CDS 140b: Homework Set 1
Due by Tuesday, January 20, 2009.

1. Determine whether the following system can have any periodic orbits:
\[ \dot{x} = y^2 + y \cos x \quad \text{and} \quad \dot{y} = 2xy + \sin x. \]

2. Use index theory to show that the system \( \dot{x} = x(4 - y - x^2) \), \( \dot{y} = y(x - 1) \) cannot have any periodic orbits.

3. Show that the system \( \dot{x} = y - x^3 \), \( \dot{y} = -x - y^3 \) cannot have any periodic orbits by considering a Liapunov function \( V = ax^2 + by^2 \) with suitable \( a, b \). Can you think of another way (i.e. without Liapunov functions) of proving this statement?

4. Consider a gradient system. Show that, away from fixed points, the trajectories of the system cross the level sets of \( V \) at right angles, and that \( V \) is decreasing along the trajectories.

5. Consider the given by \( \ddot{x} + x = \mu(1 - x^2)\dot{x} \) with \( \mu > 0 \). Use Bendixson’s criterion to show that periodic solutions necessarily have to cross \( x = \pm 1 \).