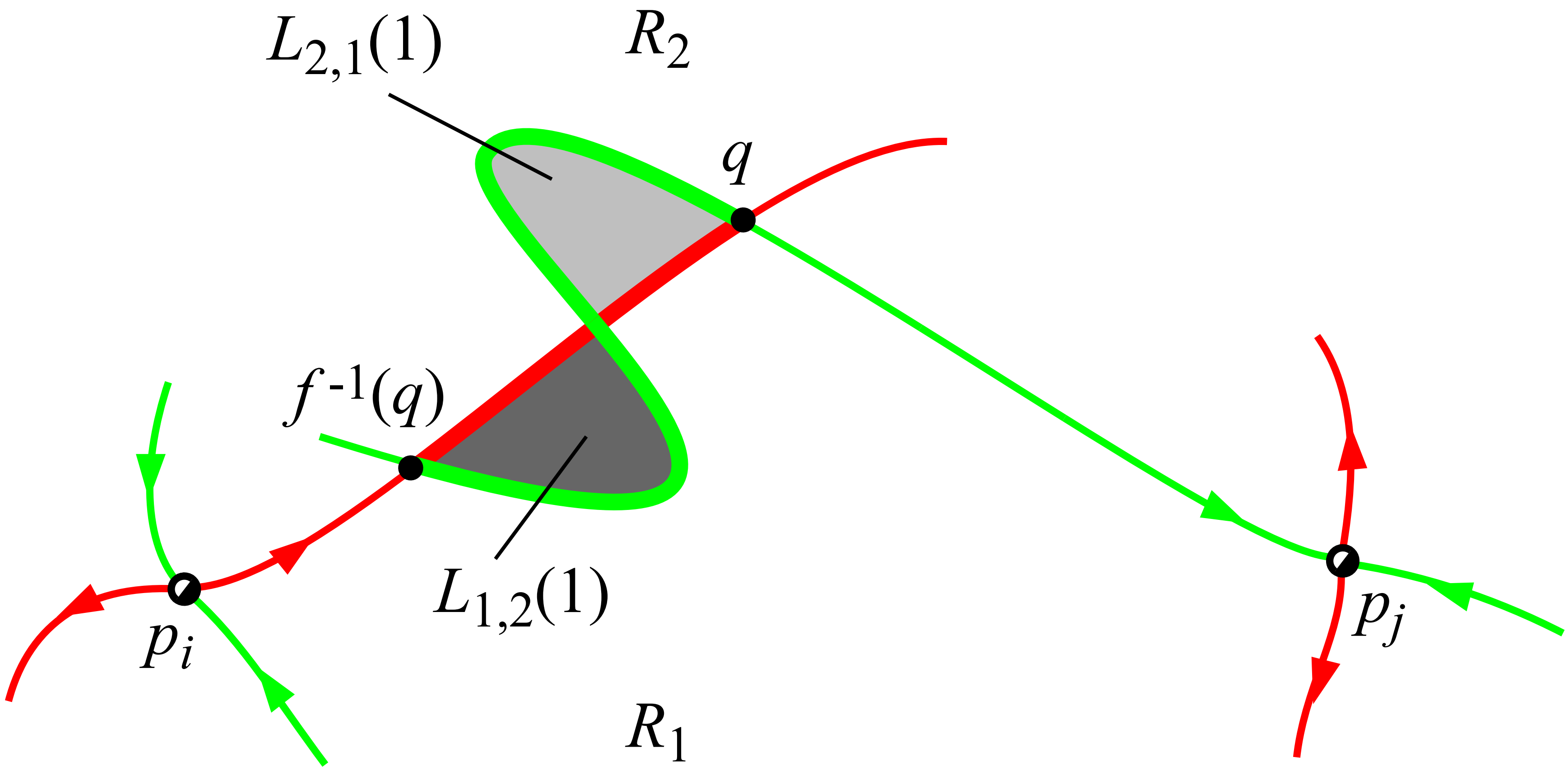
Lobe Dynamics: Partition Σ

- These boundaries divide phase space into **regions**, $R_i, i = 1, \ldots, N_R$



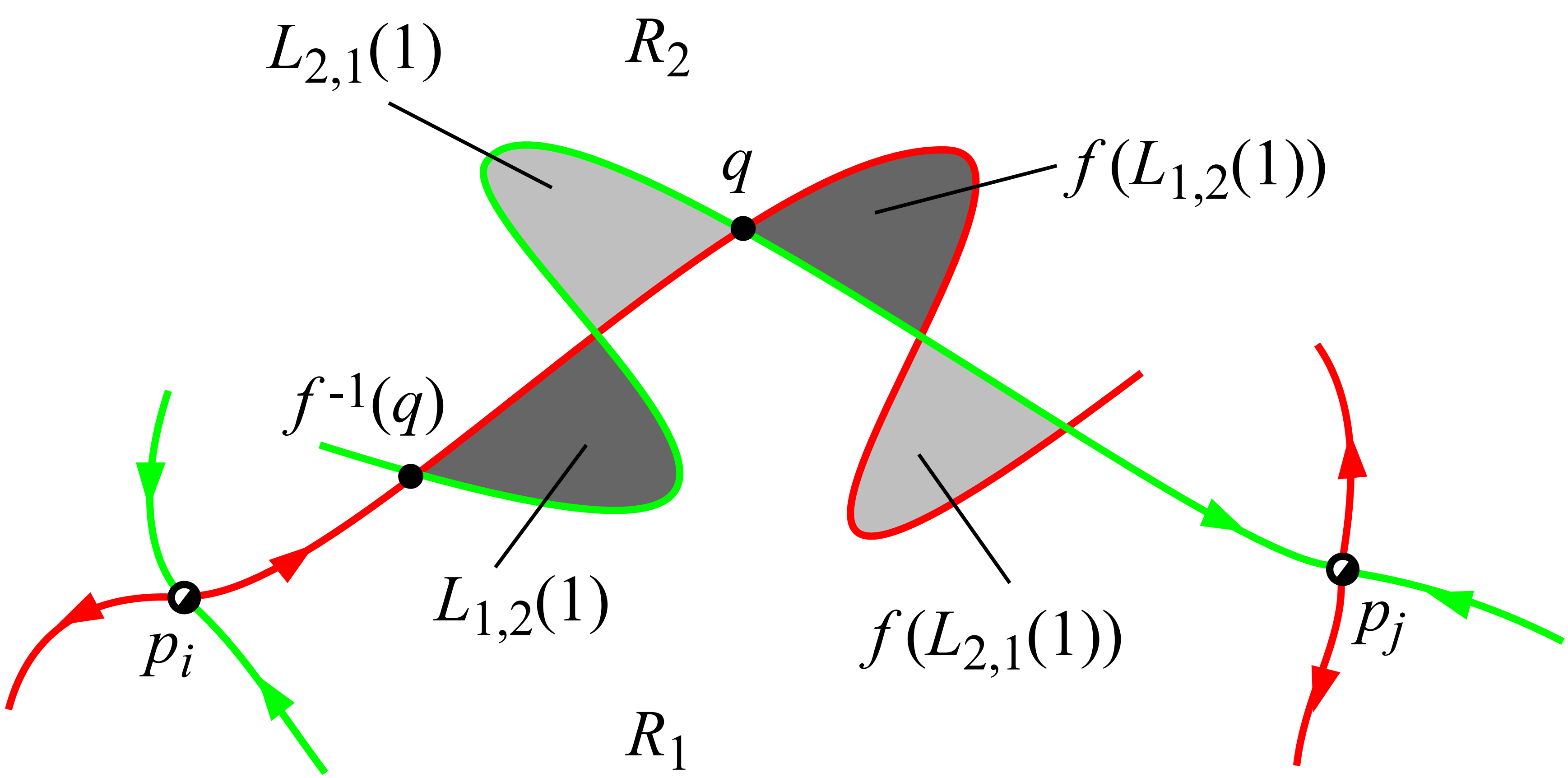
Lobe Dynamics: Turnstile

- $L_{1,2}(1)$ and $L_{2,1}(1)$ are called a **turnstile**



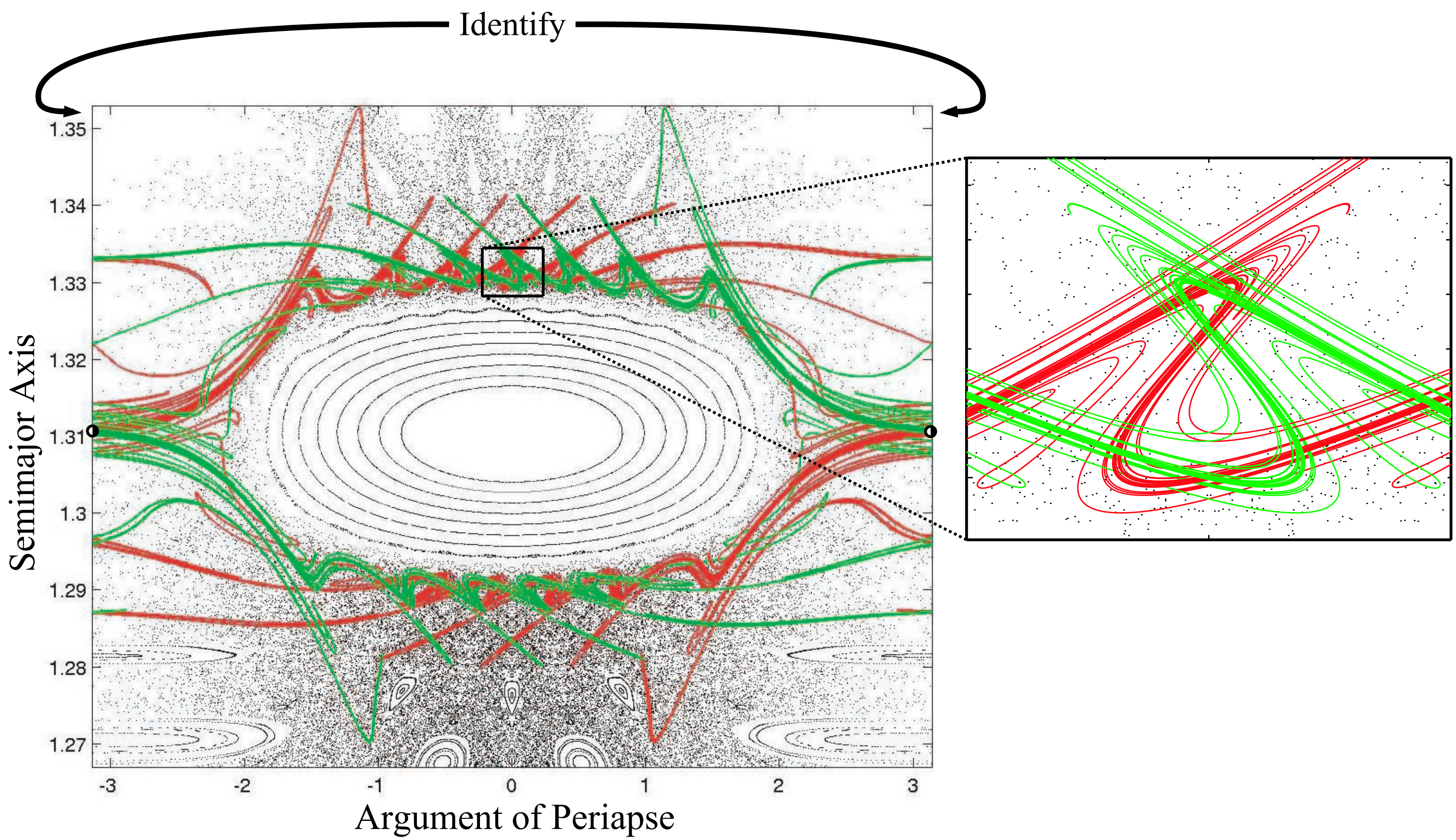
Lobe Dynamics: Turnstile

- They map from entirely in one region to another under one iteration of f



Move Amongst Resonances

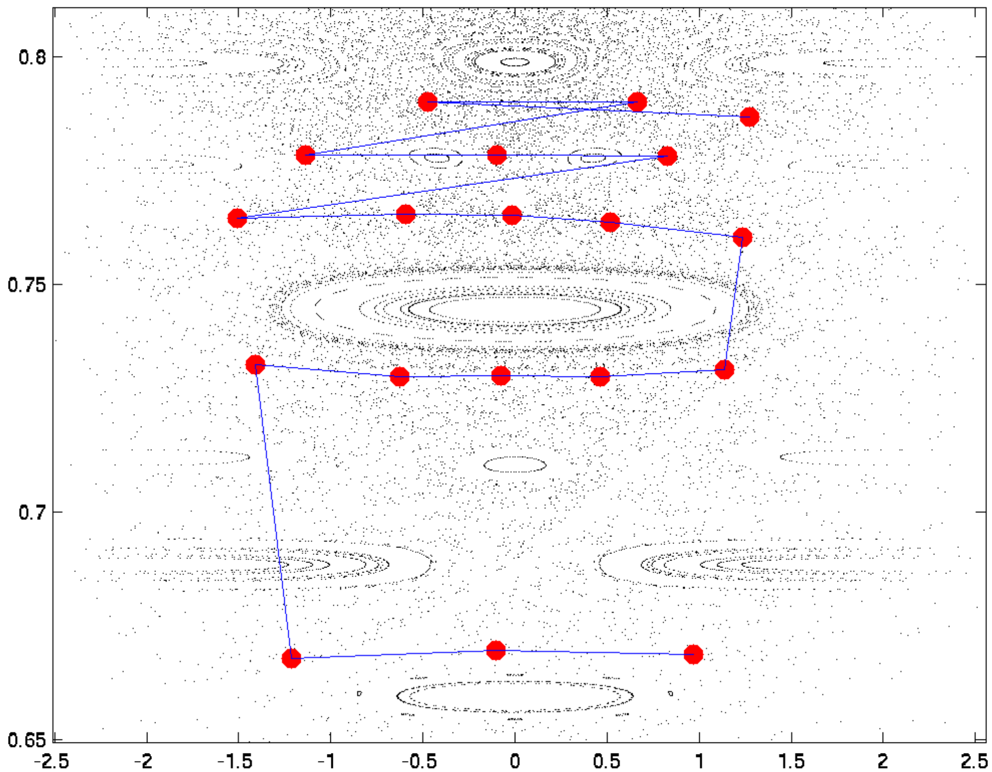
- Numerics: regions and lobes can be efficiently computed (MANGEN).



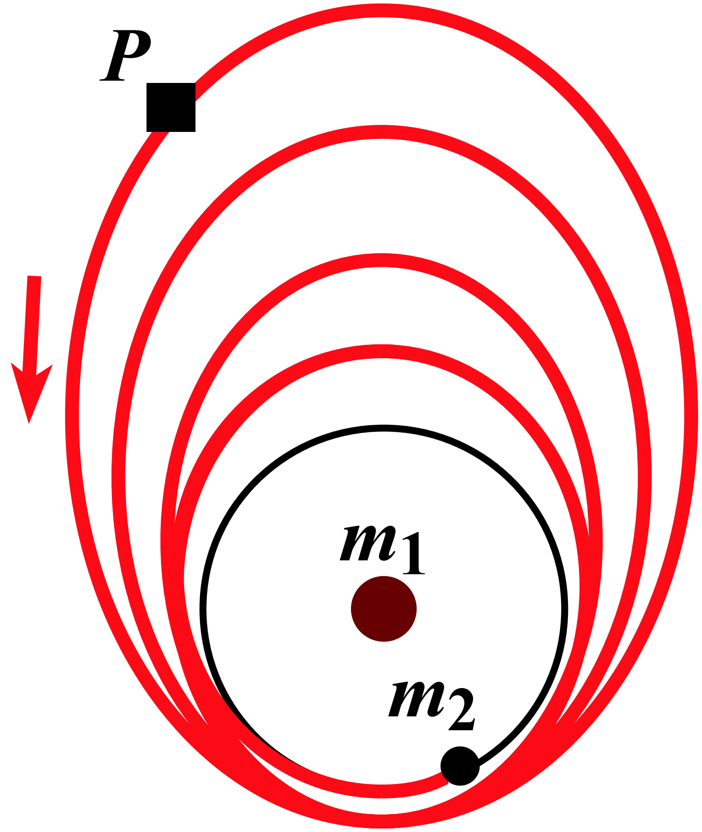
Unstable and stable manifolds in **red** and **green**, resp.

Move Amongst Resonances

- Trajectory construction:
  Large orbit changes with little fuel via **resonant gravity assists.**



Surface-of-section



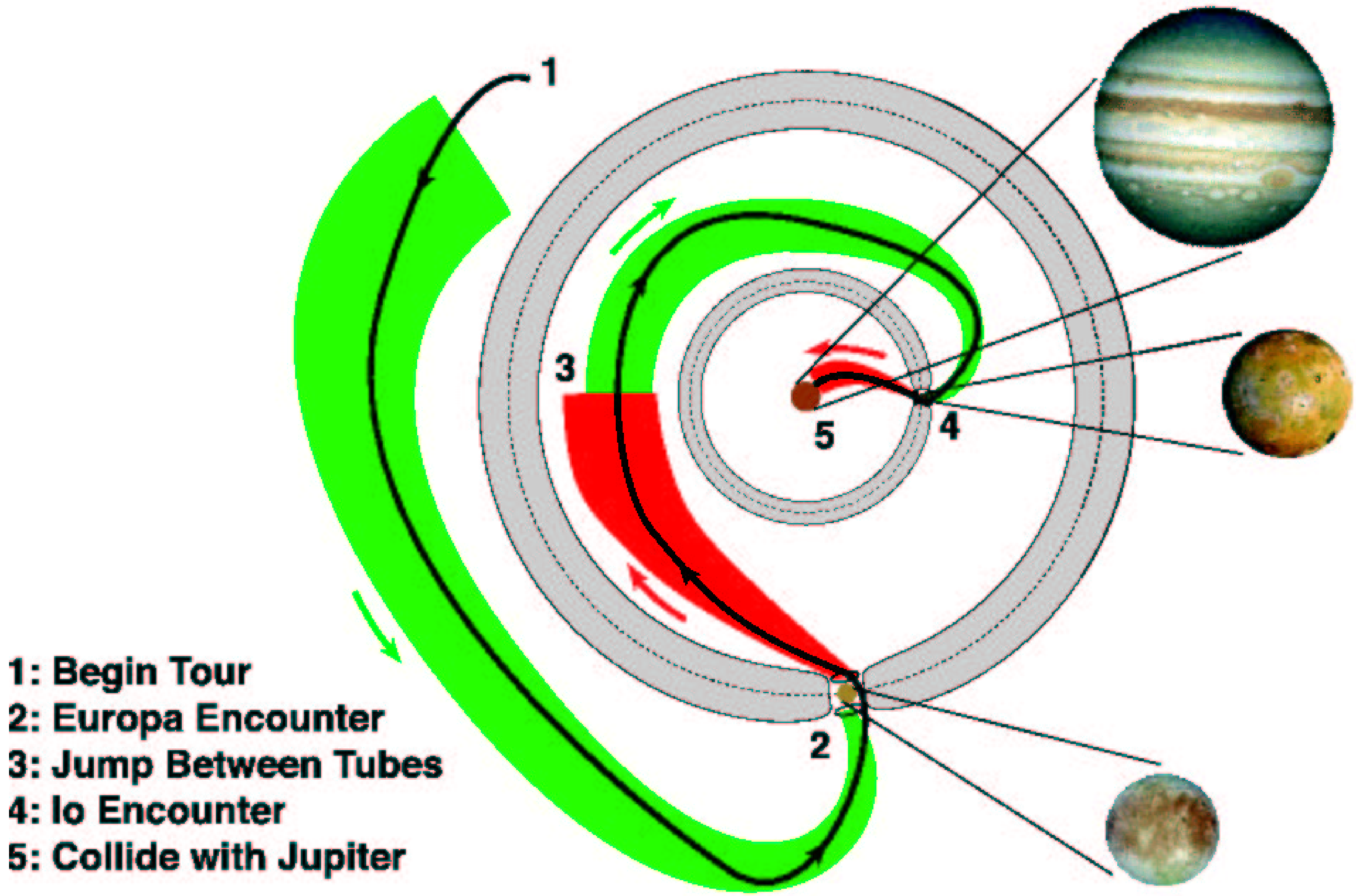
Large orbit changes

Patching Two 3-Body Sol'ns

- *Multi-Moon Orbiter (e.g., JIMO)*
  - We propose a trajectory design procedure which uses little fuel and allows a **single spacecraft to orbit multiple moons**
  - Orbit each moon for much longer than the quick fly-bys of previous missions
  - Standard "patched-conics" approach yields prohibitively high ΔV
  - Patched three-body approx. yields much lower ΔV

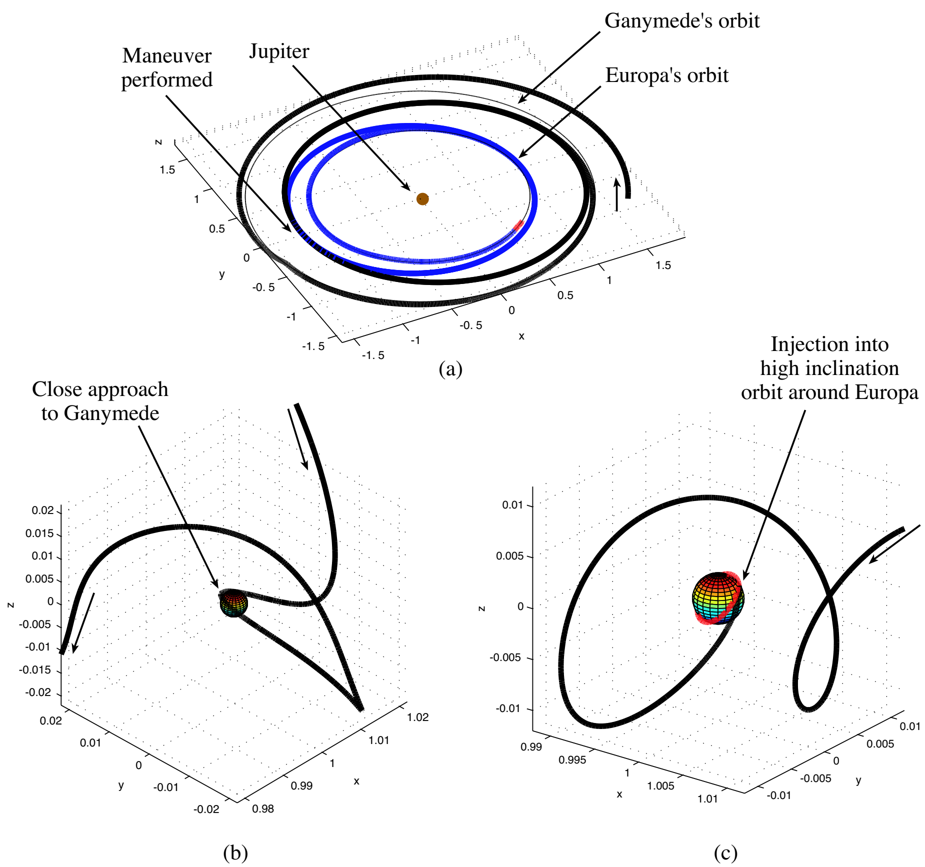
Patching Two 3-Body Sol'ns

- *Example 1:* Europa $\longrightarrow$ Io $\longrightarrow$ Jupiter



Patching Two 3-Body Sol'ns

- *Example 2:* A Ganymede-Europa Orbiter was constructed
  - ΔV of 1400 m/s was half the Hohmann transfer
  - Gómez, Koon, Lo, Marsden, Masdemont, SDR [2001]



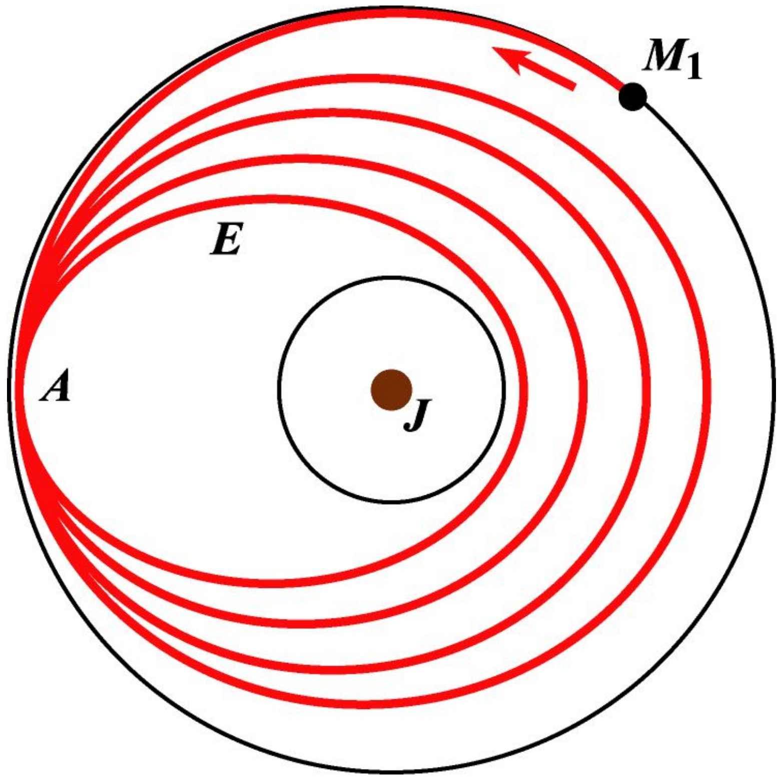
Patching Two 3-Body Sol'ns

- ***Latest example:***
  - Desirable to decrease ΔV further
  - **Resonant gravity assists** drastically reduce inter-moon transfer ΔV
  - Consider the following tour of Jupiter's moons
    - Begin in an eccentric orbit with perijove at Callisto's orbit, achievable using a patched-conics trajectory from the Earth
    - Orbit Callisto, Ganymede, and Europa

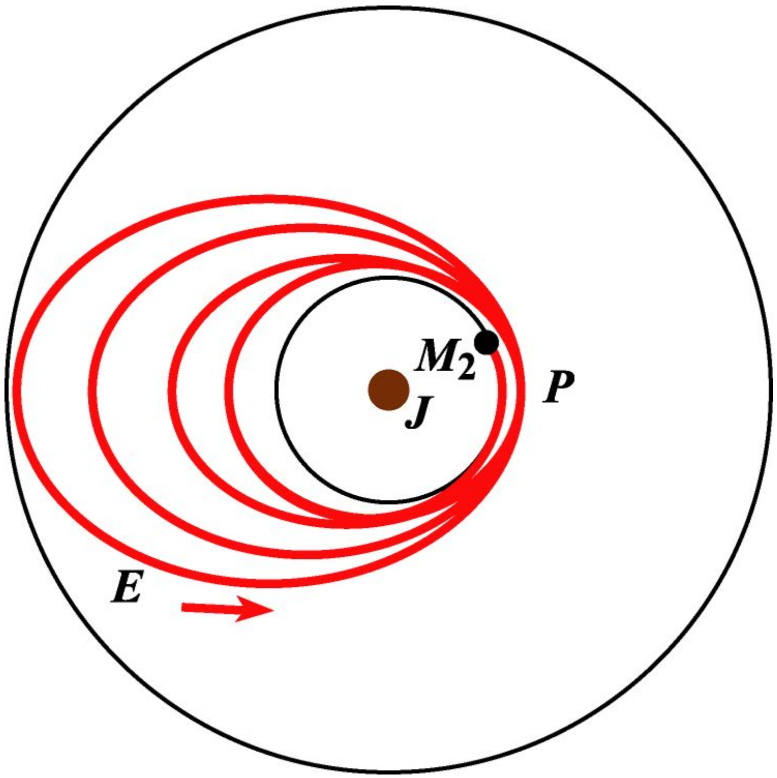
Inter-Moon Transfer

- Resonant gravity assists with outer moon M_1
- When periapse close to inner moon M_2's orbit is reached, J-M_2 system dynamics "take over"

Leaving moon M_1
Apoapse A fixed

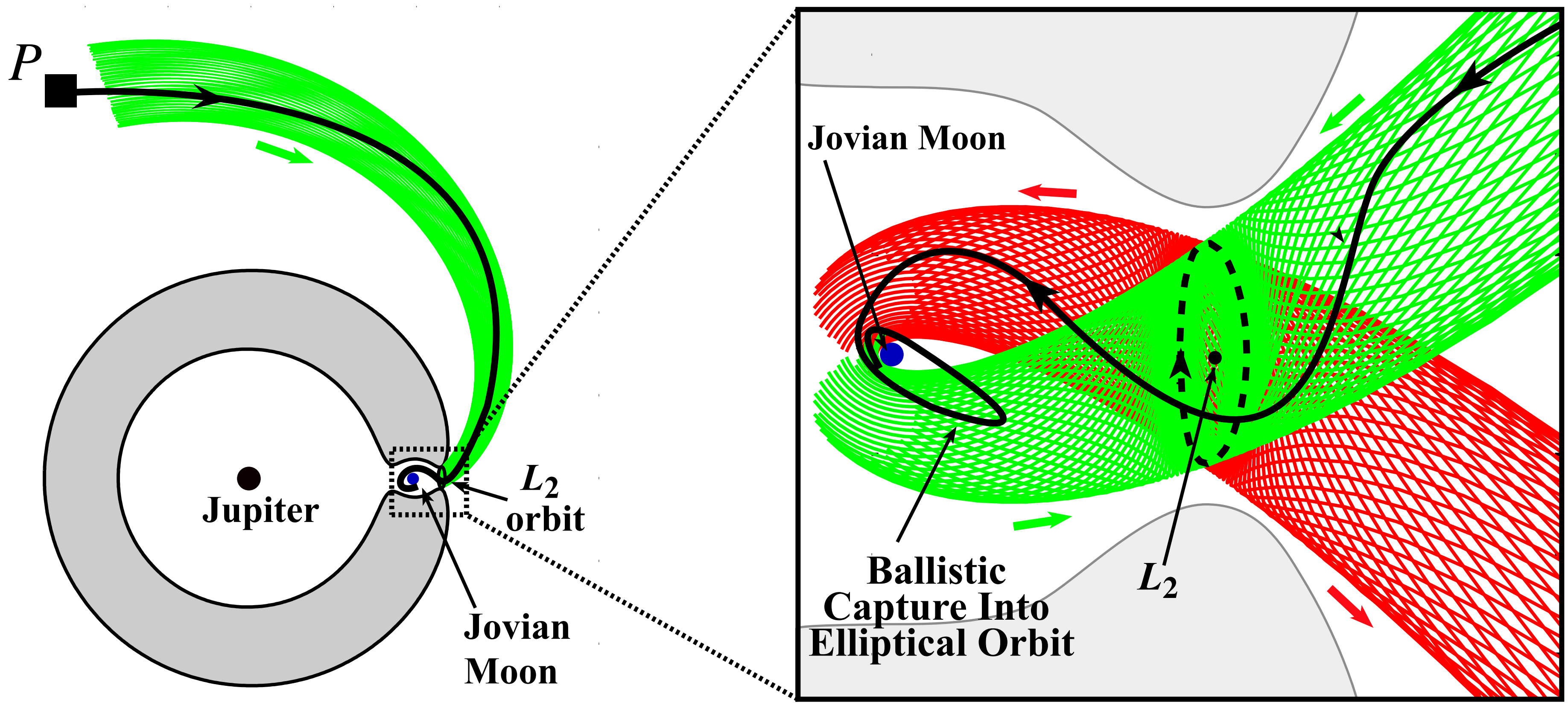


Approaching moon M_2
Periapse P fixed



Ballistic Capture

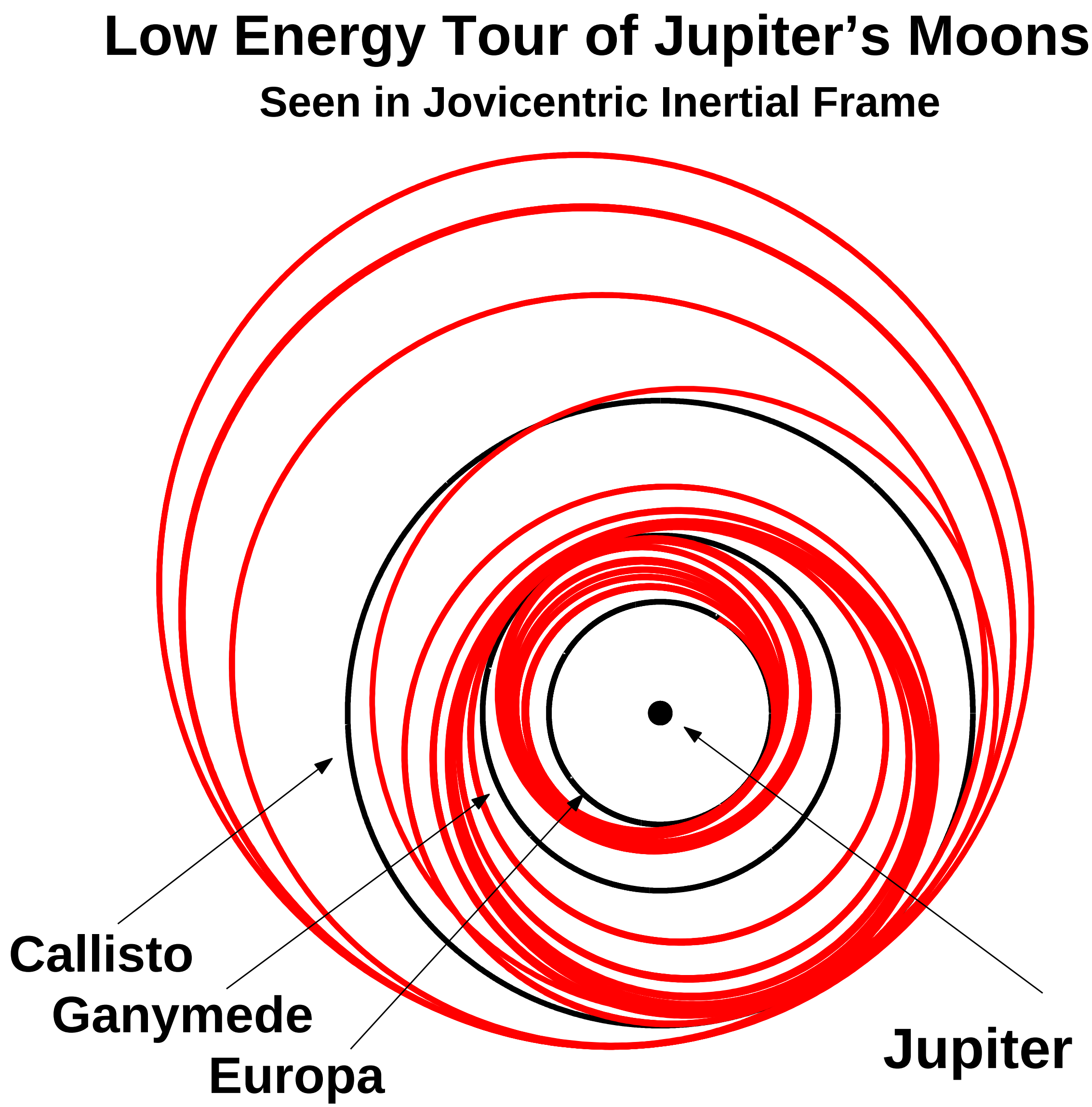
- Final phase of inter-moon transfer $\longrightarrow$ enter tube leading to ballistic capture



Tube leading to ballistic capture around a moon (seen in rotating frame)

Resulting Trajectory

- $\sum_i \Delta v_i = 22$ m/s (!!!), but flight time $\approx$ 3 years



Resulting Trajectory

- ***Results are promising***
  - … but preliminary
  - Model is a restricted bicircular 5-body problem
  - A user-assisted algorithm was necessary to produce it
  - Currently working on automated algorithm

Future Work for MMO

- ***Future challenges***
  - Consider model uncertainty, unmodeled dynamics, noise $+$ incorporation of low-thrust
  - Coordination with goals/constraints of real missions e.g., how long at each moon, radiation dose, maximum flight time
  - Decrease flight time
    Evidence suggests large decrease in time for small increase in ΔV
  - Use powerful techniques & software packages
    e.g., GAIO, MANGEN, NTG — all together?

The End

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