CDS 212 - Homework #3

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Posted on: Tuesday November 13, 2007
Due on: Tuesday November 20, 2007
Remark: Once again, please note down on your solution the amount of time you spent on this homework set.

Problem 1
Consider an LTI model $\dot{x}(t) = Ax(t) + Bu(t)$ with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3w^2 & 0 & 0 & 2w \\ 0 & 0 & 0 & 1 \\ 0 & -2w & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

where $w > 0$ is a fixed parameter. This model describes the linearized dynamics of a satellite orbiting at a nominal angular velocity w, under the effect of radial and tangential thrusters ($u_1(t)$ and $u_2(t)$, respectively).

(a) Is the linearized system asymptotically stable?

(b) Is the system reachable?

(c) Is the system reachable if the radial thruster fails?

(d) Is the system reachable if the tangential thruster fails?

Problem 2
Consider a *single output* LTI system given by:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= (C + \delta)x + Du \end{aligned}$$

where the pair (C, A) is observable. Find the smallest perturbation $\|\delta\|_2$ such that the system becomes unobservable?

Problem 3
Consider Figure 5.1 in DFT, and assume that $d = 0$ and that

$$P(s) = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{s-1}{(s+1)^2} \end{bmatrix}$$

Give a Q parametrization of all closed loop transfer functions from r to y that can be obtained by using a stabilizing, proper LTI controller C.

Problem 4
Solve the following problems from DFT:

(a) Problem 5.2.

(b) Problem 5.7.

(c) Problem 6.5.

(d) Problem 6.6.