Overview of CDS 140B:

Introduction to Dynamics

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Textbook, Other Book and Papers

Textbook

- Stephen Wiggins [2003]:
  Introduction to Applied Nonlinear Dynamical Systems and Chaos, Second Edition.

Other Books and Paper

- Ferdinand Verhulst:
  Nonlinear Differential Equations and Dynamical Systems.
- Lawrence Perko:
  Differential Equations and Dynamical Systems.
- Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross [2000]:
  Heteroclinic Connections between Periodic Orbits and Resonance Transitions in Celestial Mechanics, *Chaos*, 10, 427-469.

Goals and Description of CDS 140B

- A continuation of CDS 140A.
  80% covers basic tools from nonlinear dynamics
  - perturbation theory and method of averaging;
  - bifurcation theory;
  - global bifurcation and chaos;
  - Hamiltonian systems.
- Besides standard examples
  (van der Pol, Duffing, and Lorenz equations),
  20% applies dynamical system tools in space mission design
  - periodic and quasi-periodic orbits,
  - invariant manifolds,
  - homoclinic and heteroclinic connections,
  - symbolic dynamics and chaos.
- Examples: Genesis Discovery Mission and
  Low Energy Tour of Multiple Moons of Jupiter

Outline of Presentation

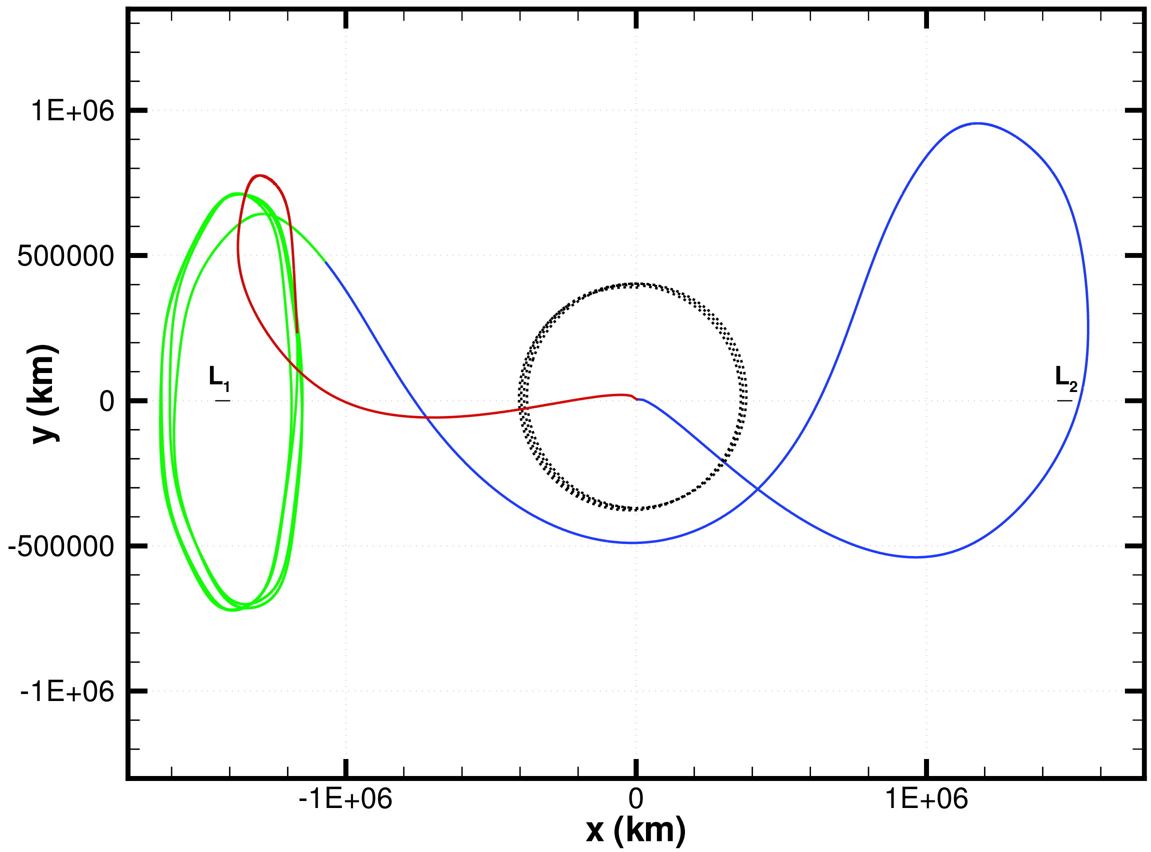
- **Main Theme**
  - how to use dynamical systems theory of 3-body problem in space mission design.
- **Background** and **Motivation**:
  - NASA's Genesis Discovery Mission.
  - Jupiter Comets.
- **Planar Circular Restricted 3-Body Problem**.
- **Major Results on Tube Dynamics**.
- **Lobe Dynamics** & **Navigating in Phase Space.**
  - A Low Energy Tour of Jupiter's Moons.
- **Conclusion** and **Ongoing Work**.
  - Two Full Body Problem and Astroid Pairs.
  - Looking into Chemical Reaction Dynamics.

Acknowledgement

- H. Poincaré, J. Moser
- C. Conley, R. McGehee, D. Appleyard
- C. Simó, J. Llibre, R. Martinez
- G. Gómez, J. Masdemont
- B. Farquhar, D. Dunham
- E. Belbruno, B. Marsden, J. Miller
- K. Howell, B. Barden, R. Wilson
- S. Wiggins, L. Wiesenfeld, C. Jaffé, T. Uzer
- V. Rom-Kedar, F. Lekien, L. Vela-Arevalo
- M. Dellnitz, O. Junge, and Paderborn group

Genesis Discovery Mission

- **Genesis spacecraft**
  - is collecting solar wind samples from a L_1 **halo orbit**,
  - will **return** them to Earth later this year for analysis.
- **Halo orbit**, **transfer**/ **return** trajectories in rotating frame.

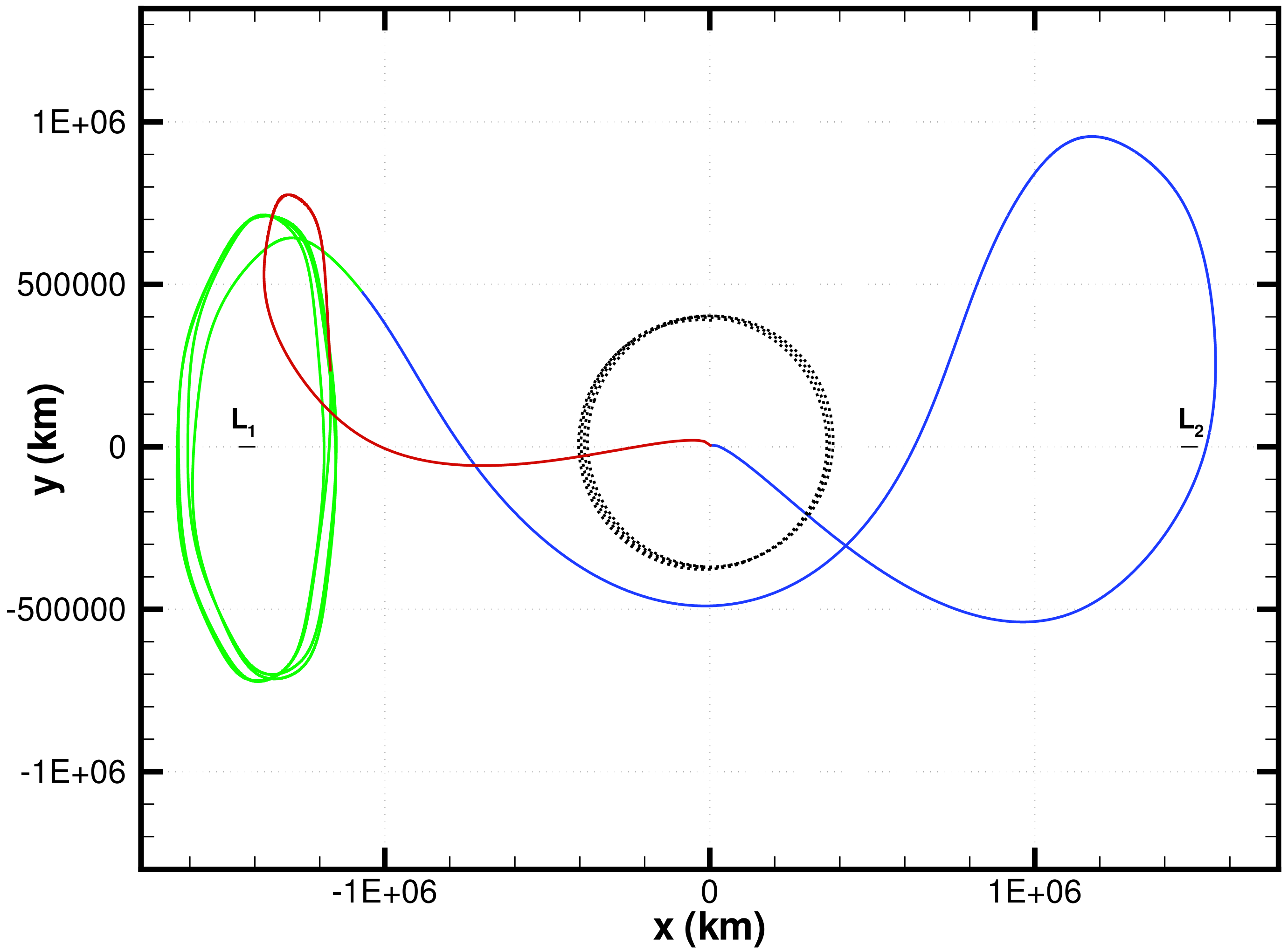


Genesis Discovery Mission

Genesis Discovery Mission

Genesis Discovery Mission

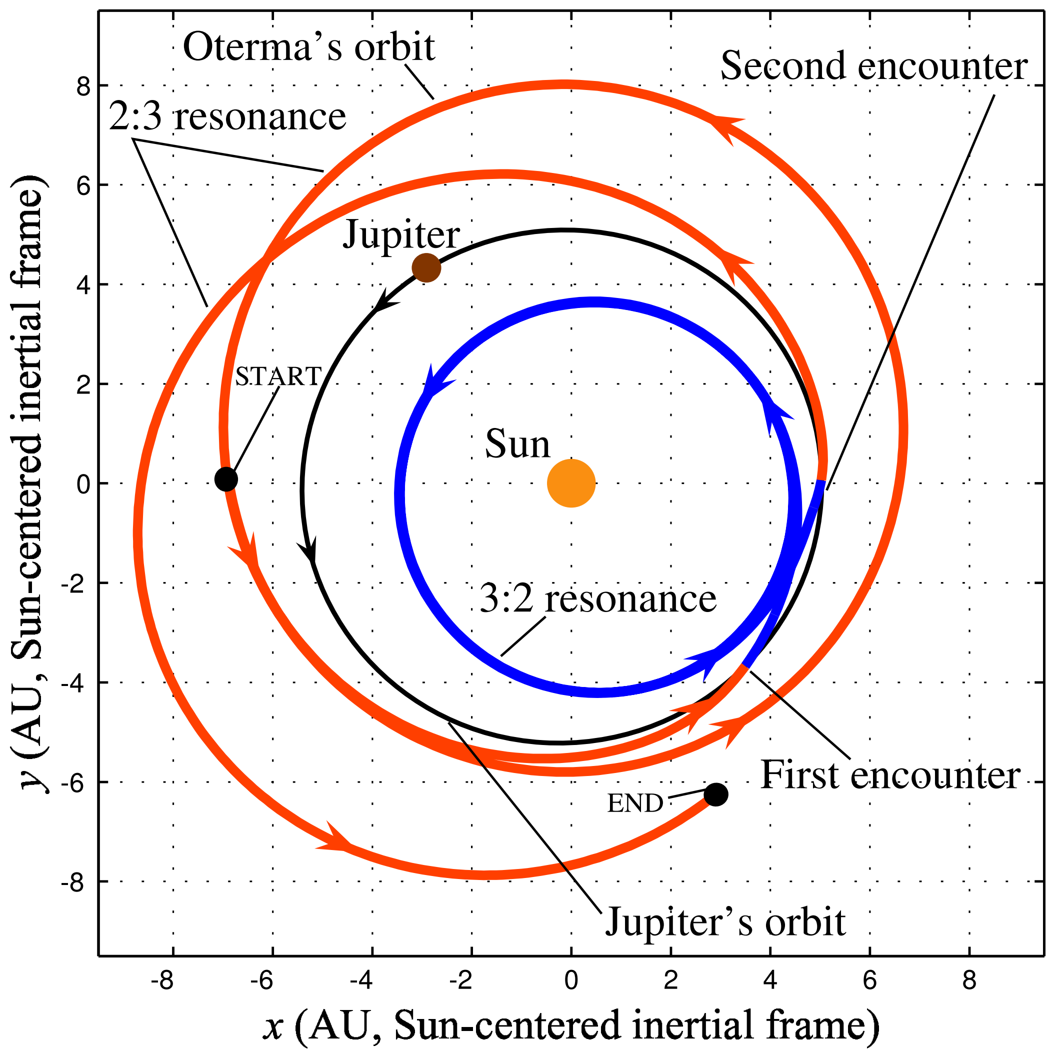
- Follows natural dynamics, little propulsion after launch.
- **Return-to-Earth portion** utilizes heteroclinic dynamics.



Jupiter Comets

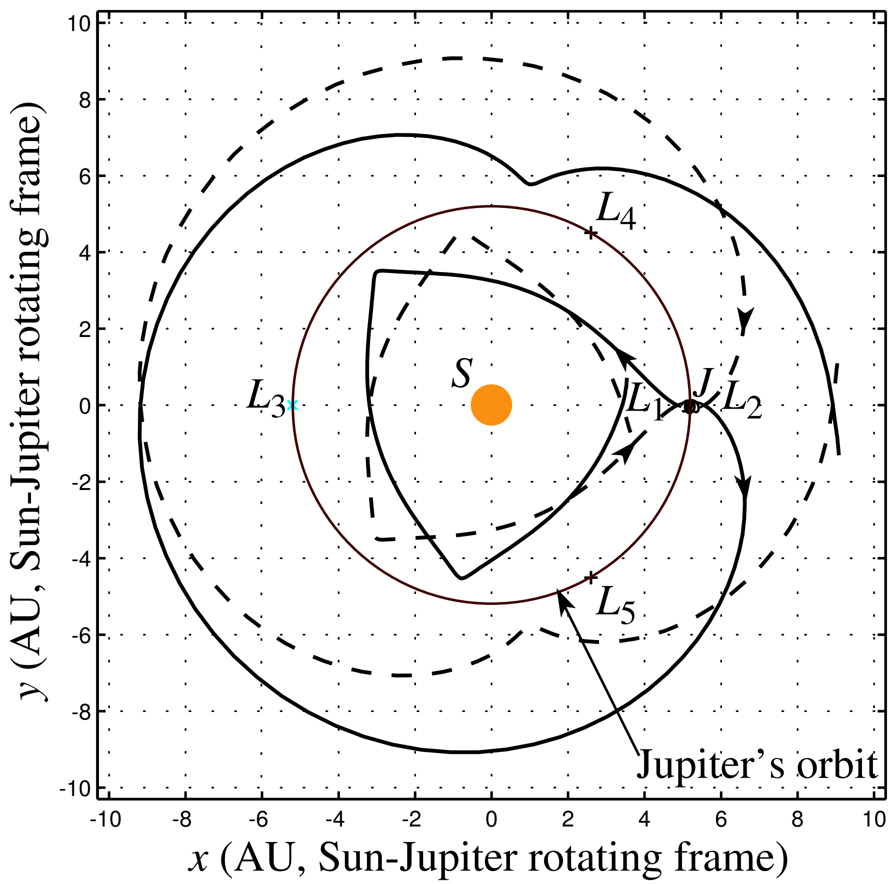
Jupiter Comets

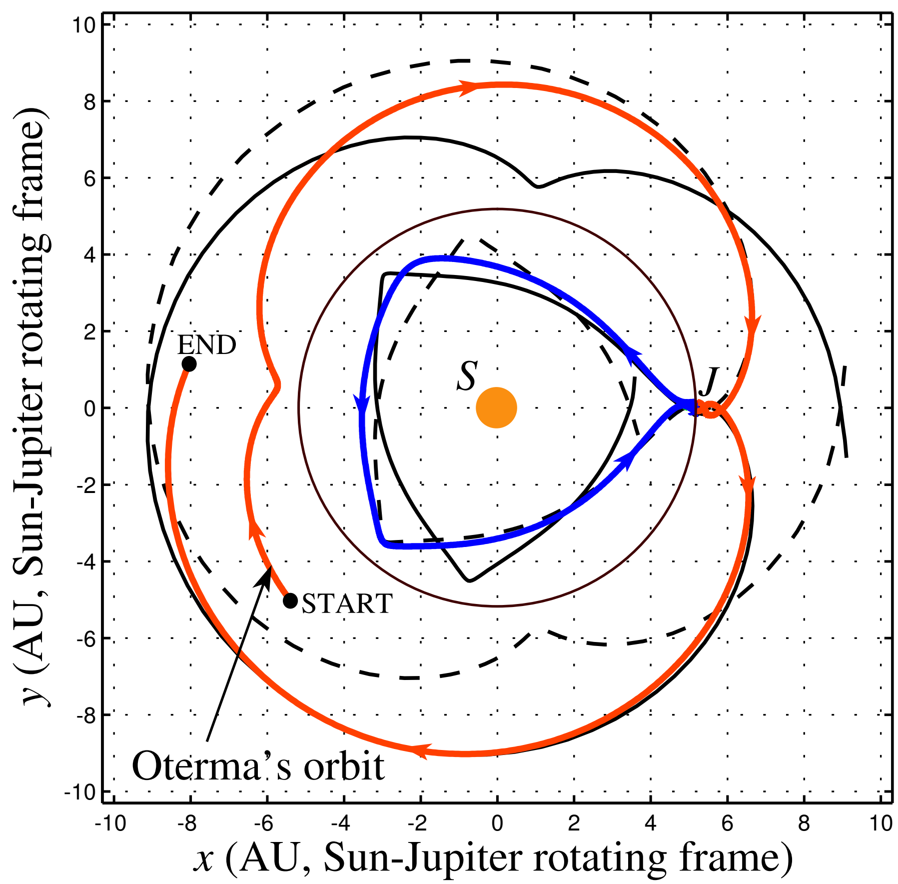
- **Rapid transition** from outside to **inside** Jupiter's orbit.
- **Captured temporarily** by Jupiter during transition.
- Exterior (**2:3 resonance**). **Interior** (**3:2 resonance**).



Jupiter Comets

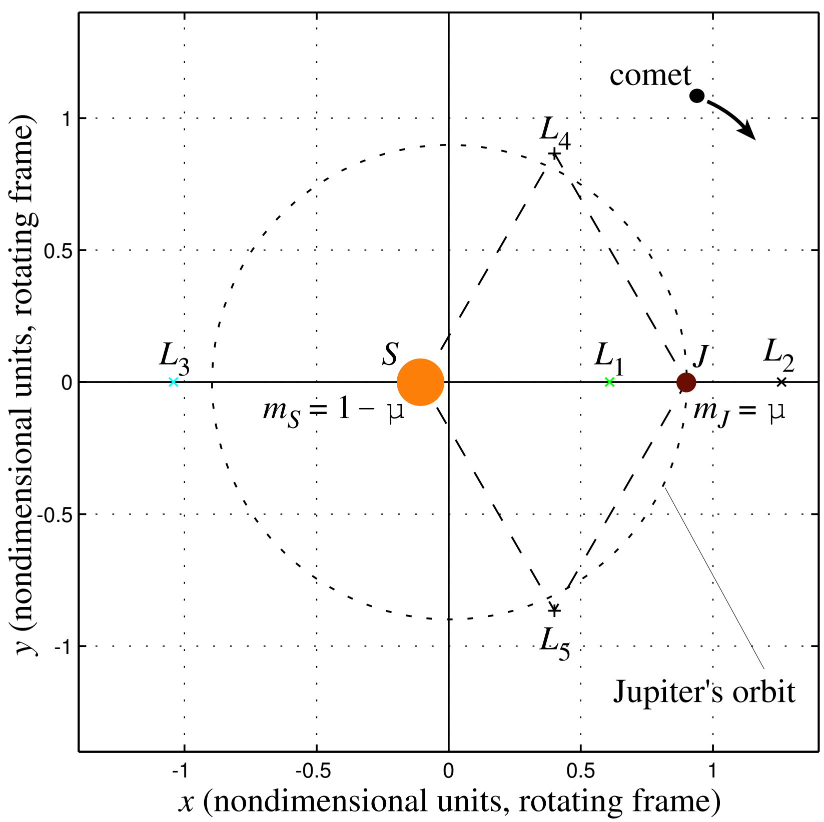
- Belbruno and B. Marsden [1997]
- Lo and Ross [1997]
  - Comet in **rotating frame** follows **invariant manifolds**.
- Jupiter comets make **resonance transition** near L_1 and L_2.





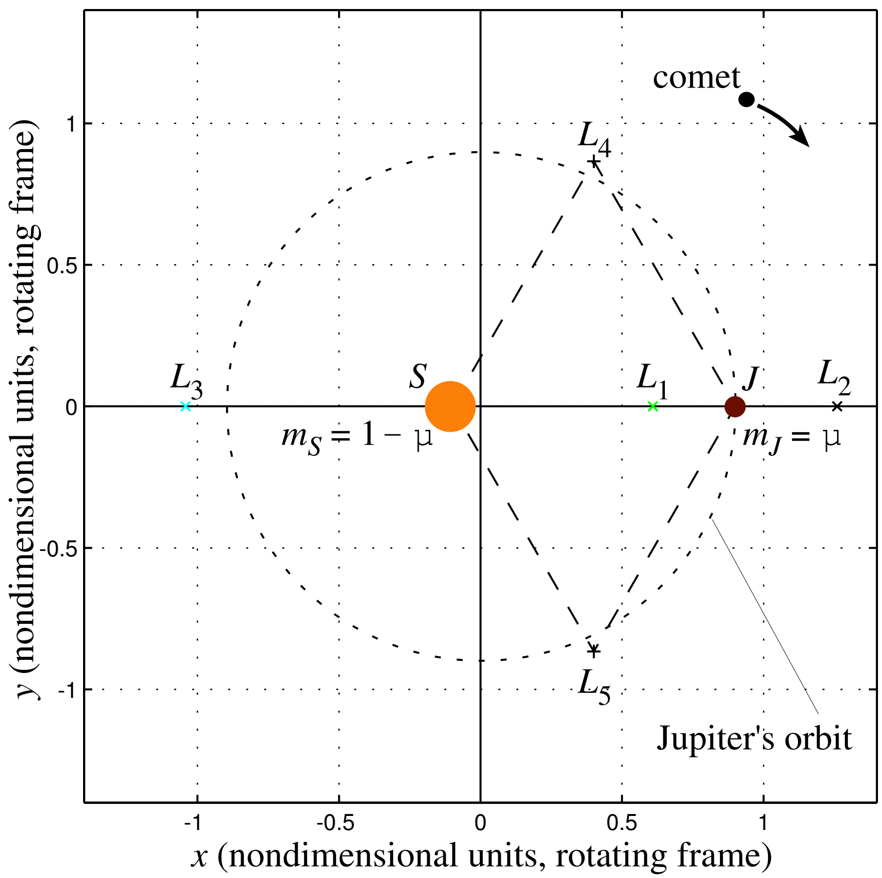
Planar Circular Restricted 3-Body Problem

- **PCR3BP** is a good starting model:
  - Comets mostly **heliocentric**, but their perturbation dominated by **Jupiter's gravitation**.
  - Their motion nearly in Jupiter's **orbital plane**.
  - Jupiter's small **eccentricity** plays little role during transition.



Planar Circular Restricted 3-Body Problem

- 2 main bodies: **Sun** and **Jupiter**.
  - Total mass normalized to 1: $m_J = \mu, \quad m_S = 1 - \mu.$
  - Rotate about center of mass, angular velocity normalized to 1.
- Choose a **rotating** coordinate system with $(0, 0)$ at center of mass, S and **J** fixed at $(-\mu, 0)$ and $(1 - \mu, 0)$.

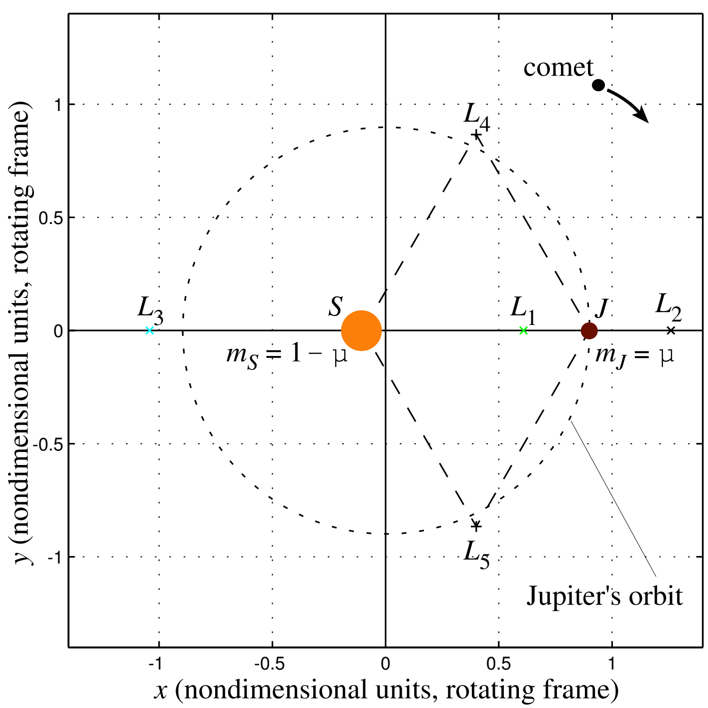


Equilibrium Points (PCR3BP)

- Comet's equations of motion are

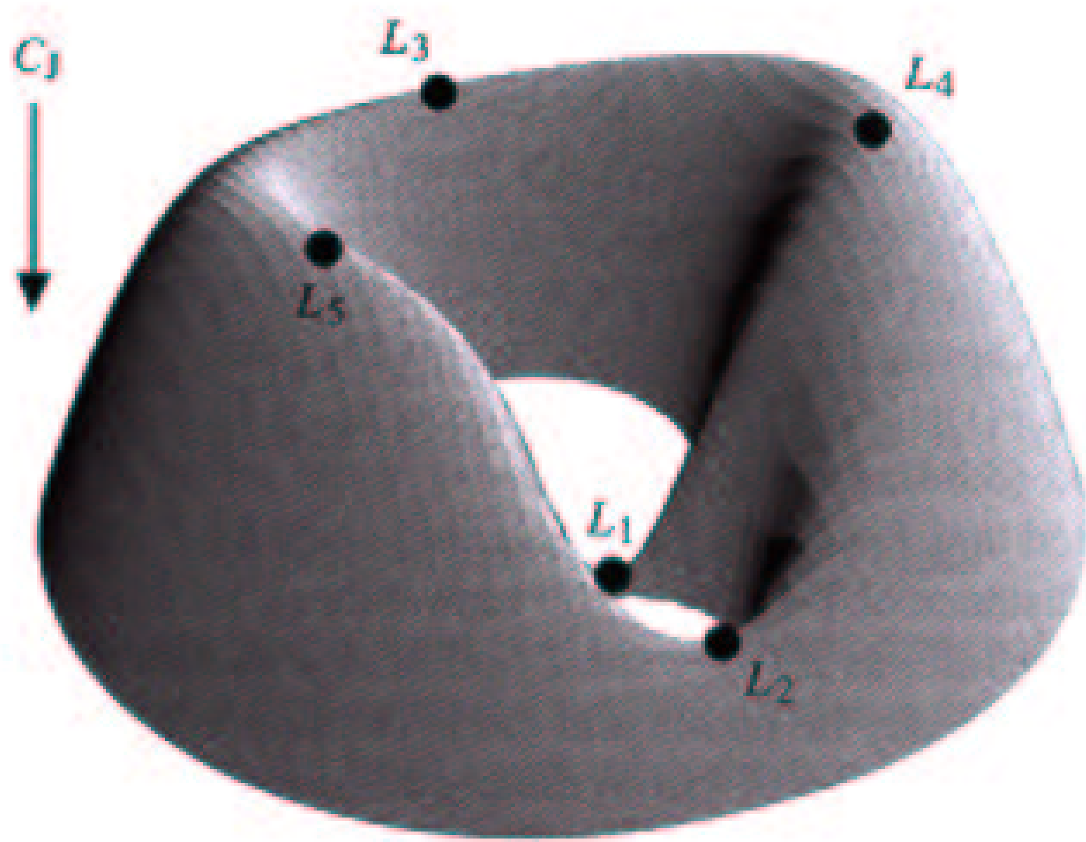
$$\ddot{x} - 2\dot{y} = -\frac{\partial U}{\partial x} \quad \ddot{y} + 2\dot{x} = -\frac{\partial U}{\partial y} \quad U = -\frac{x^2 + y^2}{2} - \frac{1-\mu}{r_s} - \frac{\mu}{r_j}$$

- Five equilibrium points:
  - 3 **unstable** equilibrium points on S-J line, L_1, L_2, L_3.
  - 2 equilateral equilibrium points, L_4, L_5.



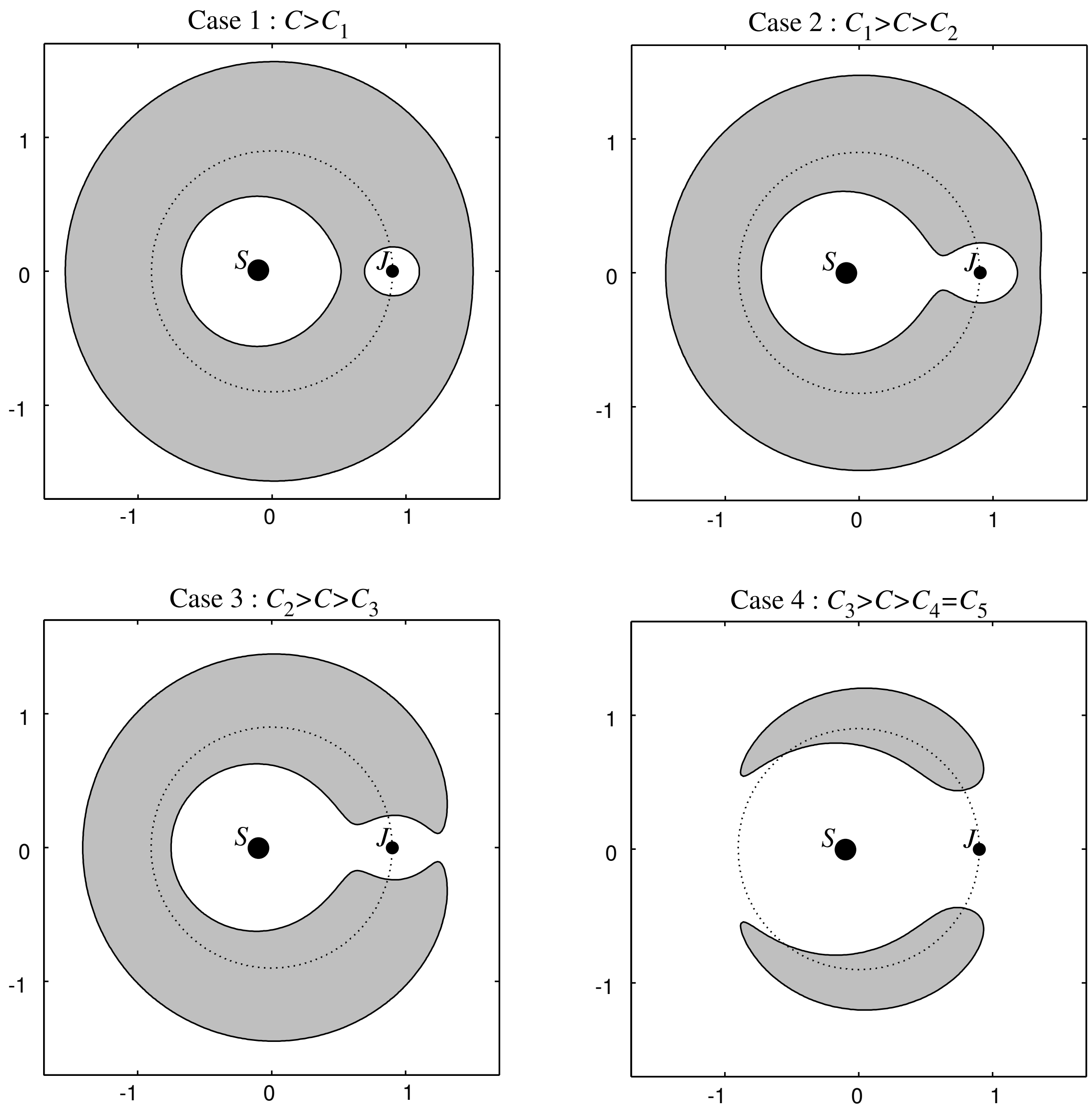
Hill's Realm (PCR3BP)

- **Energy integral**: $E(x, y, \dot{x}, \dot{y}) = (\dot{x}^2 + \dot{y}^2)/2 + U(x, y)$.
- E can be used to determine (**Hill's**) **realm** in position space where comet is energetically permitted to move.
- **Effective potential**: $U(x, y) = -\frac{x^2+y^2}{2} - \frac{1-\mu}{r_s} - \frac{\mu}{r_j}$.



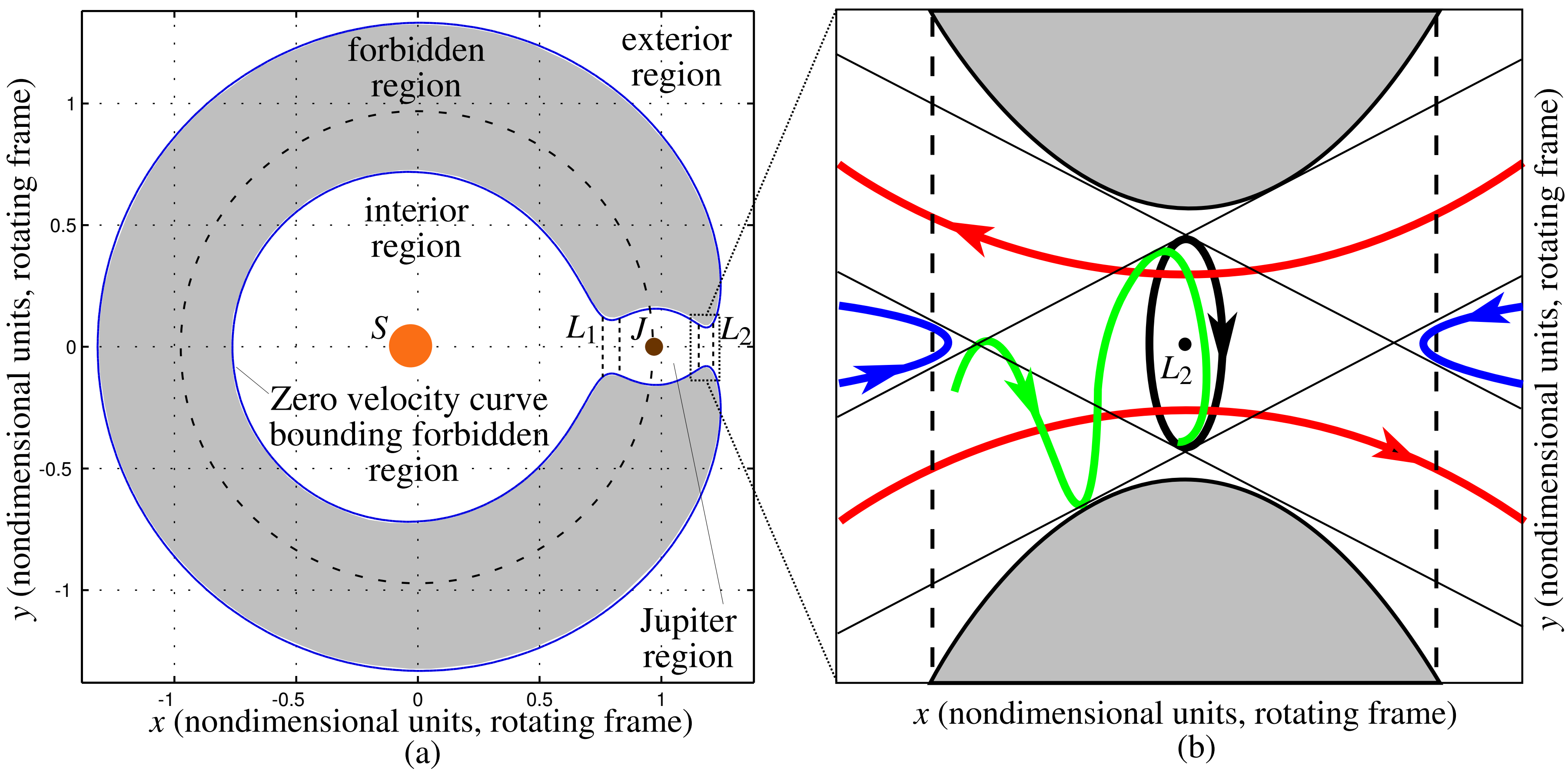
Hill's Realm (PCR3BP)

- To fix **energy value** E is to fix **height** of plot of $U(x, y)$. **Contour plots** give **5** cases of Hill's realm.



The Flow near L_1 and L_2

- For **energy value** just above that of L_2, **Hill's realm** contains a **"neck"** about L_1 & L_2.
- Comet can make **transition** through these equilibrium realms.
- 4 types of orbits: **periodic**, **asymptotic**, **transit** & **nontransit**.



The Flow near L_1 and L_2: Linearization

- [Moser] All the qualitative results of the linearized equations carry over to the full nonlinear equations.
- Recall equations of PCR3BP:

$$\begin{aligned} \dot{x} &= v_x, & \dot{v}_x &= 2v_y + U_x, \\ \dot{y} &= v_y, & \dot{v}_y &= -2v_x + U_y. \end{aligned} \tag{1}$$

- After linearization,

$$\begin{aligned} \dot{x} &= v_x, & \dot{v}_x &= 2v_y + ax, \\ \dot{y} &= v_y, & \dot{v}_y &= -2v_x - by. \end{aligned} \tag{2}$$

- Eigenvalues have the form $\pm\lambda$ and $\pm i\nu$.
- Corresponding eigenvectors are

$$\begin{aligned} u_1 &= (1, -\sigma, \lambda, -\lambda\sigma), \\ u_2 &= (1, \sigma, -\lambda, -\lambda\sigma), \\ w_1 &= (1, -i\tau, i\nu, \nu\tau), \\ w_2 &= (1, i\tau, -i\nu, \nu\tau). \end{aligned}$$

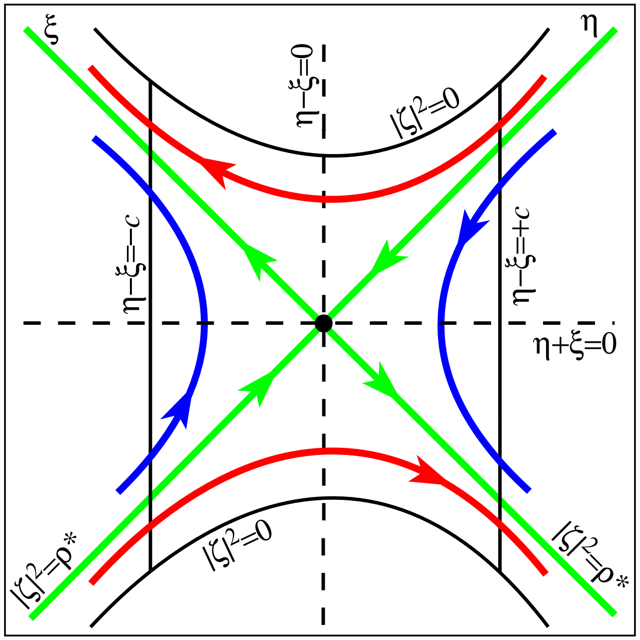
The Flow near L_1 and L_2: Linearization

- After **linearization** & making **eigenvectors** as new coordinate axes, equations assume simple form

$$\dot{\xi} = \lambda\xi, \qquad \dot{\eta} = -\lambda\eta, \qquad \dot{\zeta}_1 = \nu\zeta_2, \qquad \dot{\zeta}_2 = -\nu\zeta_1,$$

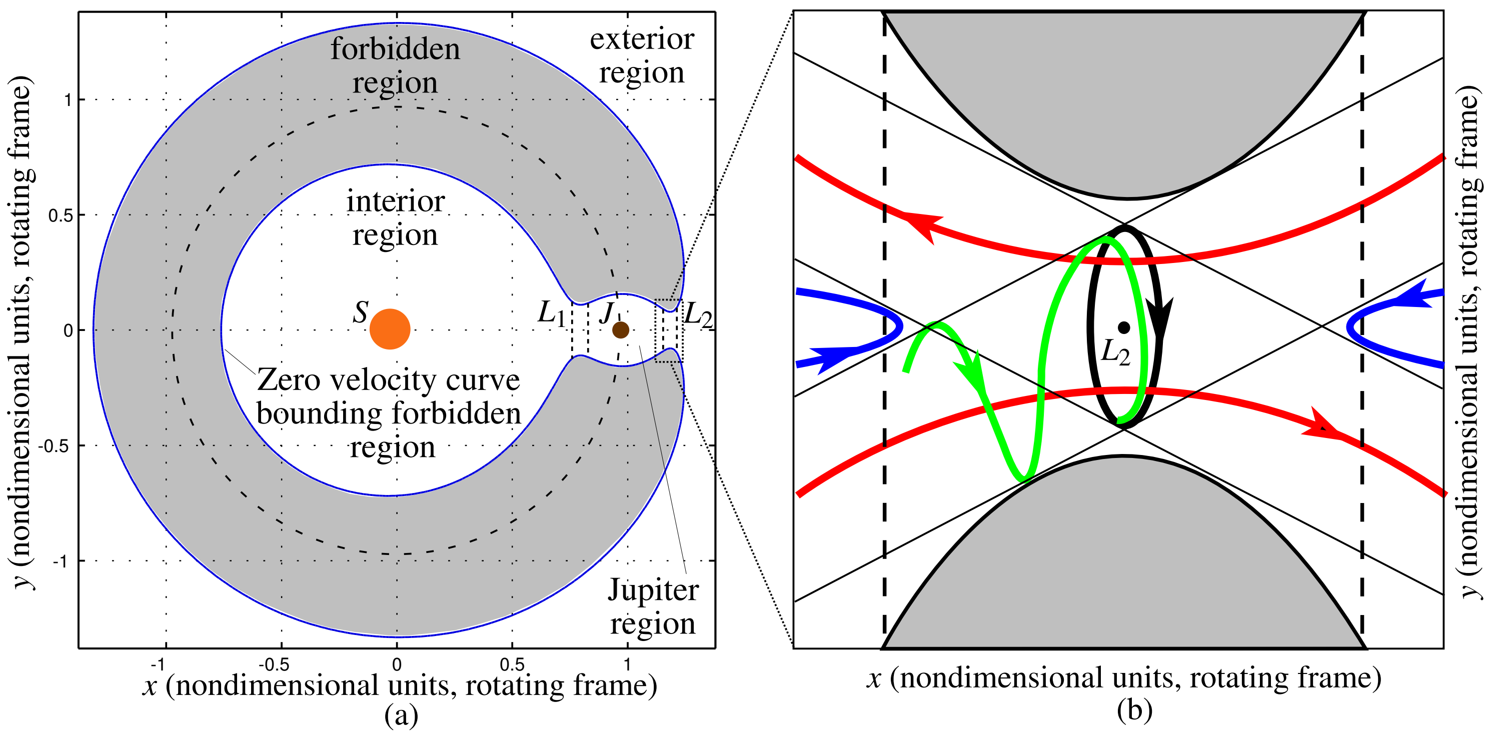
with **energy function** $E_l = \lambda\eta\xi + \frac{\nu}{2}(\zeta_1^2 + \zeta_2^2)$.

- The **flow** near L_1, L_2 have the form of **saddle**×**center**.



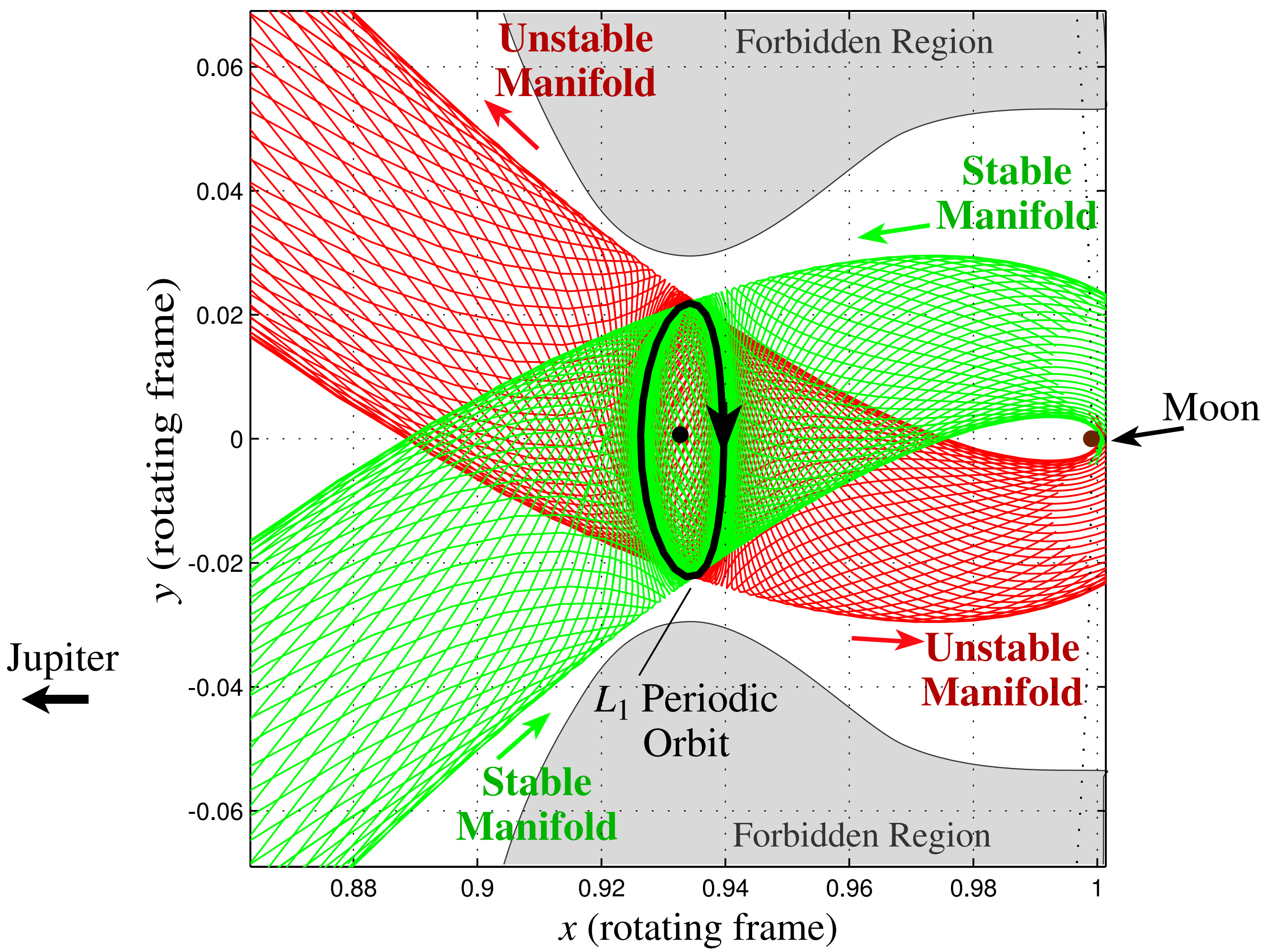
Appearance of Orbits in Position Space

- **4 types** of orbits: Shown are
  - A **periodic** orbit.
  - A typical **aymptotic** orbit.
  - 2 **transit** orbits.
  - 2 **non-transit** orbits.



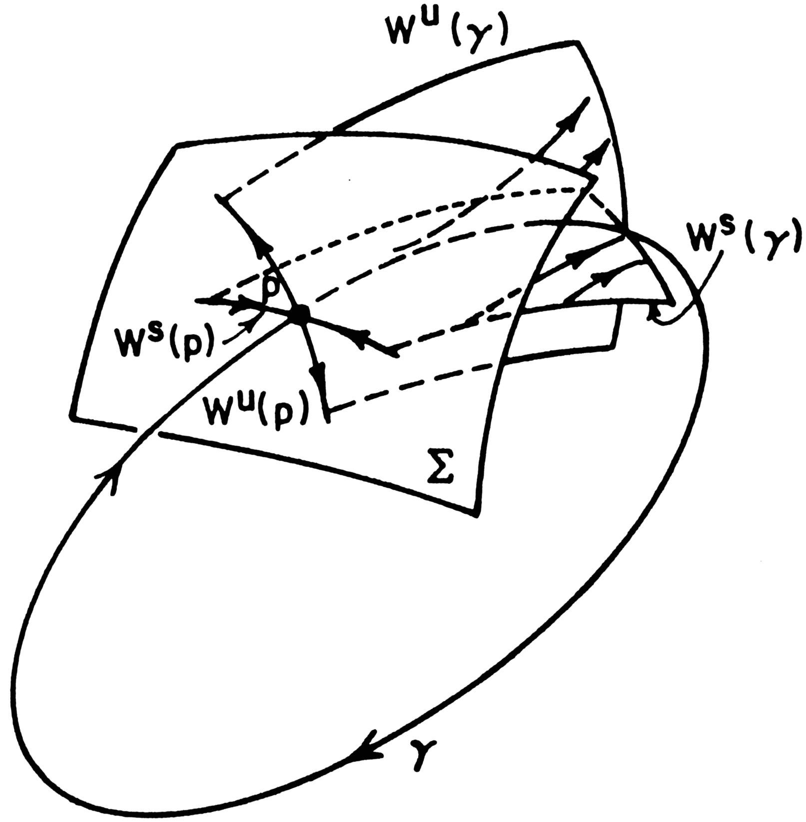
Invariant Manifolds as Separatrix

- **Stable** and **unstable** **manifold tubes** act as **separatrices** for the flow in $\mathcal{R}$:
  - Those inside the tubes are **transit** orbits.
  - Those outside of the tubes are **non-transit** orbits.



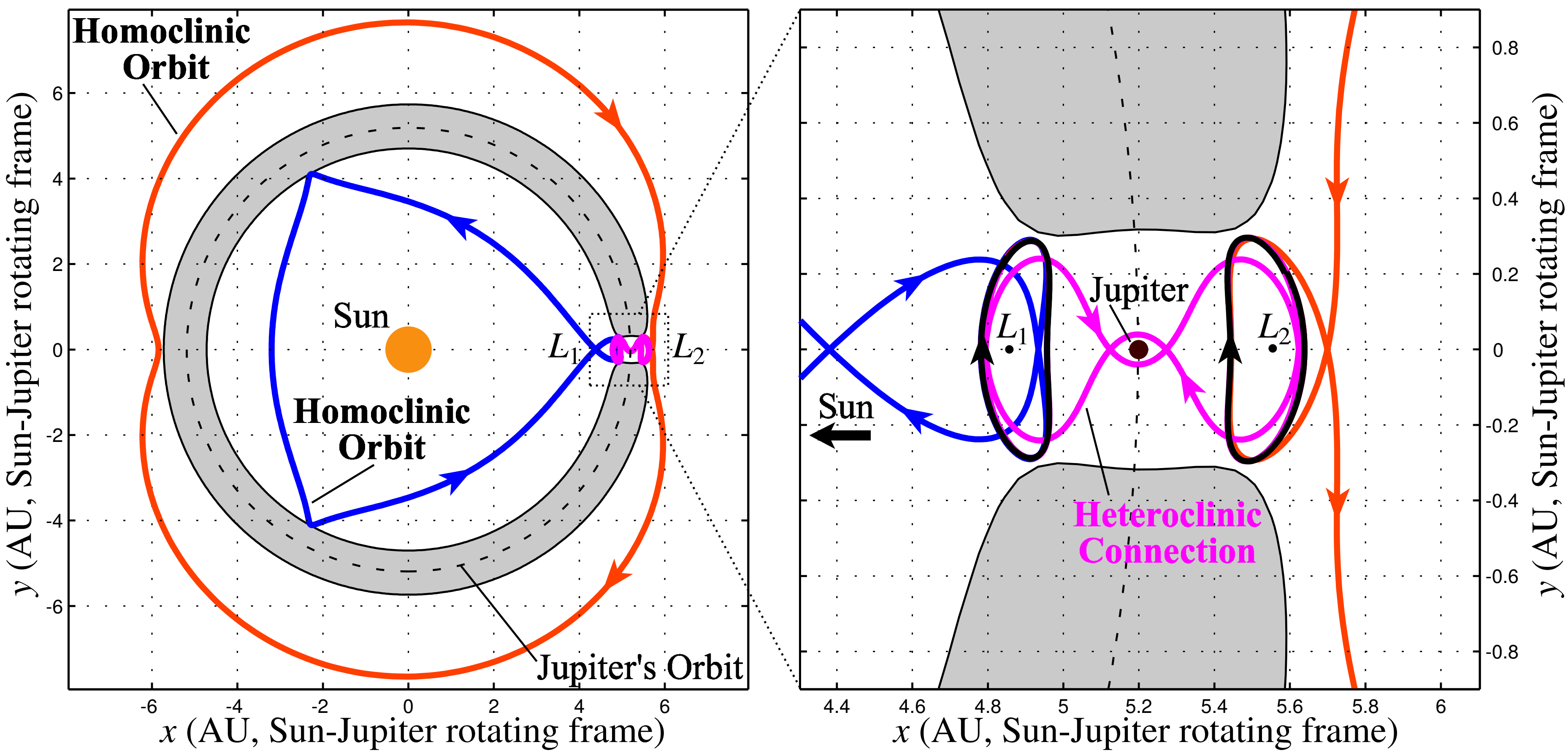
Computation of Periodic Orbit & Invariant Manifolds

- Periodic orbit can be computed by Lindstedt-Poincaré method.
- Invariant manifold can be computed by finding eigenvectors of monodromy matrix.



Major Result (A): Heteroclinic Connection

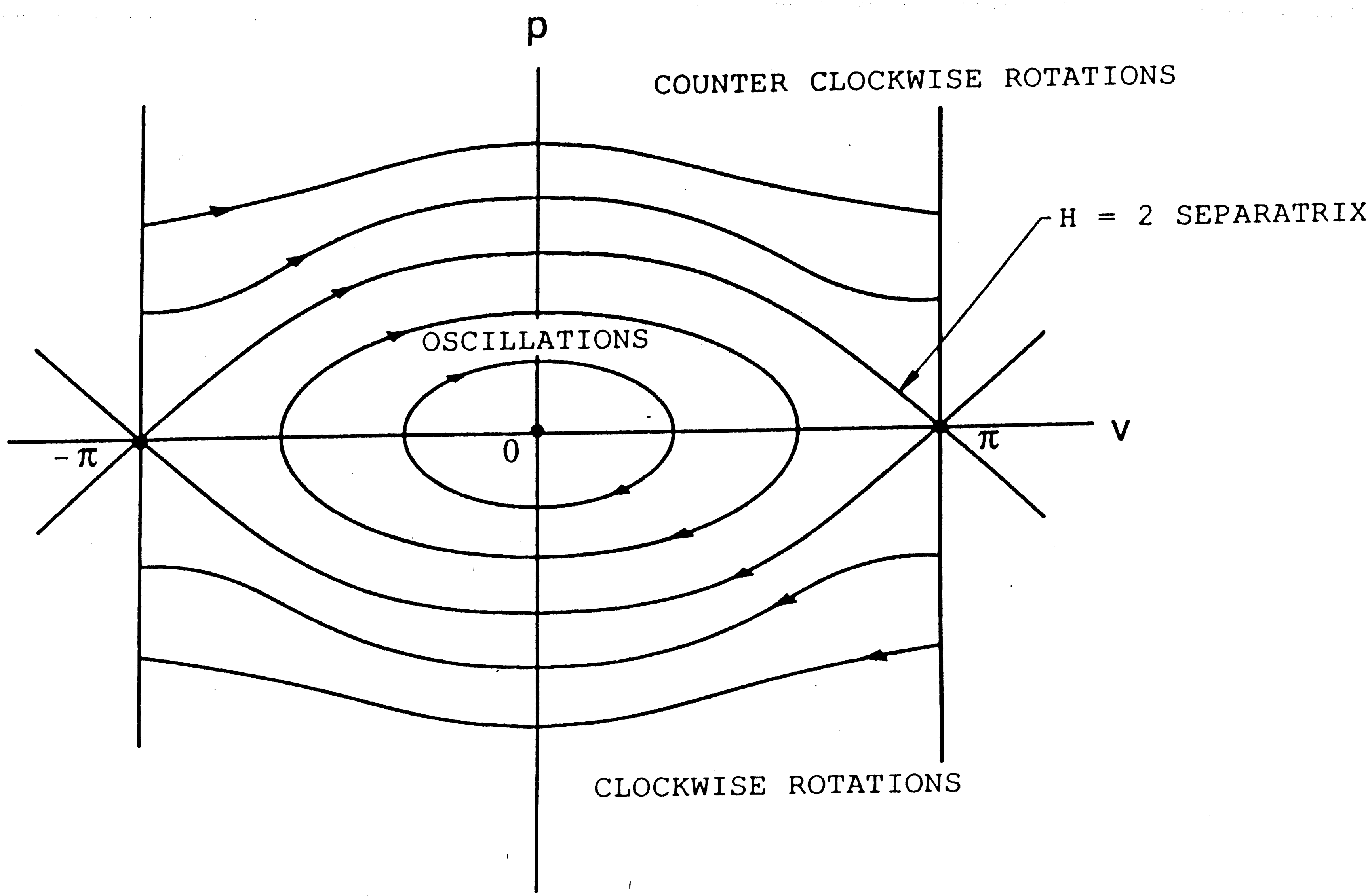
- Found **heteroclinic connection** between pair of periodic orbits.
- Found a large class of **orbits** near this (homo/heteroclinic) ***chain***.
- Comet can follow these ***channels*** in rapid transition.



Major Result (A): Heteroclinic Connection

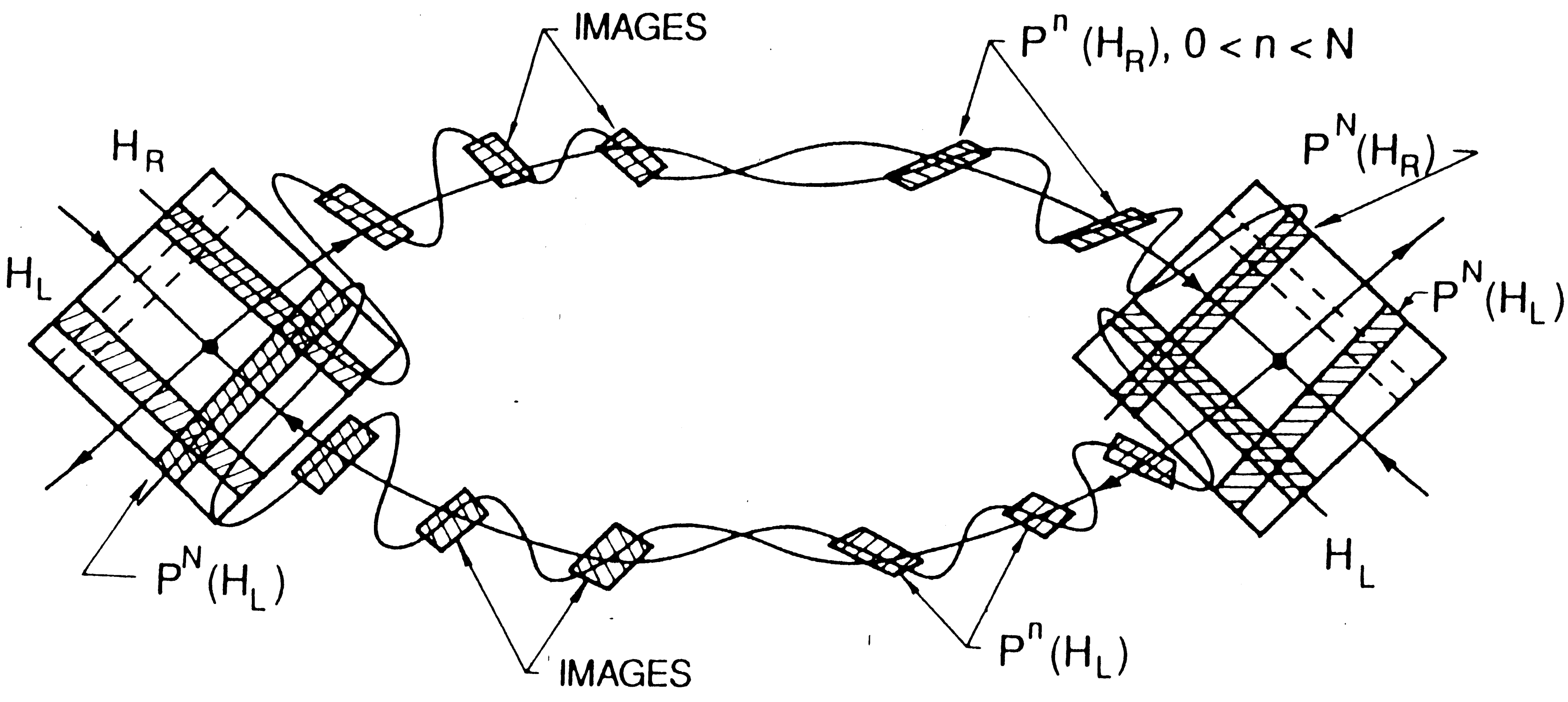
Review of Horseshoe Dynamics: Pendulum

- Homoclinic orbits separate librational motion from rotational motion.



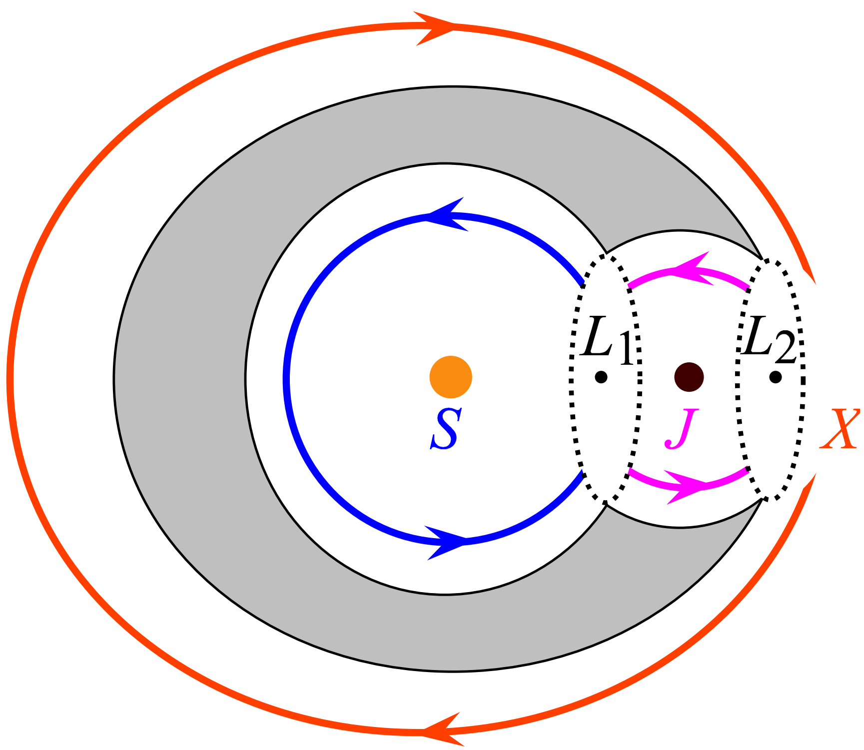
Review of Horseshoe Dynamics: Forced Pendulum

- Periodic forcing leads to transversal intersection and chaotic dynamics.
- For any bi-infinite sequence of itenarary $(\ldots, R, L; R, R, \ldots)$, there is an orbit whose whereabout matches the sequence.



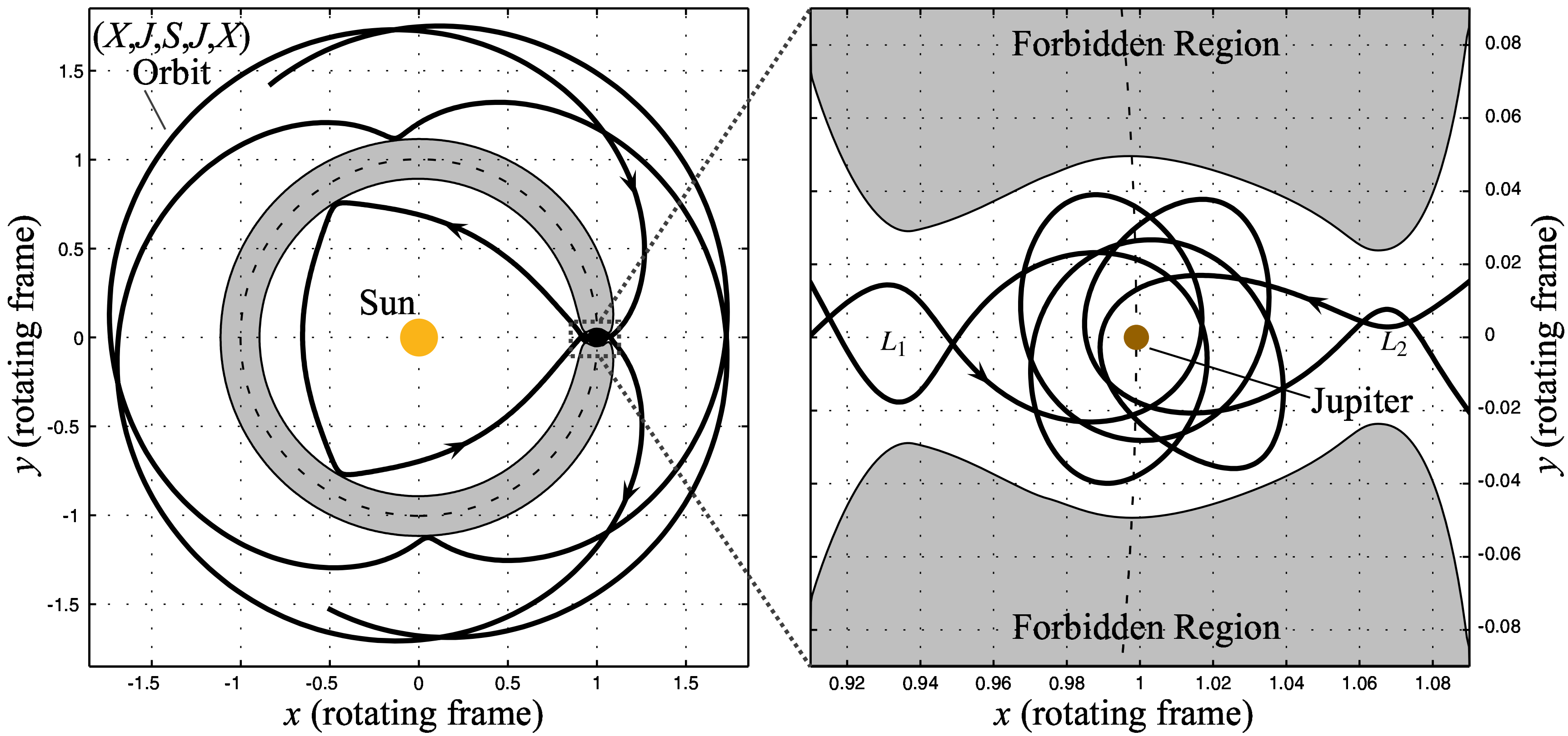
Major Result (B): Existence of Transitional Orbits

- **Symbolic sequence** used to label itinerary of each comet orbit.
- ***Main Theorem***: For any admissible **itinerary**, e.g., $(\ldots, \mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X}, \ldots)$, there exists an orbit whose **whereabouts** matches this **itinerary**.
- Can even specify **number of revolutions** the comet makes around Sun & Jupiter (plus L_1 & L_2).



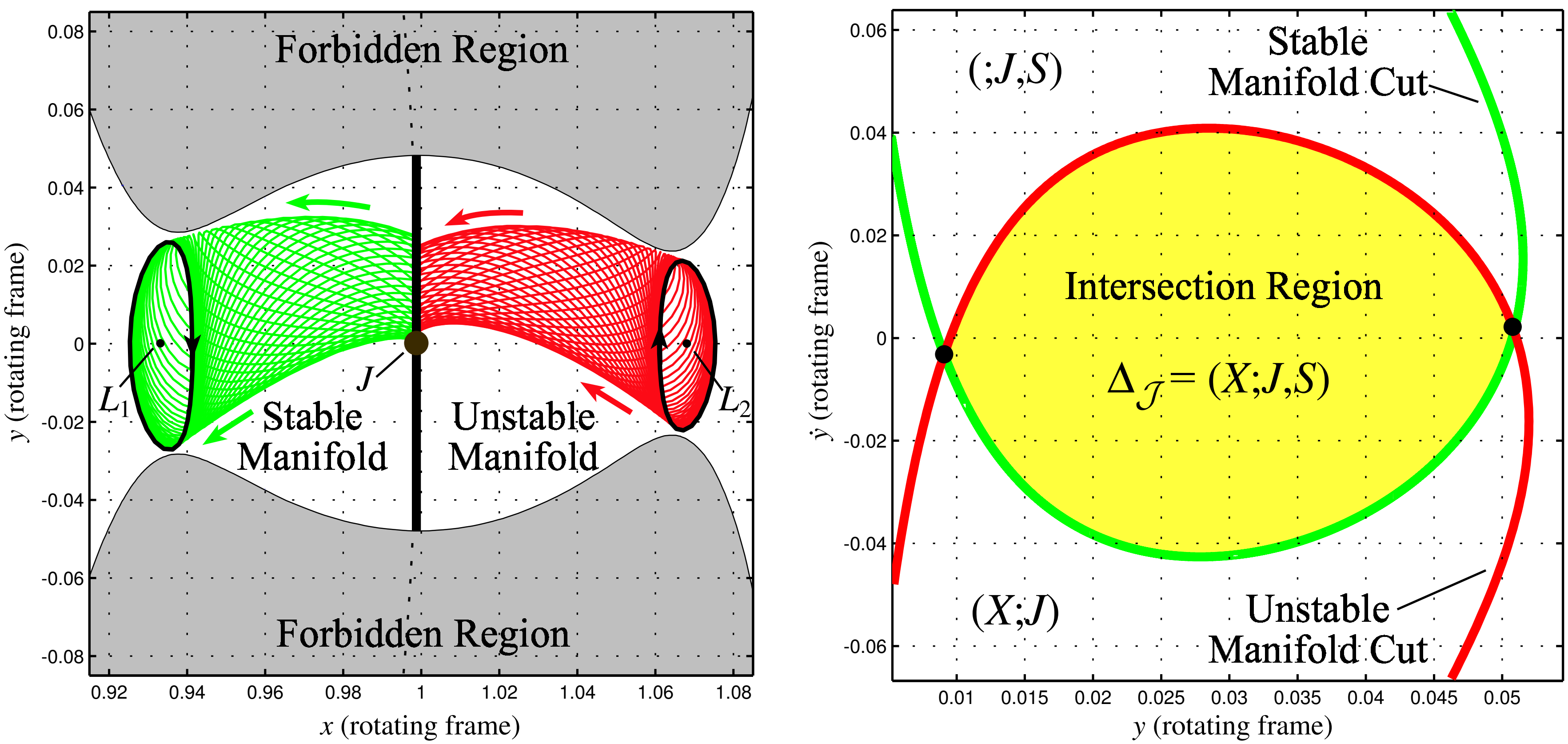
Major Result (C): Numerical Construction of Orbits

- Developed procedure to construct orbit with **prescribed itinerary**.
- Example: An orbit with itinerary $(\mathbf{X}, \mathbf{J}; \mathbf{S}, \mathbf{J}, \mathbf{X})$.



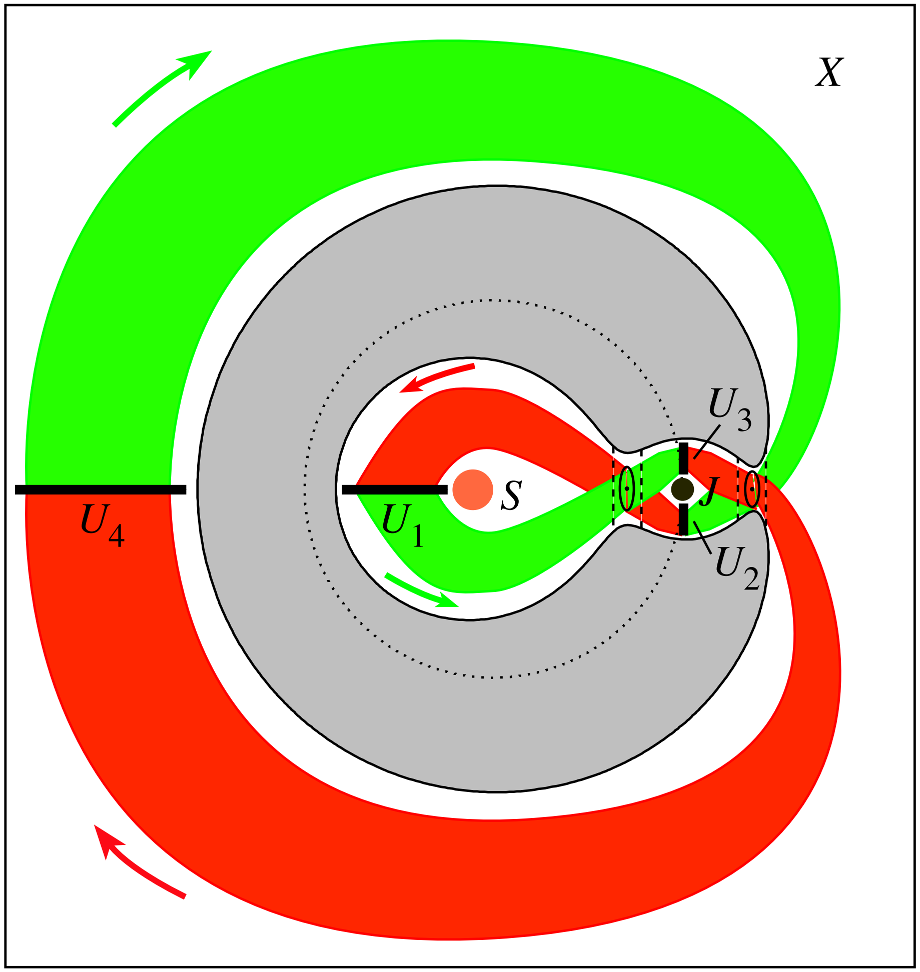
Details: Construction of $(\mathbf{J}, \mathbf{X}; \mathbf{J}, \mathbf{S}, \mathbf{J})$ Orbits

- Invariant manifold **tubes** separate transit from nontransit orbits.
- **Green curve** (Poincaré cut of L_1 **stable manifold**).
  Red curve (cut of L_2 **unstable manifold**).
- Any point inside the intersection region Δ_J is a $(\mathbf{X}; \mathbf{J}, \mathbf{S})$ orbit.



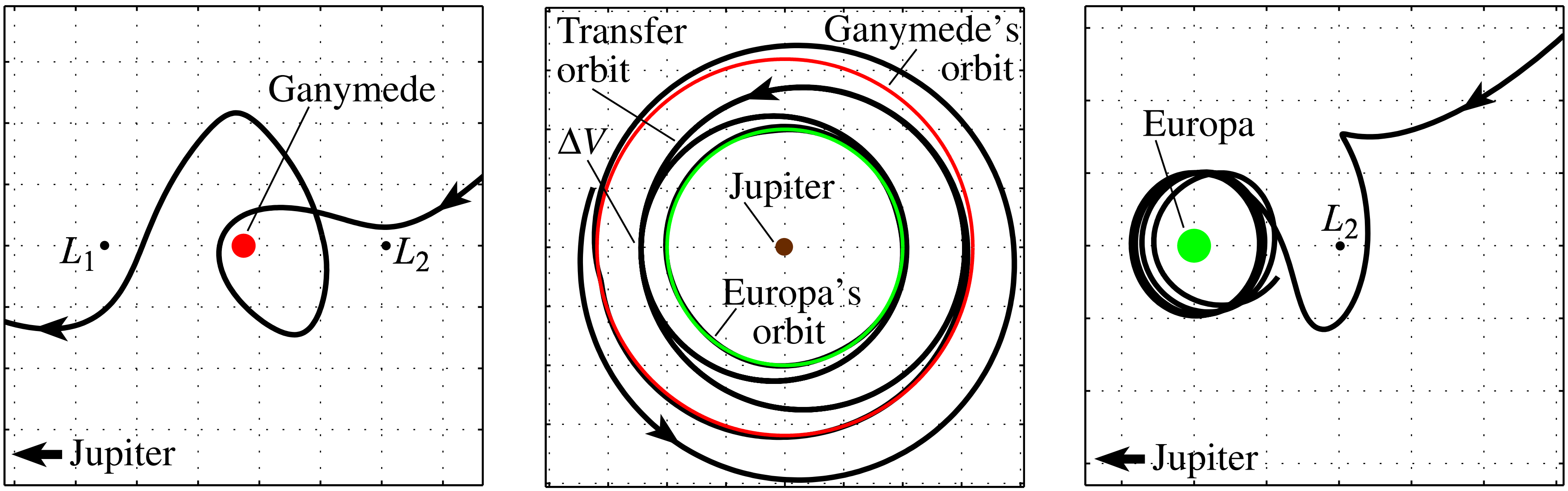
Details: Construction of $(\mathbf{J}, \mathbf{X}; \mathbf{J}, \mathbf{S}, \mathbf{J})$ Orbits

- The desired orbit can be constructed by
  - Choosing appropriate **Poincaré sections** and
  - linking invariant **manifold tubes** in right order.



Petit Grand Tour of Jupiter's Moons

- Used **invariant manifolds** to construct trajectories with interesting characteristics:
  - Petit Grand Tour of Jupiter's moons. 1 orbit around **Ganymede**. 4 orbits around **Europa**.
  - A ΔV nudges the SC from **Jupiter-Ganymede** system to **Jupiter-Europa** system.
- Instead of **flybys**, can orbit several moons for **any duration**.

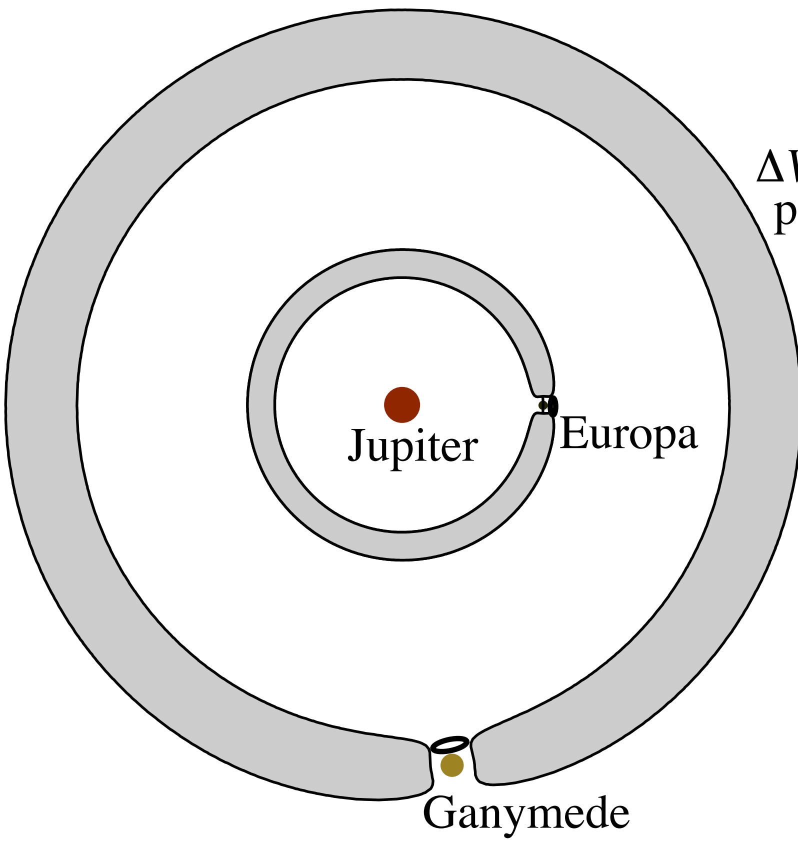


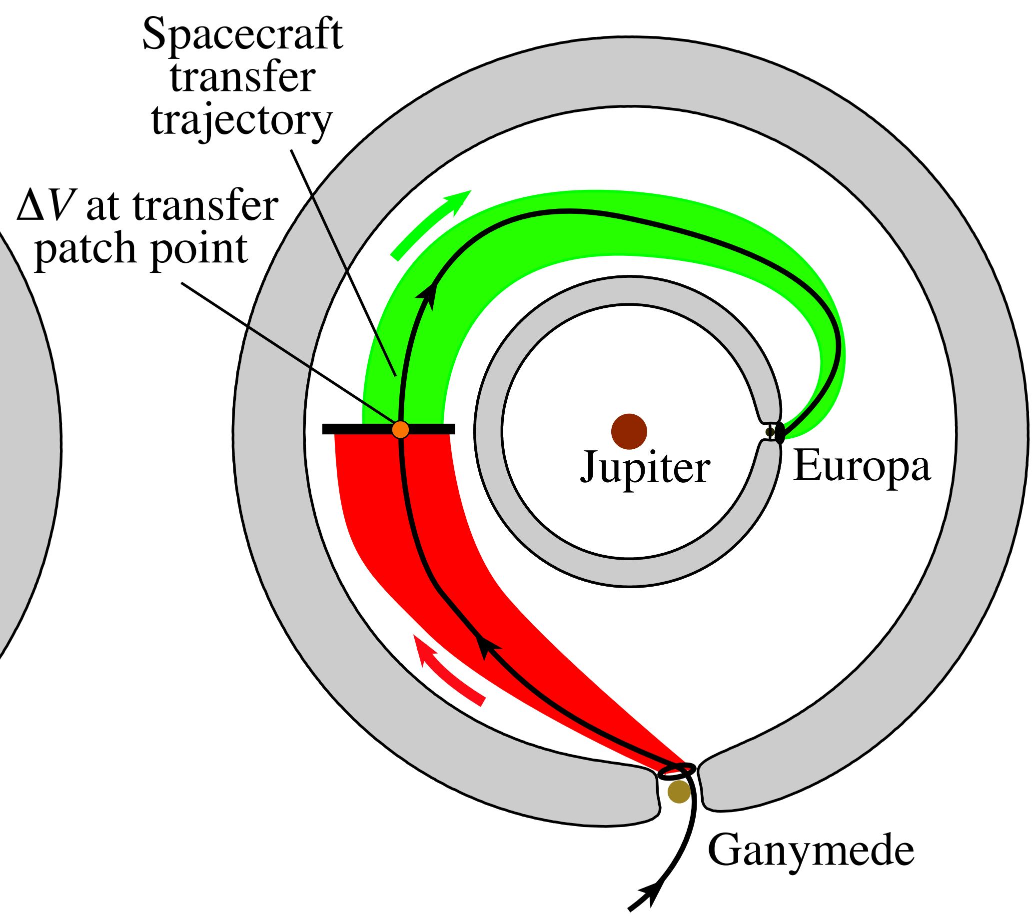
Petit Grand Tour of Jupiter's Moons

Petit Grand Tour of Jupiter's Moons

Petit Grand Tour of Jupiter's Moons

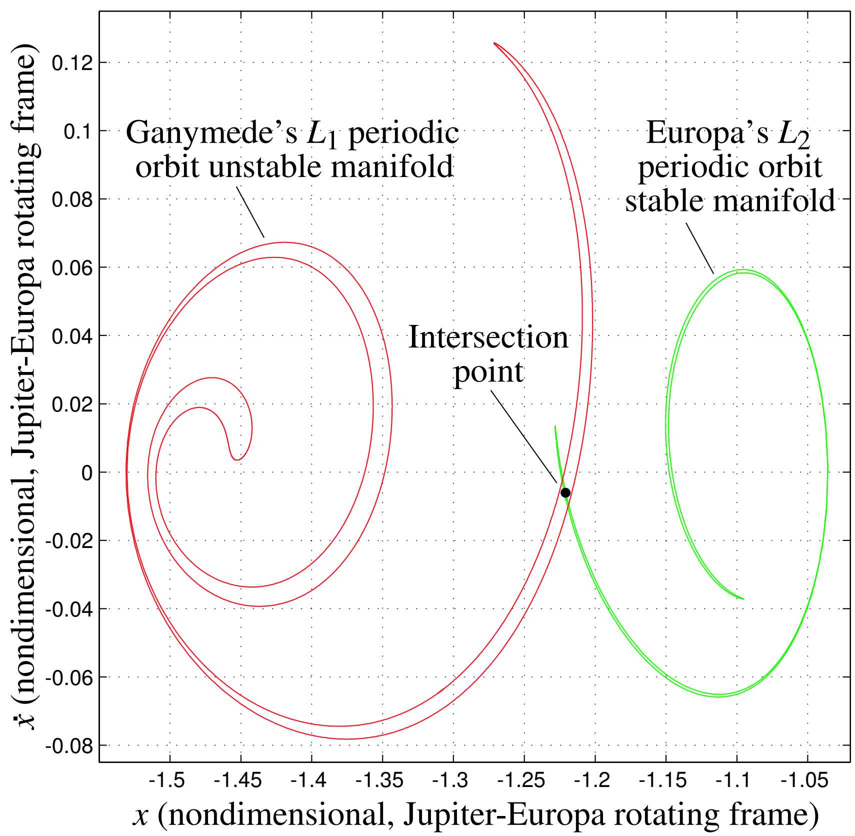
- Jupiter-Ganymede-Europa-SC 4-body system approximated as 2 **coupled 3-body systems**
- **Invariant manifold tubes** of **spatial** 3-body systems are linked in right order to construct orbit with desired itinerary.
- Initial solution refined in **4-body model**.



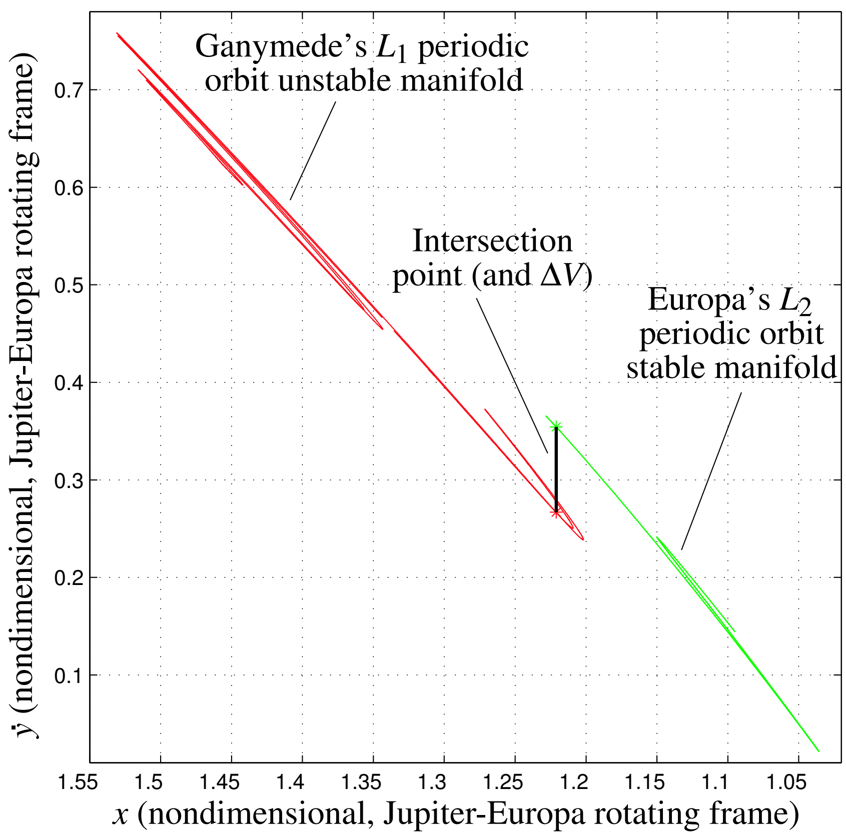


Intersection of Tubes: Poincaré Section

- Poincaré section: Vary configuration of the moons until,
  - $(x, \dot{x})$-plane: Intersection found!
  - $(x, \dot{y})$-plane: Velocity discontinuity since energies are different.
  - A rocket burn ΔV of this magnitude will make transfer trajectory **"jump" from one tube to the other**.



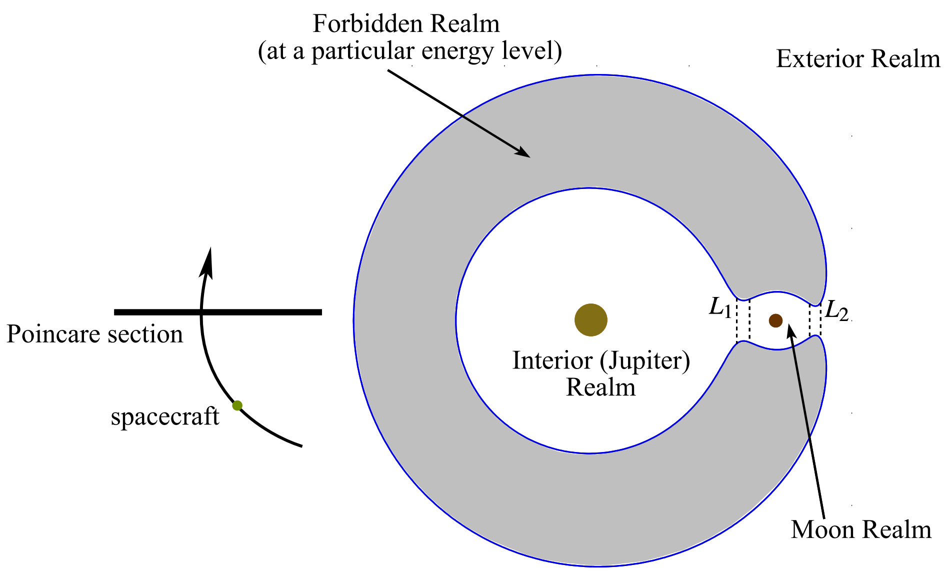
(a)



(b)

Look for Natural Pathways to Bridge the Gap

- **Tubes** of two 3-body systems **may not intersect** for awhile. May need large ΔV to "jump" from one tube to another.
- Look for **natural pathways** to bridge the gap
  - between z_0 where tube of one system **enters** and z_2 where tube of another system **exits** (into Europa realm) by "hopping" through **phase space**.



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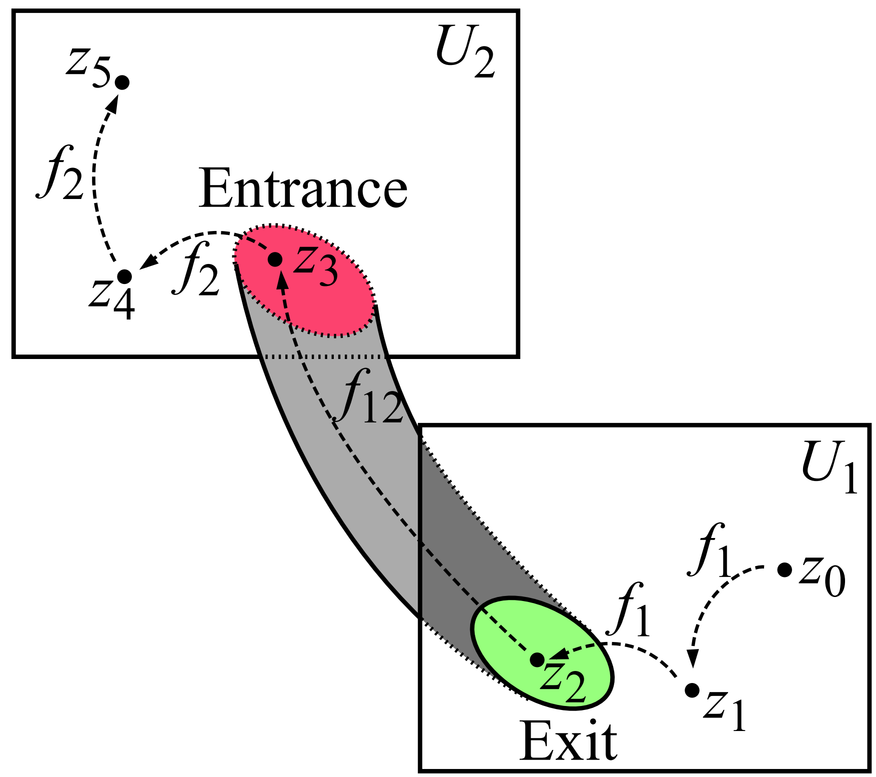
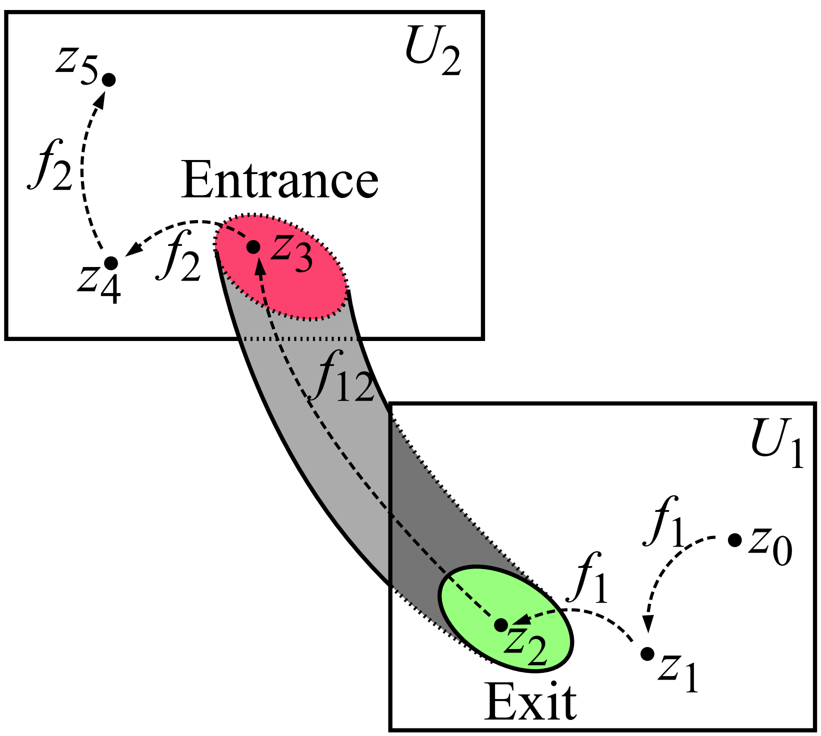


(b)

Transport in Phase Space via Tube & Lobe Dynamics

- By using
  - **tubes** of rapid transition that connect realms
  - **lobe dynamics** to hop through phase space,

  New tour only needs $\Delta V = 20$m/s (50 times less).



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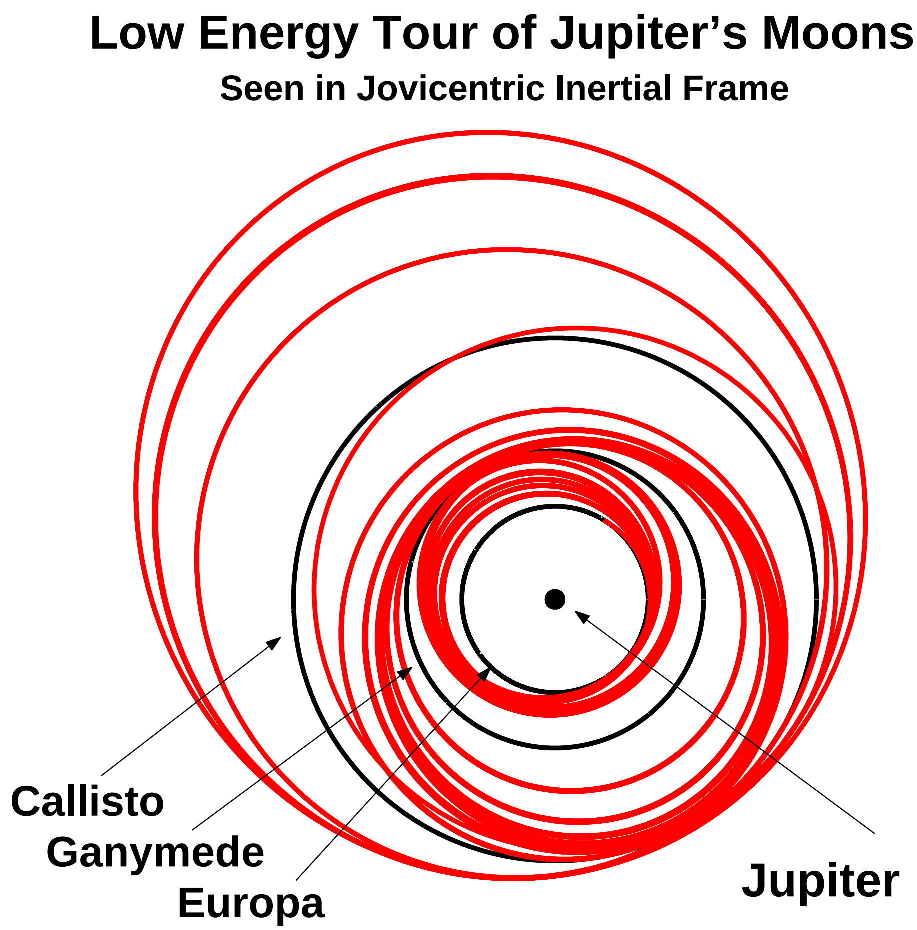
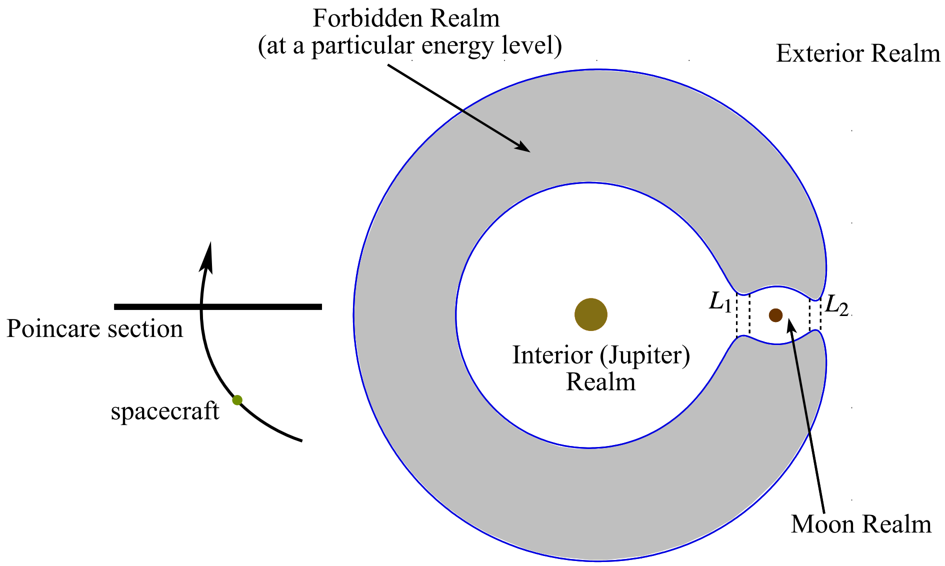


(b)

Tube Dynamics: Mixed Phase Space

- Poincaré section reveals **mixed phase space**:
  - **resonance regions** and
  - "**chaotic sea**".



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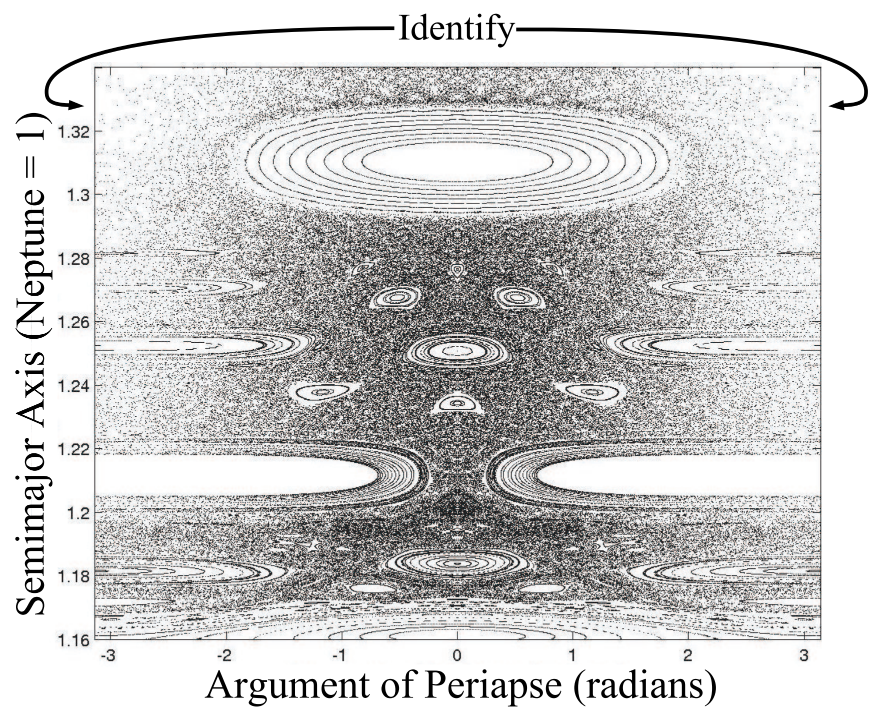
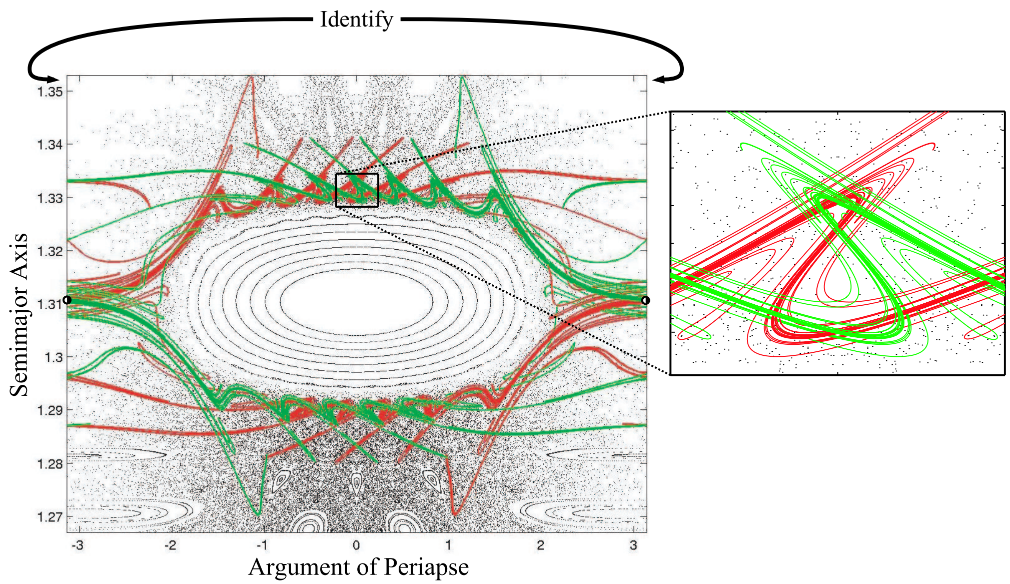
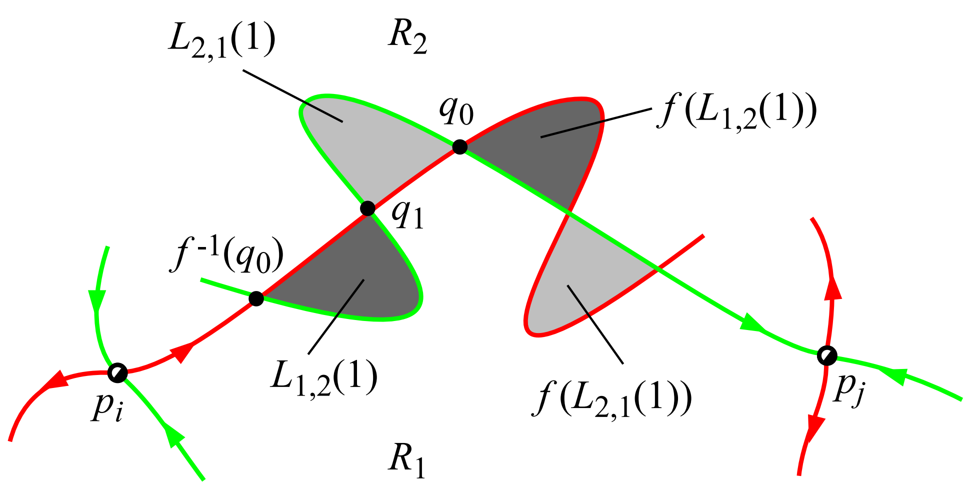


(b)

Transport between Regions via Lobe Dynamics

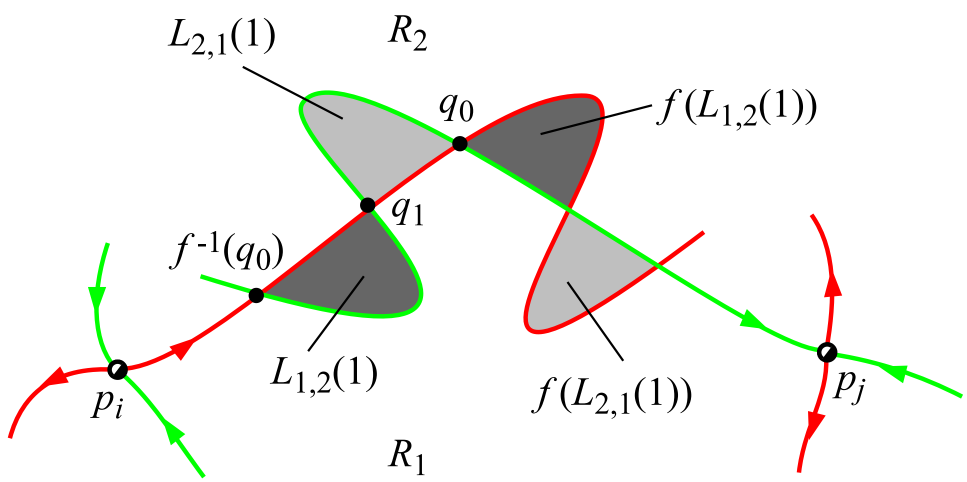
- **Invariant manifolds** divide phase space into resonance regions.
- Transport between regions can be studied via **lobe dynamics**.





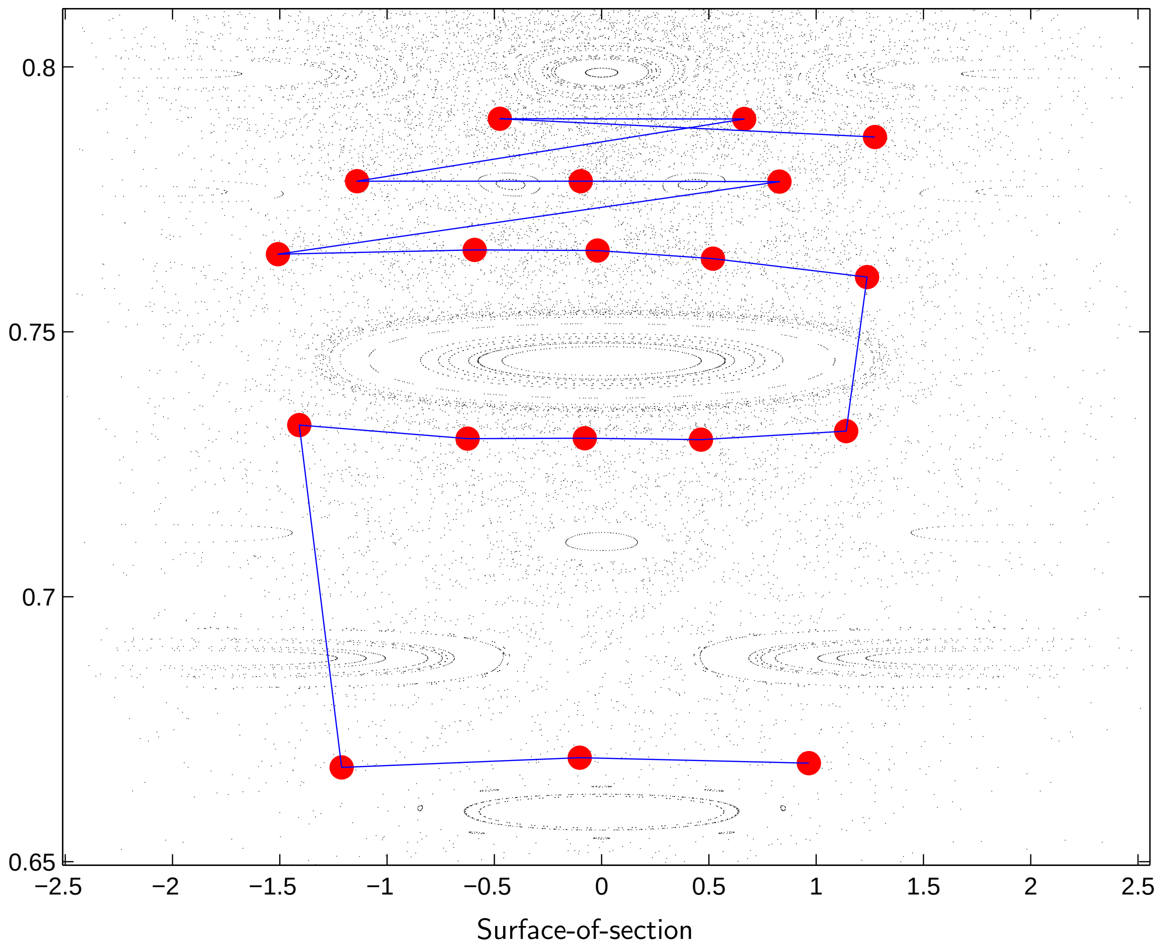
Transport between Regions via Lobe Dynamics

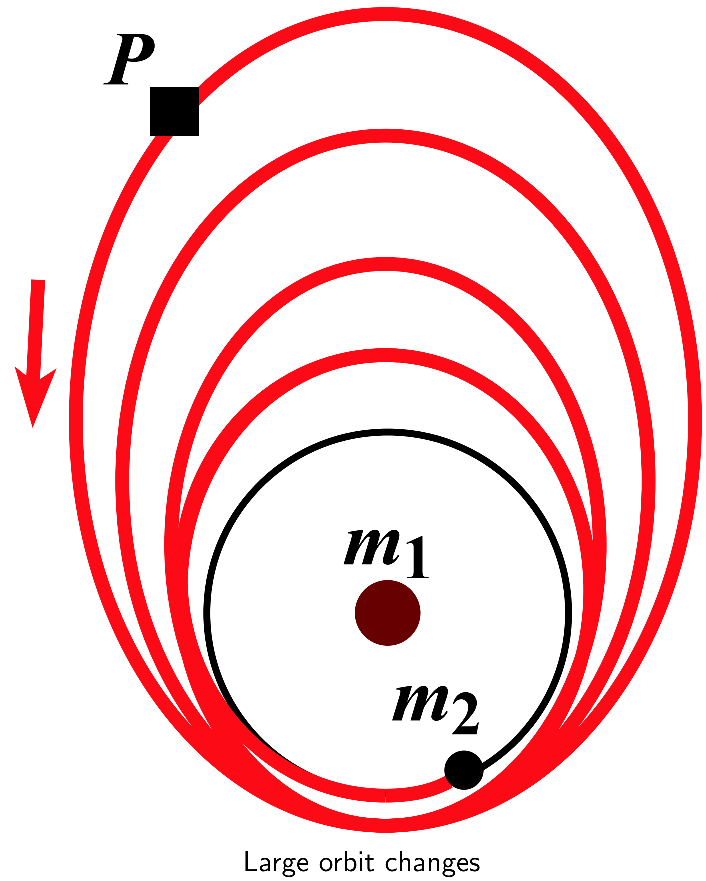
- Segments of **unstable** and **stable** manifolds form **partial barriers** between regions R_1 and R_2.
- $L_{1,2}(1), L_{2,1}(1)$ are **lobes**; they form a **turnstile**.
  - In one iteration, only points from R_1 to R_2 are in $L_{1,2}$
  - only points from R_2 to R_1 are in $L_{2,1}(1)$.
- By studying pre-images of $L_{1,2}(1)$, one can find efficient way from R_1 to R_2.



Hopping through Resonaces in Low Energy Tour

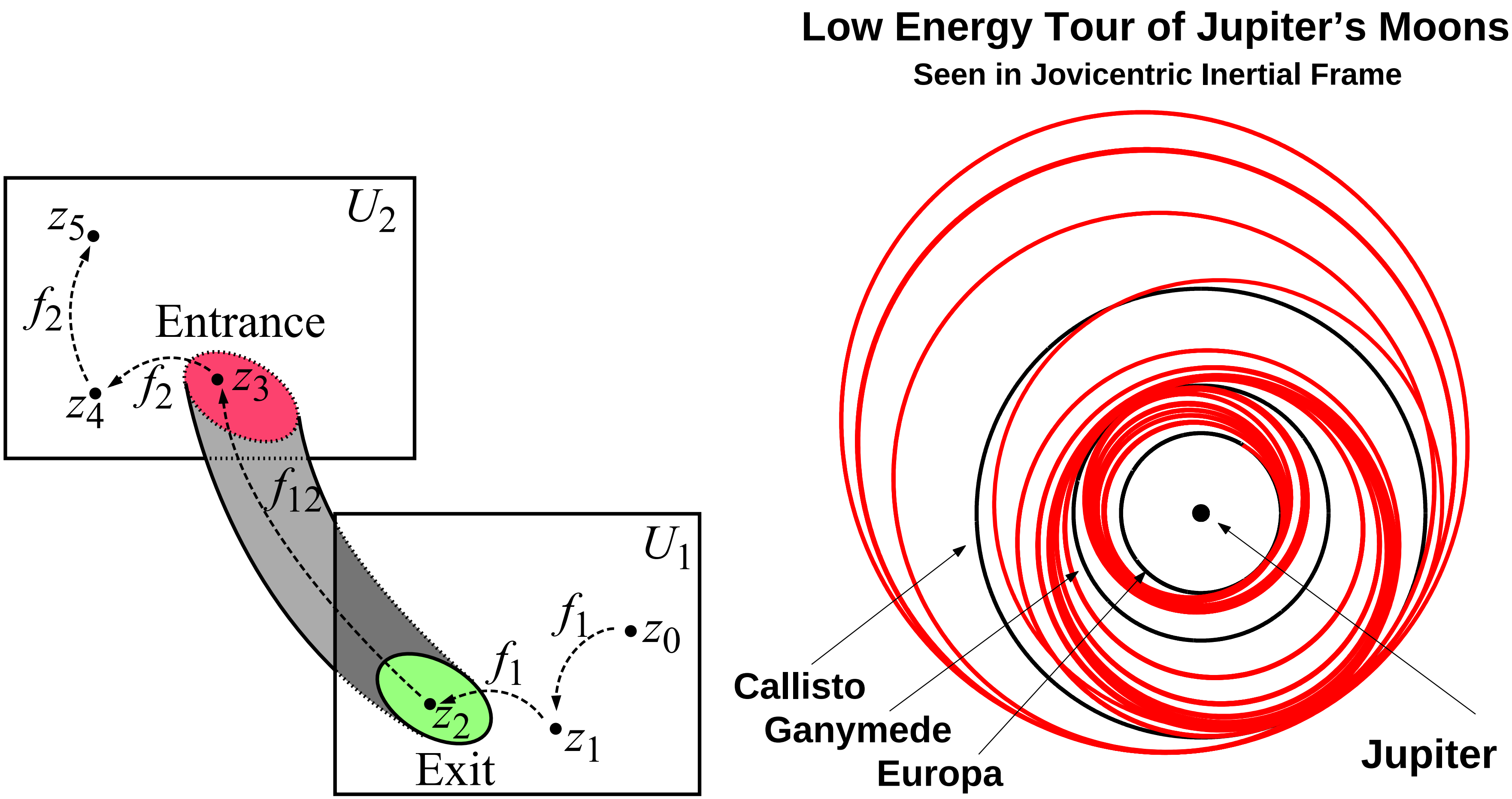
- Guided by lobe dynamics, **hopping** through resonances (essential for low energy tour) can be performed.





Tube/Lobe Dynamics: Transport in Solar System

- To use **tube** dynamics/**lobe** dynamics of **spatial** 3-body problem to **systematically** design low-fuel trajectory.
- Part of our program to study transport in solar system using **tube** and **lobe dynamics**.

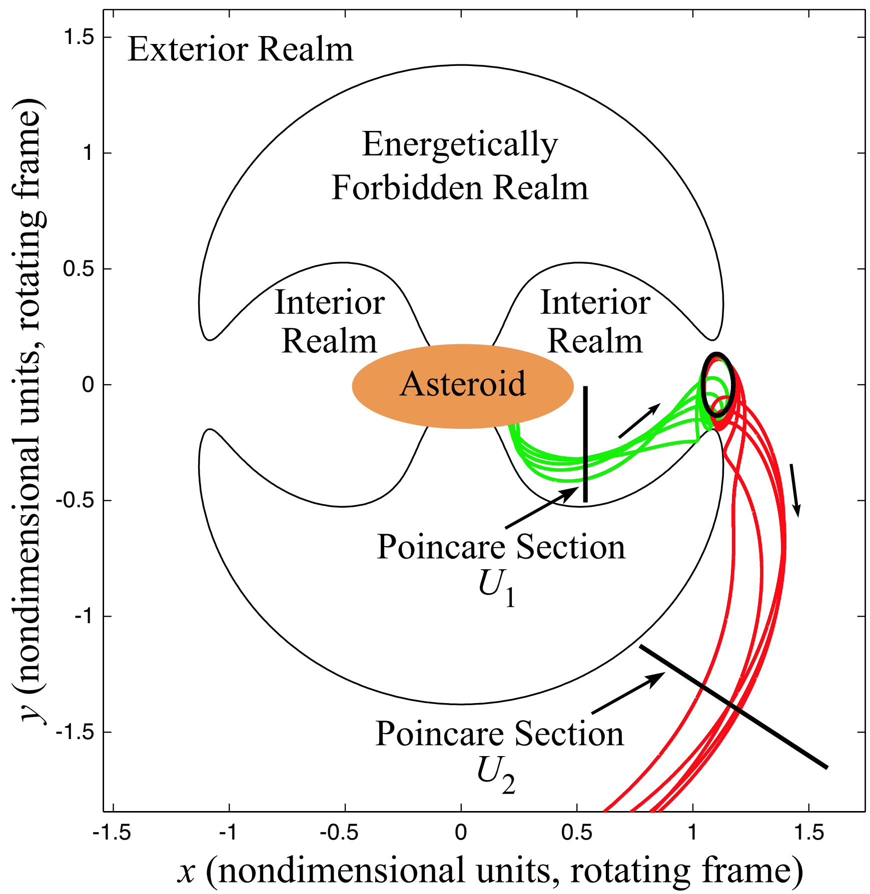


(a) (b)

Tube/Lobe Dynamics: 2FBP/Asteroid Pairs

- To study dynamical interacton between 2 rigid bodies where their **rotational** and **translational** motions are coupled.
  - formation of binary asteroids (shown below are Ida and Dactyl)
  - evolution of asteroid rotational states

(a)



(b)

Multi-scale Dynamics of Biomolecules (ICB)

- Carry out **model reduction** (POD) for simple biomolecules, keeping chains of molecules and more complex systems in view.
- Indentify **conformations** in simple molecular models using techniques of *AIS* and **set oriented** methods.
- Compute **transport rates** between different conformations by both **lobe dynamics** and **set oriented methods**.
- Collaborate with Institute of Collaborative Bio-Technologycd (ICB).