CDS270: Optimization, Game and Layering in Communication Networks

Lecture 9: Random Access Games and Medium Access Control Design

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Agenda

- Contention-based medium access control (contention control)

- A game theoretic approach to contention control
  - Random access game
  - A case study
  - Utility and reverse-engineering
  - Conclusions
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Medium access control (MAC)

- Wireless channel is shared medium and interference-limited
- Medium access control: coordinate channel access
  - Reduce/avoid interference/collision
  - Efficient utilization of wireless spectrum
  - Quality of Service control

a multiple access network
Two kinds of methods

- **Schedule-based**
  - Establish transmission schedules \textit{a priori} or dynamically
  - Usually requires centralized implementation
  - High complexity, not practical in real networks

- **Contention-based**
  - Wireless nodes contend for the channel
  - Simple, distributed implementation
  - High statistical multiplexing gain
  - Aloha, CSMA/CA, 802.11 DCF, ...
Aloha

- Very simple: if a node has a packet to send, it just transmits
- Listen for an amount of time
  - If an ACK is received, done.
  - Otherwise, resend the packet
- Low-delay in light-load scenarios
- Low channel utilization ($\leq 18\%$)
  - Collision window is equal to transmission time (TT) plus propagation delay (PD)
Slotted Aloha

- Time is slotted
  - slot duration is equal to transmission time plus maximum propagation delay
- Begin transmission at the slot boundaries
- Higher channel utilization ($\leq 1/e$)
  - Collision window is a point -- the slot boundary
Carrier Sensing multiple access (CAMA)

- Infer channel state through carrier sensing
  - Sense carrier before transmission
  - If idle, transmit the whole packet
  - Wait for ACK
- Higher channel utilization
  - Collision window is equal to maximum propagation delay
- When finding a busy channel
  - Non-persistent: sense the channel again after a random amount of time; if idle, send immediately
  - P-persistent: sense continuously; if idle, send with probability p
Contestation/collision resolution

- What to do upon a collision
  - If the colliding nodes transmit immediately when the channel is idle after a collision, another collision is guaranteed

- Two collision resolution mechanisms
  - Persistence: transmit with a probability $p$
  - Backoff: wait for a random amount of time bounded by $CW$ before retransmission

- Contention resolution algorithm (i.e., how to decide $p$ and $CW$ values dynamically in response to contention) is the key
CSMA/CD

- Collision detection (CD): immediately stop the transmission when sensing a collision
  - Detect at the senders
  - Not wait for an ACK

- Contention resolution: Binary exponential backoff
  - Wait a random amount of time bounded by CW before retransmission
  - Double CW upon every collision
  - Packet collision is the feedback signal

- Invented for Ethernet
Why collision avoidance (CA)?

CD is difficult in wireless networks: sender cannot effectively distinguish incoming weak signals from noise and the effects of its own transmission.

Hidden terminal problem
Approaches for CA

- Randomized “backoff”
  - Slotted contention period
  - Operation
    - Each node selects a random backoff number
    - Waits that number of slots while sensing the channel
    - If channel stays idle and reaches zero then transmit
    - If channel becomes active wait until transmission is over then resumes backoff counter again
$B_1 = 5$

$B_2 = 15$

$B_1 = 25$

$B_2 = 20$

$B_2 = 10$

$B_2 = 15$

$CW = 32$
Use of RTS (request-to-send) and CTS (clear-to-send) exchange

Before sending a packet, the sender first sends a RTS. The receiver responds with a CTS. Nodes hearing RTS or CTS then know that the channel will be busy for the duration of the request (indicated by Duration ID in the RTS and CTS).

Virtual carrier sensing: nodes will adjust their Network Allocation Vector (NAV) -- time that must elapse before a station can sense channel for idle status.
Wireless 802.11 DCF (basic)

- DCF stands for distributed coordination function
- A CSMA/CA medium access protocol
  - CSMA: sense before transmission
  - CA: random backoff to reduce collision probability
    - when transmitting a packet, choose a backoff interval in the range [0, CW-1]
  - Count down the backoff interval when medium is idle
    - count-down is suspended if medium becomes busy
  - Transmit when backoff interval reaches 0
Contention resolution: contention window $CW$ is adapted dynamically depending on collision occurrence

- Binary exponential backoff: double $CW$ upon every collision
- Set to base value ($CW=32$) after a successful transmission
- Packet collision is the feedback signal
- **Slotted system: Inter Frame Spacing**
  - **SIFS (Short Inter Frame Spacing)**
    - highest priority, for ACK, CTS
  - **DIFS (Distributed Coordination Function IFS)**
    - lowest priority, for asynchronous data service

![Diagram of slotted system with timing notations: DIFS, SIFS, medium busy, contention, and next frame over time (t).]
DCF basic access method

source

Data

SIFS

destination

Ack

other

NAV

DIFS

Defer access

DIFS

Random backoff time

CW

Random backoff time
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Contestion control

- A distributed strategy to access/share wireless channel
- Control theoretic point of view
  - A contention resolution algorithm: dynamically adjusts persistence probability or contention window in response to the contention
  - A feedback mechanism: updates a contention measure and sends it back to wireless nodes
Two contention resolution mechanisms

- Persistence: access the channel with some persistence probability
- Backoff: wait for a random amount of time bounded by the contention window before a transmission

Different MAC methods differ in terms of

- how they adjust persistence probability or contention window
- What contention measure they use
IEEE 802.11 DCF (revisited)

- Uses a binary contention signal: packet collision or successful transmission
- Uses a backoff mechanism
  - Doubles contention window upon a collision (binary exponential backoff)
  - Sets it to the base value upon a successful transmission
Performance problems/limitations of 802.11 DCF

- Throughput degradation in high-load scenarios because of excessive collisions
  - Set to the base contention window is too drastic
- Short-term unfairness due to the oscillation in contention window
  - Directly caused by binary exponential backoff
  - Unavoidable because of binary contention signal
- Performance deterioration in adverse channel conditions
  - Cannot distinguish collisions from corrupted frames
- Not easy to adapt to channel variations
Observations

- For high efficiency and better fairness, need to stabilize the network into a steady state which sustains appropriate window sizes
  - Require continuous feedback signal
- Need to use a contention measure whose estimation is not based on packet collisions, and decouple contention control from handling failed transmissions
Objective

- To provide an analytical framework to study contention/interaction among wireless nodes
- To design medium access methods
  - Stabilize the network around a steady state with a target fairness (or service differentiation) and high efficiency (high throughput, low collision)
  - Decouple contention control from handling failed transmissions
**Methodology**

- Study the contention/interaction among wireless nodes in game theory framework
- Design MAC according to distributed strategy update algorithm achieving Nash equilibrium
  - Not intended to model selfish behaviors of wireless nodes
  - But to capture information and implementation constraints encountered in real networks
  - Design game to guide individual nodes to seek an equilibrium that achieves system-wide performance objectives
Random access game

- Consider a set $N$ of greedy wireless nodes in a single-cell wireless LAN
- Each node $i$ attains a utility $U_i(p_i)$ when it accesses the channel with probability $p_i$
  - $U_i(\cdot)$ is assumed to be a continuously differentiable, increasing, concave function with the curvatures bounded away from zero, i.e., $-1/U_i''(p_i) \geq 1/\lambda > 0$
  - If persistence mechanism is used, $p_i$ is just persistence probability.
  - If backoff mechanism is used, $p_i$ is related to a constant contention window $cw_i$ according to $p_i = 2/(cw_i + 1)$
Definition: A random access game $G$ is defined as a triple $G := \{N, (s_i)_{i \in N}, (u_i)_{i \in N}\}$

- $N$ is a set of players (wireless nodes)
- Strategy $s_i = \{p_i \mid p_i \in [v_i, w_i]\}$ with $0 \leq v_i \leq w_i \leq 1$
- Payoff function $u_i(p) := U_i(p_i) - p_i q_i(p)$ with $q_i(p) := 1 - \prod_{j \in N / \{i\}} (1 - p_j)$

- Wireless nodes interact through collisions
- Homogeneous users and heterogeneous users
Nash equilibria

- Denote the strategies of all nodes other than \( i \) by \( p_{-i} \). A vector of access probability \( p \) is a Nash equilibrium if, for all nodes \( i \),
  
  \[ u_i(p_i, p_{-i}) \geq u_i(\bar{p}_i, p_{-i}) \text{ for all } \bar{p}_i \in s_i \]

- **Theorem**: There exists a Nash equilibrium for random access game \( G \).

- **Proof**: strategy spaces \( s_i \) are compact convex sets, and the payoff functions \( u_i \) are continuous and concave in \( p_i \).
- At Nash equilibrium, $p_i$ either takes value at boundaries of the strategy space or satisfies
  \[ U_i'(p_i) = q_i(p) \]
- Nontrivial Nash equilibria: for all nodes $i$, $p_i$ satisfies the above equality.
- Trivial Nash equilibria, otherwise.
Nontrivial Nash equilibria

- **Theorem**: Random access game $G$ has nontrivial Nash equilibrium if, for each node $i$, inverse function $(U'_i)^{-1}(q_i)$ maps any $q_i \in [0,1]$ into a point $p_i \in s_i$.

- **Proof**: define $p_i = B_i(p) := (U'_i)^{-1}(q_i(p))$, then $B(p) := (B_1(p), B_2(p), \ldots, B_{|M|}(p))$ maps the strategy space into itself. The theorem follows from the Brouwer’s fixed point theorem.
Theorem: if additionally $\Gamma_i(p_i)$ is a monotone function in $s_i$ for all $i$, then random access game has a unique nontrivial Nash equilibrium.

- Define idle probability $\gamma(p) = \Pi_{i \in N} (1 - p_i)$, and $\Gamma_i(p_i) := (1 - p_i) (1 - U_i(p_i))$. At nontrivial Nash equilibrium, $\Gamma_i(p_i) = \gamma(p)$.

- $\Gamma_i(p_i) = \Gamma_j(p_j)$ for any $i, j \in N$.

- Proof by contradiction.
Definition: A Nash equilibrium $\mathbf{p}$ is said to be a symmetric equilibrium if $p_i = p_j$ for all $i, j \in N$, and an asymmetric equilibrium otherwise.

- If a system of homogeneous users has an asymmetric Nash equilibrium, all its permutations are Nash equilibria.
- The symmetric equilibrium must be unique.
Corollary: Random access game $G$ has a unique nontrivial Nash equilibrium which is symmetric among each class of users.

- Guarantees the uniqueness of nontrivial Nash equilibrium.
- Guarantees fair sharing of wireless channel among the same class of wireless nodes.
- Provides service differentiation among different classes of wireless nodes.
Dynamics

- Studies how interacting players (wireless node) could converge to a Nash equilibrium
- Difficult problem: “game theory lacks a general and convincing argument that a Nash outcome will occur”
- In the setting of random access
  - Players can observe the outcome of the action of others
  - Players do not have direct knowledge of other player actions or payoffs
- Consider repeated play of the random access game, and look for strategy update mechanism that achieves Nash equilibrium.
Best response strategy

\[ p_i(t + 1) = B_i(p(t)) := \arg \max_{p \in s_i} (U_i(p) - pq_i(p(t))) \]

Theorem: If function \( B^{(2)}(p) \) has a unique fixed point in the strategy space, then best response strategy converges to unique nontrivial Nash equilibrium of random access game \( G \).
Gradient play

\[ p_i(t + 1) = [p_i(t) + f_i(p_i(t))(U_i'(p_i(t)) - q_i(p(t)))]^{s_i} \]

Theorem: Gradient play converges to the unique nontrivial Nash equilibrium of random access game G if stepsize \( f_i(p_i) < 1/(\lambda + |N| - 1) \).

Proof by Lyapunov method.
Random access games provide a general analytical framework to model a large class of system-wide quality-of-service models via the specification of per-node utility functions.

System-wide fairness or service differentiation can be achieved in a distributed manner as long as each node executes a contention resolution algorithm that is designed to achieve the Nash equilibrium.
After each transmission
{
    /* Wireless node observes $n$ idle slots before a transmission*/
    isum ← isum + n
    ntrans ← ntrans + 1
    if ( ntrans \geq \text{maxtrans} ){
        /*compute the estimator*/
        \bar{p} ← \frac{\text{isum}}{\text{ntrans}}
        q_i ← \frac{(1 - (\bar{p} + 1) p_i)}{(\bar{p} + 1)(1 - p_i))}
        /*update access probability*/
        p_i ← p_i + f_i(p_i)(U_i(p_i) - q_i)
        /*update contention window*/
        cw_i ← \frac{(2 - p_i)}{p_i}
        /*reset variables*/
        isum ← 0
        ntrans ← 0
    }
}
- Adapt to continuous feedback signal, and stabilize the network around a steady state specified by Nash equilibrium
  - controllable performance objective and better short-term fairness
- Equation-based control, adjust contention window according to how far the current state to the equilibrium
  - Result in simpler dynamics, achieve better contention control and higher throughput
- Can decouple contention control from handling failed transmissions
A case study

Define random access game $G_1$ with the following utility

$$U_i(p_i) = \frac{1}{a_i} \left( \frac{(a_i - 1)w_i}{a_i} \ln(a_i p_i - w_i) - p_i \right),$$

where $0 < w_i < 1$ and $p_i \in [2w_i/(1+a_i), w_i]$. 
Nash equilibrium and dynamics

- **Theorem**: if \( a_i w_i < 1 \), random access game \( G_1 \) has unique nontrivial Nash equilibrium. Moreover, the unique nontrivial Nash equilibrium of \( G_1 \) is symmetric among each class of users.

- **Gradient play**
  \[
  p_i(t + 1) = \left[ p_i(t) + f_i(p_i(t))\left( \frac{w_i - p_i(t)}{a_i p_i(t) - w_i} - q_i(p(t)) \right) \right]^{z_i}
  \]
  \[
  cw_i(t) = \frac{2 - p_i(t)}{p_i(t)}
  \]

- **Theorem**: Suppose \( a_i w_i < 1 \), the system described by the above equations converges to the unique nontrivial Nash equilibrium of random access game \( G \) if \( f_i(p_i) < 1/(\lambda + |N| - 1) \).
MAC design

- Make two key modifications to 802.11 DCF
  - Each node $i$ estimates its conditional collision probability $q_i$
    $$\bar{n} \leftarrow \beta \bar{n} + (1 - \beta) \frac{isum}{ntrans}$$
    $$q_i \leftarrow \frac{1 - (\bar{n} + 1) p_i}{(\bar{n} + 1)(1 - p_i)}$$
  - Adjusts its contention window $cw_i$ according to gradient play
    $$p_i \leftarrow p_i + f_i(p_i)(U_i(p_i) - q_i)$$
    $$cw_i \leftarrow \frac{2 - p_i}{p_i}$$
Throughput comparison

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Number of Nodes

Throughput (Mbps)

- our design (ω=0.0606, a=14.576)
- 802.11 DCF (CWmin=32, CWmax=256)
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Collision overhead comparison

Graph 1:
- 'our design (ω=0.0606, a=14.576)
- '802.11 DCF (CWmin=32, CWmax=256)

Graph 2:
- 'our design (ω=0.0606, a=14.576)
- '802.11 DCF (CWmin=32, CWmax=256)
Fairness comparison

Normalized Window Size

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- our design ($\omega=0.0606$, $a=14.576$)
- 802.11 DCF (CWmin=32, CWmax=256)
Service differentiations

$n_1$: number of class 1 nodes ($w_1 = 0.06$ and $a_1 = 15$)
$n_2$: number of class 2 nodes ($w_2 = 0.04$ and $a_2 = 15$)
$n_1$: number of class 1 nodes ($w_1 = 0.04$ and $a_1 = 10$)

$n_2$: number of class 2 nodes ($w_2 = 0.04$ and $a_2 = 20$)
Utility and reverse-engineering

- Utility functions ⇔ Equilibria of random access games and the stable operating points of MAC protocols

- Reverse-engineering
  - The stable operating point defines an implicit relation  $p_i = F_i(p_i, q_i)$
  - Exists a unique continuously differentiable function $F_i$ such that  $q_i = F_i(p_i)$
  - Define the utility functions as  $U_i(p_i) = \int F_i(p_i) dp_i$ , with which we can define a random access game. MAC can be interpreted as a distributed strategy update algorithm to achieve Nash equilibrium of the random access game.
Conclusions

- Presented game theoretic framework for contention control
  - Define a general game theoretic model to capture interaction/contention among wireless nodes
  - Capable of modeling a large class of system-wide QoS models via the specification of per-node utility functions.
  - Design MAC according to distributed strategy update algorithm achieving Nash equilibrium.
  - Study a concrete random access game and medium access control design and show it achieves superior performance than the standard protocol
  - Provides an analytical framework to understand the equilibrium and dynamics properties of different MAC protocols and their interactions