CDS 140b: Project Ideas (Hamiltonian Dynamics)

The following is a non-exhaustive list of possible project ideas for CDS-140b. The idea is that you choose a topic during one of the guest lectures, and that you write a short summary of it, demonstrating the insight into the subject matter. You are also free to suggest a topic of your own choosing, provided there is a clear link with dynamical systems theory and CDS-140b.

To keep everything within bounds, I suggest a limit of four pages for the paper, though you are allowed to go over that if you think it necessary. The due date for the project paper is ***Tuesday, March 10***. It is strongly suggested that you write the paper in LaTeXand submit it via email (`jv@caltech.edu`).

Given the fact that the deadline for the project is only four weeks away, let me know as soon as you choose a particular topic.

Project Ideas.

- *Symplectic/Poisson integrators.* As discussed in class, conventional integration schemes often fail to preserve the Poisson or symplectic structure of the system, leading to qualitatively inaccurate behaviour over long time scales. Over the past two decades, new integrators have been developed that remedy this defect. These integrators are known collectively as *geometric integrators*. The idea of the project is to gain insight into some of the simplest geometric second-order schemes, both from a theoretical point of view (variationality, preservation of the symplectic structure, etc.) as well as by applying these integrators to some elementary toy problems and observing these benefits in practice. See the paper at this location:

  `http://www.cds.caltech.edu/~marsden/bib/2004/02-LeMaOrWe2004a/`

- *Dynamics of point vortices and chaotic motions.* In fluid dynamics, a *point vortex* is intuitively a point around which the fluid rotates (think of the eye of a hurricane, shrunk down to a point). An ensemble of N point vortices forms a Hamiltonian system and can be analyzed using the methods we studied in class. You can solve these equations of motion for $N = 1, 2$, prove integrability for $N = 3$, and/or observe non-integrability numerically as soon as $N > 3$. See Newton [2001].

- *In-depth study of the Lorentz attractor.* The Lorentz attractor is a feature of a celebrated dynamical system in three dimensions. Rather than settling down on a limit cycle or converging to a stable fixed point, the trajectories of this system have very complicated asymptotical behavior. In Hirsch et al. [2004], this system is studied from a dynamical-systems point of view, and the link is made with foundational concepts such as the Baker's Transform and the Smale Horseshoe.

- *Homoclinic chaos.* When looking at the stability of periodic orbits, we constructed a Poincaré map such that the periodic orbit gives rise to a fixed point of the Poincaré-map. For this fixed point, analogues of the stable and unstable manifold can be defined. However, in contrast to continuous-time dynamical systems, the stable and unstable manifold of the Poincaré map can intersect, leading to a very intricate *homoclinic tangle*, and hence chaos. The idea of the project is to read up on the construction of a homoclinic tangle, and observe its features in some simple dynamical systems. See Tabor [1989].

- *KAM theory.* We have hurried through the description of the KAM theorem in just a few lectures, but this is a huge topic. In a survey paper, Meiss [1992] highlights some of the salient features of perturbed Hamiltonian systems without going through too many proofs (which are often extremely technical). Depending on your preference, different topics can be discussed; see class for more details.

References

Hirsch, M. W., S. Smale, and R. L. Devaney [2004], *Differential equations, dynamical systems, and an introduction to chaos*, volume 60 of *Pure and Applied Mathematics (Amsterdam)*. Elsevier/Academic Press, Amsterdam, second edition.

Meiss, J. D. [1992], Symplectic maps, variational principles, and transport, *Rev. Modern Phys.* **64**, 795–848.

Newton, P. K. [2001], *The N-vortex problem. Analytical techniques*, volume 145 of *Applied Mathematical Sciences*. Springer-Verlag, New York.

Tabor, M. [1989], *Chaos and integrability in nonlinear dynamics.* A Wiley-Interscience Publication. John Wiley & Sons Inc., New York. An introduction.